Mathematical Proceedings

Cambridge Philosophical Society, Cambridge, England

PROCEEDINGS

OF THE

CAMBRIDGE PHILOSOPHICAL SOCIETY.

VOLUME XI.

October 29, 1900—May 19, 1902.

Cambridge:
AT THE UNIVERSITY PRESS,
AND SOLD BY
DEIGHTON, BELL & CO. AND MACMILLAN & BOWES, CAMBRIDGE.
C. J. CLAY AND SONS, AVE MARIA LANE, LONDON.
1902

CONTENTS.

VOL. XI.

PLATES.

PROCEEDINGS

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Apparatus for the Analysis of the Gases in Small Quantities of Blood. By JOSEPH BARCROFT, King's College. [With Plate I.]

The present communication consists of an account of an apparatus for the quantitative estimation of the oxygen and carbonic acid in the blood coming from a particular organ.

The general principle of analysis used in all estimations of the two gases is that of exposing the blood to a vacuum and removing the gases which it gives off by means of a mercurial pump.

So far, however, this principle has been only applied with accuracy when considerable quantities of blood (30—50 c.c.) have been available for each analysis, and in the case of blood drawn direct from a living animal it has been necessary to take it from some large vessel when the rate of flow is rapid.

These experimental difficulties have excluded the possibility of analysing the blood from special organs except in a few cases. Among these cases may be mentioned the experiments on the respiration of muscle by Ludwig, Schmidt and Sczelcow.

The object of the research for which my apparatus was devised is a comparison of the arterial and venous bloods entering and leaving the submaxillary gland, according as the gland is or is not secreting saliva.

In this case the samples of blood available are, as a rule, about in volume 8 c.c., though sometimes only 5 c.c., and the length of time taken to collect such a sample is 3—4 minutes.

The object before me then has been to construct an apparatus in which the working error is about $\frac{1}{6}$th of that of existing ones,

or to put the matter in another way, my object has been to secure the same error in the figures when worked out as percentages as others have done, while using only a fifth of the blood for analysis.

The most obvious source of error is, of course, the contamination of the blood gases with small quantities of air, and it will be apparent that, in an apparatus the parts of which are detachable, it is almost impossible to avoid small leakages.

In the present apparatus there are no joints to be made, as the whole is in one piece. This arrangement would be simple were it only necessary to collect one sample of blood; the present apparatus however provides for the collection of nine samples, six from a vein, and three for comparison from an artery.

With regard to the errors of the apparatus, the greatest is that of measuring the blood, as the surface of the mercury beneath it is sometimes broken by films of blood; the extreme error here is ·05 c.c. Another error is produced by the fact that a film of blood is left over the glass in a tube leading to the chambers and in the burette. This is a very variable quantity and depends in the first place upon the individual apparatus, and in the second place upon the tendency of the blood to coagulate. If the blood is very fluid, and the apparatus carefully made with the tube straight and of 2 millimetres diameter, the amount of blood then left as a film is about ·05 c.c. Each sample will therefore be contaminated to that extent by the previous one.

In regard to the collective error of the whole apparatus, the following analysis of defibrinated blood will serve as a guide.

Analysis of three samples of the same defibrinated blood:

CO_2	23·7	23·9	24·0 c.c.
O	20·4	20·2	20·6 c.c.
N	1·9	1·7	1·9 c.c.

On another occasion three samples gave respectively,

CO_2	52·6	52·6	52·7 c.c.
O	11·9	11·9	11·9 c.c.
N	1·6	2·4	1·9 c.c.

Description of Apparatus. The Pump.

The Toepler Pump has been employed, without any modification except that the drying apparatus is so arranged that the sulphuric acid can be renewed without introducing more air into the apparatus than what is dissolved in the acid.

A three-way tap, Fig. 1 (*c*), leads to a water-pump which gives a vacuum, almost up to the tension of aqueous vapour.

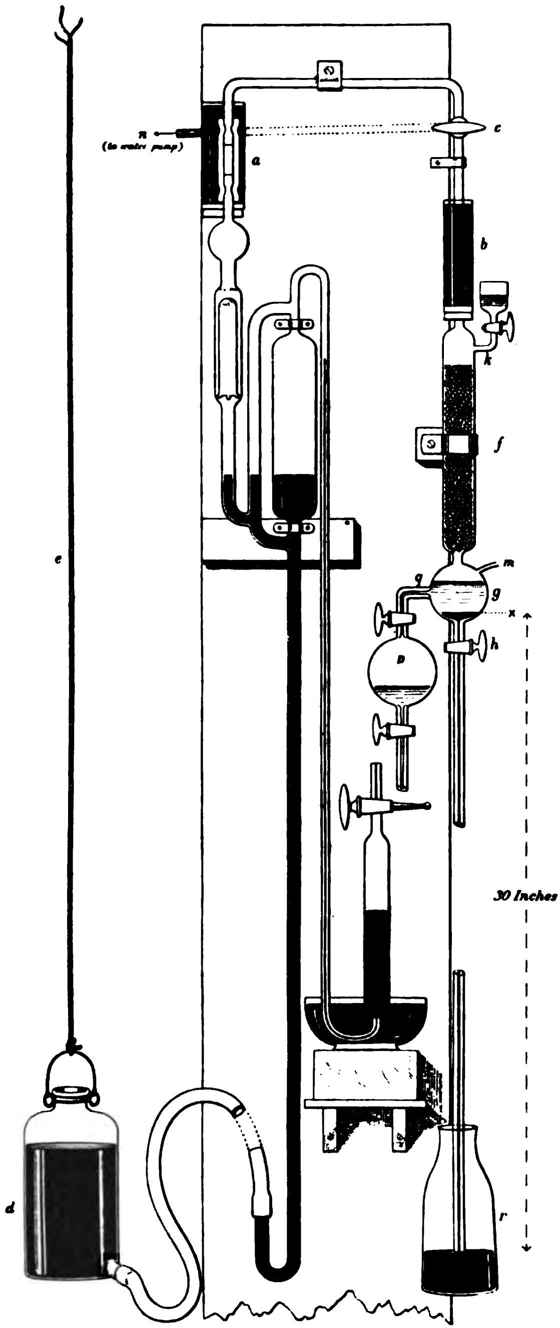


FIG. 1.

In order to reduce the strain on the pump, due to the weight or disposition of the rest of the apparatus, two permanent rubber joints have been introduced. These are marked (*a*) and (*b*) on Fig. 1; (*a*) is shewn in detail. The glass ends are joined with stout pressure tubing, smeared inside and out with Canada balsam. The tubing is bound on with 'grips'. These 'grips' I had made to obviate the risk of breaking the pump, which occurs in the binding on of wire. One of them is shewn incidentally in Fig. 2.

For drying the gases I have used a tower about eighteen inches in height, containing pumice wet with sulphuric acid. The sulphuric acid can readily be renewed when spent, being poured in at *k*. In the lower part of the tower some sulphuric acid is allowed to lie, at once helping the drying process and rendering obvious any leak in the tap (*h*).

If, as is necessary, more than four or five samples of blood are to be analysed successively, the entire acid in the drying chamber must be replaced without introducing air; the apparatus may be set up as figured in Fig. 1. The vessel *r* contains mercury, and is so placed that with a vacuum in the pump the mercury rises to about the level *x*. The sulphuric acid is put in at the top of the tower and flows over the pumice. Sufficient is put in to fill the bulb *g* to the level of the tube *q*. Much of the aqueous vapour from the gases coming along *m* condenses on the surface of the sulphuric acid, forming a film of dilute acid. By raising the vessel *r* and opening the tap connecting *g* with the vacuous chamber, *p*, this dilute acid can be disposed of. Similarly, if fresh acid be put in at the top of the tower, the excess can be put into *p*, otherwise *m* would become closed up.

The Receiver.

This part of the apparatus divides itself into that for measuring the blood, and that for exposing it to a vacuum; both of which appear illustrated in the Figs. 2—8. Fig. 2 refers to the measuring burette. This burette consists simply of a vertical graduated tube *B* of about 11 c.c. capacity connected at its lower end by means of pressure tubing with a small mercury reservoir, *J*. At its upper end the graduated tube, *B*, terminates in a glass tap, *A*, of peculiar construction. The stopper is shewn in detail in Fig. 3. Leading to the tap are two horizontal tubes, *C* and *D*. Of these *C* is connected with the cannula from which the blood is to be drawn, whilst *D* is joined at the point *f* to the froth bulbs shewn in Fig. 4. From Fig. 3 it will be obvious that the passage *F* may be continuous with either *C* or *D*, or with neither of these,

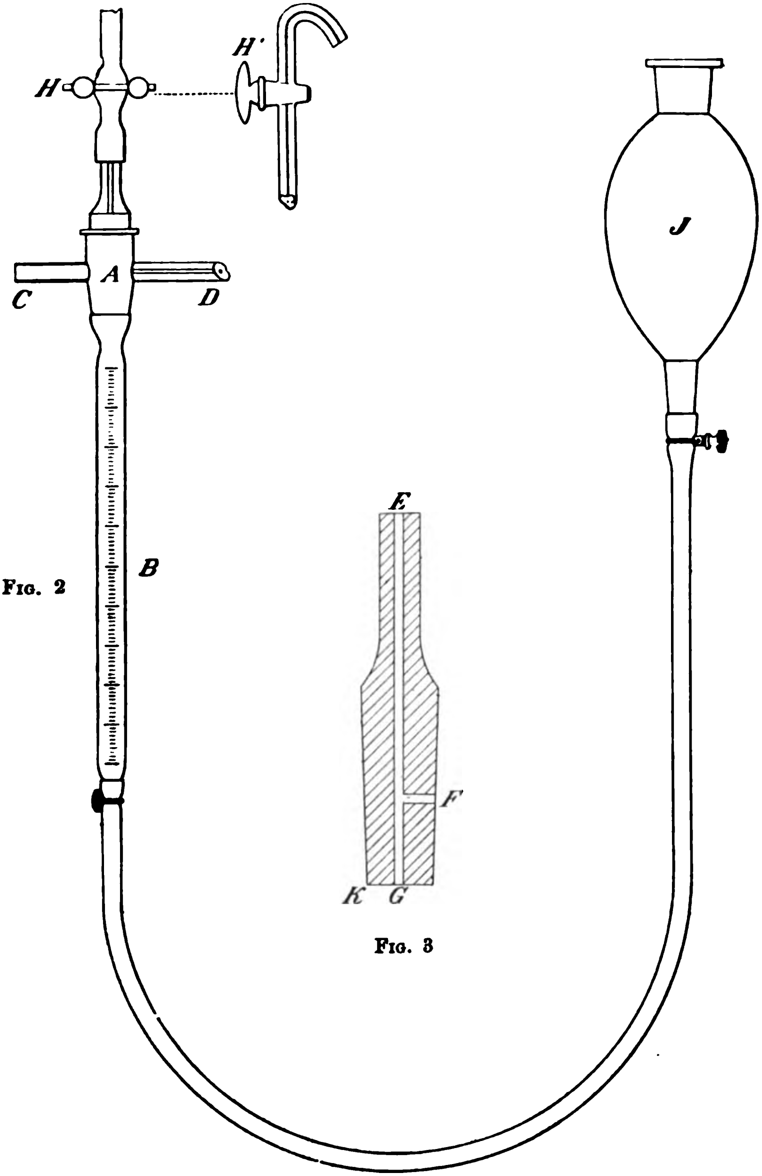


Fig. 2

Fig. 3

according as the stopper is turned. Besides the passage F, the stopper is bored from end to end (E, G). The orifice E is drawn off into a tube which is closed by a tap H. Perhaps the mechanism will be made most clear by a description of the actual manipulation in the measuring of a sample of blood. The whole burette, and its connections, are filled with mercury by raising the reservoir J, the tap H is opened so that the mercury may rise and fill the passages, G and E, in the stopper A. (The bottom of the stopper has been carefully ground with sharp angles at K to prevent bubbles of air getting caught at the sides.) H is then closed and the stopper turned so that F is continuous with C. Thus the passage F and the tube C are filled with mercury. By means of a small piece of india-rubber tube C is connected with the cannula which is already full of blood. F is now again made continuous with C, the reservoir J is lowered a little, and thus some blood, and any air that is in the cannula finds its way into B. Perhaps 1 c.c. of blood may be taken at this juncture. The stopper is now turned through 90° so that F becomes blind. J is raised, H is opened, and when the blood and air have been expelled H is closed again. After this preliminary the experiment may begin. Blood is drawn into B by the process just described, the surface of the mercury in J being kept at the same level as that of the mercury in B, except when some special manipulation is required. When about 8 c.c. has been collected, and the reading on the graduated scale of B taken, that stopper is turned through 180°, so that F becomes continuous with D. As soon as this takes place all the blood is sucked into the vacuous receiver and some mercury is allowed to follow to wash the blood over.

The reading of the blood-mercury surface on the graduated scale has been taken. The question arises, "Supposing this reading to be, say, 7 c.c., how much blood is actually delivered into the receiver? What constant must be added to this reading, which starts from an arbitrary zero, to get the exact amount of blood which is subjected to the vacuum?" The answer to this question is: The burette must be calibrated by taking out the stoppers from beneath each of the froth chambers successively, and allowing the blood measured in the burette to flow out of the apparatus through the stopperless tap into a graduated tube. The reading in the tube may then be compared with that of the burette. In such a calibration the first reading in the graduated tube will be fractionally lower than the successive ones. To correct for this, a small bulb is placed at the end of what will hereafter be described as the "blood-main," so that a film of blood may be made along the glass before the first sample is taken into one of the vacuous chambers.

Passing from the measuring apparatus to that which repre-

sents the receiver in the usual forms of gas-pump, the blood can be allowed to pass into any one of a number of vacuous chambers, and the gases from any chamber can be drawn off at will into the pump.

Fig. 4 represents the apparatus in question in which six samples of blood can be analysed.

For purposes of description the whole of this apparatus will be spoken of as a "receiver," the part between the tap (*a*) and the tap (*b*) (that part which concerns itself solely with one sample of blood) as a "chamber," each chamber being made up of three "froth bulbs." The two parallel tubes *cd* and *ef* are each termed in the following description a "main," following the analogy which the drawing presents to a system of electric lighting. The tube *cd* is termed the "gas-main," and *ef* the "blood-main." The receiver stands in the vertical plane, mounted on a wooden stand, which is screwed on to the stand of the pump (Fig. 8). The "blood-main," made of two-millimetre bore tubing, is horizontal at the bottom, the "gas-main" made of broad tubing is at the top. Leading from the one to the other, there are arranged "in parallel" six chambers *A* (1), *A* (2), etc. The junction of each chamber with the "blood-main" is a tap b_1, b_2, etc. These taps are of special construction, and are shewn in detail in Figs. 5, 6, and 7. They are three-way taps, and a glance at the figure will shew that when the stopper is turned with the handle parallel to the "main," there is a path for the blood to run straight through the stopper, the tube in which becomes neither more nor less than part of the "main" itself. When the tap is turned through an angle of 90° as shewn in Fig. 6, any blood or mercury running along the main to the tap must go into the corresponding chamber, and there only. When it is required to transfer blood from the burette to any chamber, say *b* (4), the stoppers of *b* (1), *b* (2) and *b* (3) (that is, of those taps situated between *b* (4) and the point *f* which is attached to the burette) are turned as in Fig. 7, the stopper of the tap *b* (4) is turned as in Fig. 6, and one continuous tube from the burette to the chamber *A* (4) is formed, along which the blood, and some mercury to wash it over, must necessarily travel. Once the blood and a cubic centimetre or two of mercury have gone into the "chamber" the tap *b* (4) is closed, the handle being turned at an angle of 45° to the "main," and as the chambers are used in the reverse order to that in which they are numbered in Fig. 4, this tap is never again touched during the experiment.

In the apparatus described the capacity of each chamber is 200 cubic centimetres. With receivers of this size it is necessary to hold warm sponges round the uppermost froth-bulb while the blood is entering the chamber, otherwise the whole chamber fill-

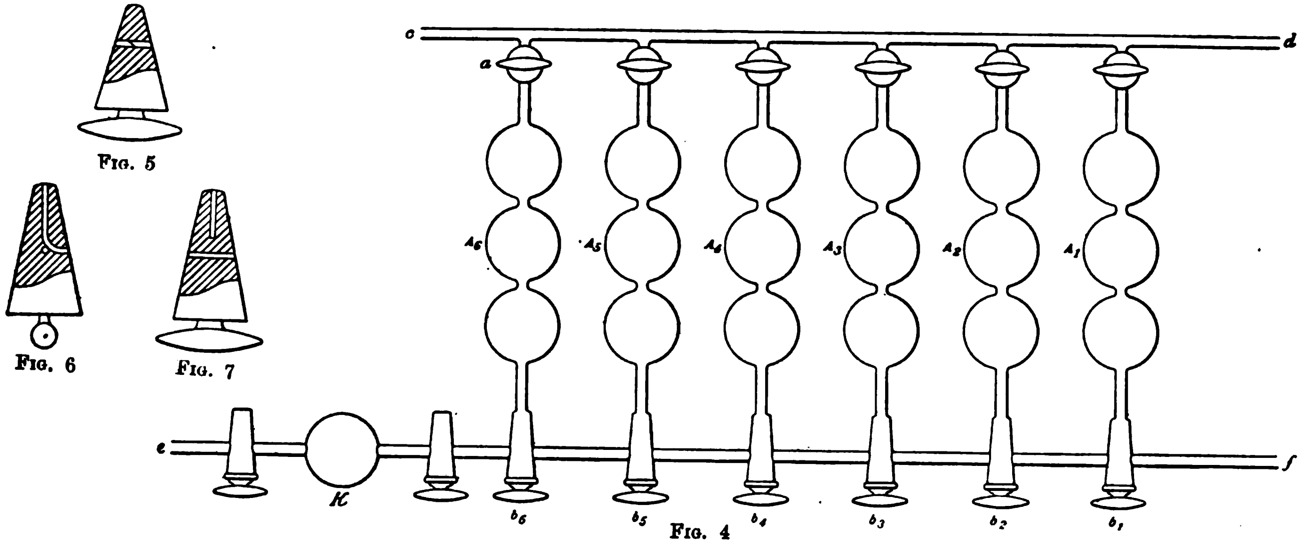


Fig. 5

Fig. 6

Fig. 7

Fig. 4

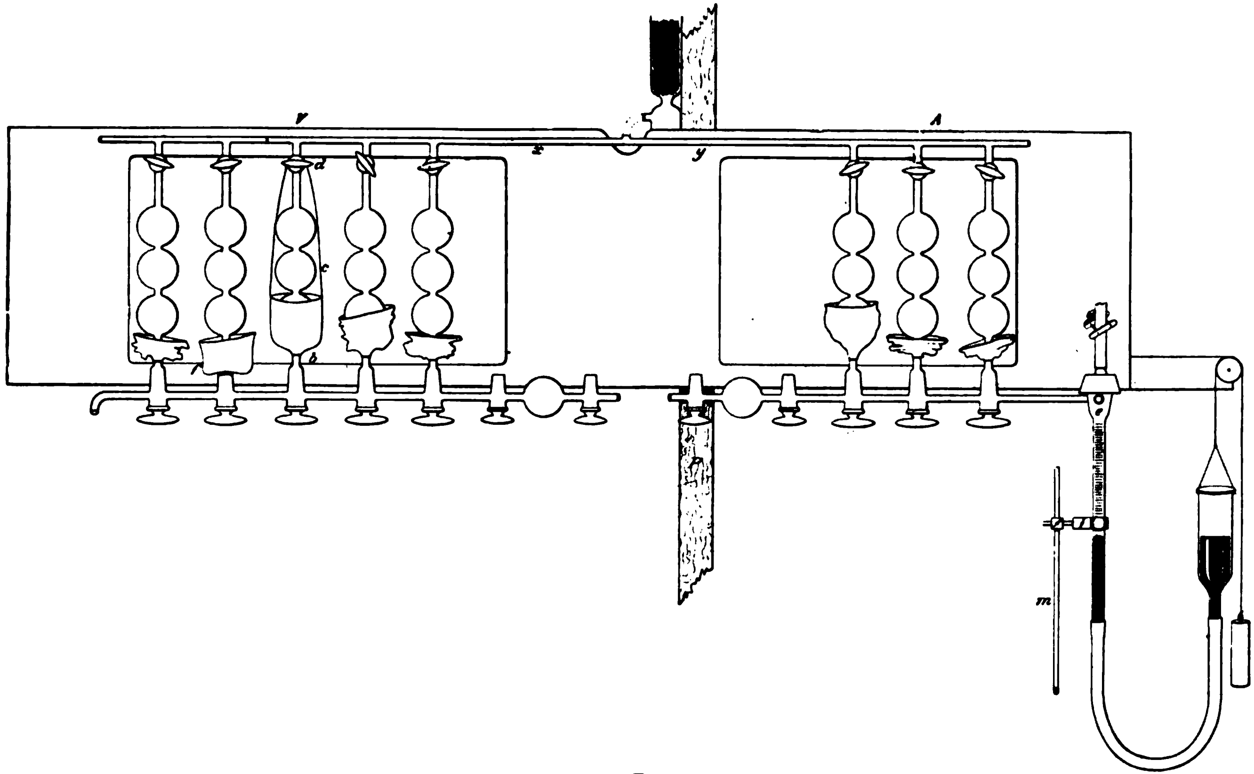


Fig. 8.

with froth. In use the end (d) of the gas-main is drawn off; (c) is blown on to the tube leading to the gas-pump. The end (f) of the blood-main is blown on to the measuring burette. To pump off the gases it is only necessary to open the two-way taps between the chamber and the gas-main and set the gas-pump going.

Fig. 8 represents the apparatus as it appears when actually in use, as also does the photograph, Plate I. The stand is made to hold two receivers A and V, used for arterial and venous blood, respectively. The receivers have, from time to time, had a variable number of chambers, V being shewn with six and A with three in the plate.

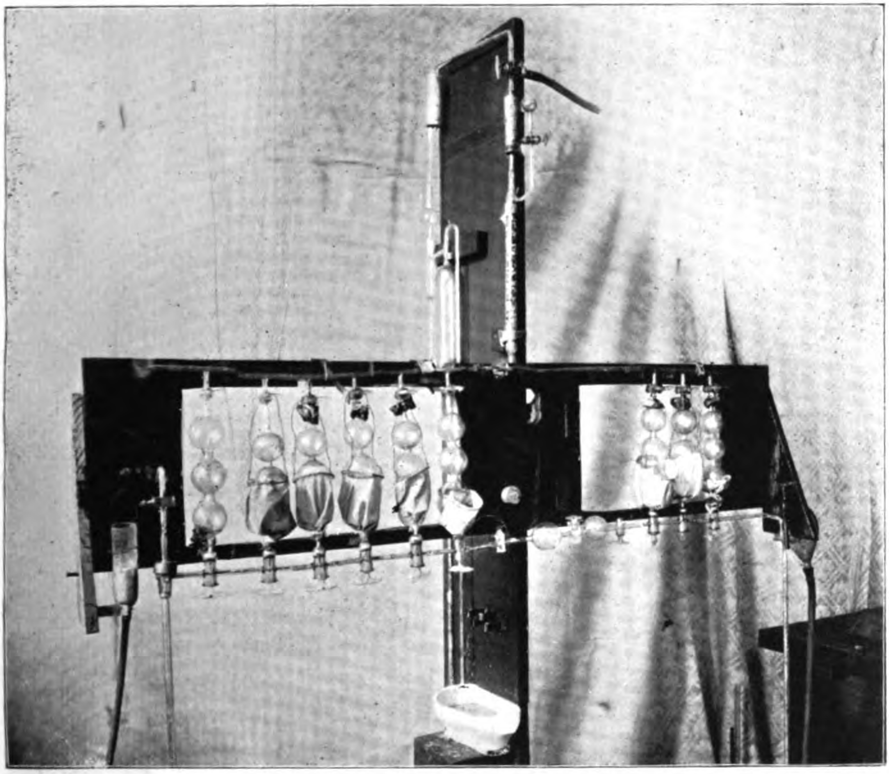
The tube leading from the drying chamber of the pump (seen in perspective and dotted in but really horizontal, or if inclined, inclined in the other direction in order to let any aqueous vapour which condenses fall into the lower part of the drying towers) is supplied with a horizontal T-piece (xy). At x and y respectively the gas-mains of the receivers are blown on, making one horizontal tube the whole length of the apparatus. The part of the blood-main adjacent to the burette is bent through an angle of 90° in the horizontal plane, in order to allow the tube at the other side of the burette to project towards the table on which the animal is lying. The end of this tube is shewn at e.

It is convenient to suspend the "receivers" by strips of chamois leather and drawing-pins, instead of any more rigid support, which might throw a strain on the apparatus.

Surrounding the lower "bulb" of each chamber is shewn, in various positions, a bag. This bag is made out of water-proof material bent on to a ring of wire at the top and gathered in at the bottom. This bag is to hold warm water or warm sponges.

The Gas Analysis Apparatus.

In conjunction with this apparatus, the analysis of the gases has been carried out by an apparatus devised and described by Dr John Haldane, F.R.S. The description will be found in the *Journal of Physiology*, Volume XXII., p. 456.



To face p. 10.

On the Structure and Classification of the Cheilostomatous Polyzoa.

By SIDNEY F. HARMER, Sc.D., King's College.

[*Read* 29 October 1900.]

The first part of Busk's Report on the "Challenger" Polyzoa[1] contains an account of a cavity, separated from the general body-cavity by a delicate membrane, in the zooecia of *Siphonicytara* and *Gephyrophora*, two genera of Cheilostomatous Polyzoa. In the former the cavity opens to the exterior by the so-called "median pore." Although nothing is said of any external opening in the latter, the important suggestion is made "that this division of the zooecial cavity into two compartments by a flexible membranous diaphragm will be found pretty generally in all zooecia of which the wall is wholly rigid, and that it is intended for the purpose of allowing the compression of the perigastric cavity necessary to effect the protrusion of the polypide." The exact details of the mechanism are not further discussed; and Busk states, on p. 175 of his Report, that the nature of median pores is "at present wholly unknown."

Four years later Jullien published an important note[2] on the same subject; without, however, referring to Busk's statements. The results arrived at were, (i) the operculum, in certain Cheilostomes which have a rigid body-wall, is not continuous proximally with the adjacent calcareous wall of the zooecium; (ii) its proximal border does not form the hinge-line, which is situated more distally; (iii) the interval between the proximal border of the operculum and the calcareous wall is the opening of a chamber which lies in the body-cavity and can be dilated by the admission of water in order to compensate for the protrusion of the polypide from its zooecium; the chamber opens widely to the exterior when the operculum is open. This "compensation-sac" was described by Jullien in greater detail in another paper[3], *Catenicella alata* and *C. ventricosa* being the species principally studied; but its mode of action and its full significance were not thoroughly appreciated in this or in any of Jullien's later papers.

[1] Part xxx., 1884, pp. 101, 168.
[2] "Sur la sortie et la rentrée du polypide dans les zooecies dans les Bryozoaires Cheilostomiens Monodermiés," *Bull. Soc. Zool. France*, XIII. 1888, p. 67.
[3] "Observations anatomiques sur les Caténicelles," *Mém. Soc. Zool. France*, I. 1888, p. 274.

Waters describes the compensation-sac, without understanding its mode of action, in *Calwellia sinclairii*, *C. bicornis*, *Urceolipora dentata*, *Onchoporella bombycina* (?), and *Ichthyaria oculata* (?); his account[1] appearing in the same year as Jullien's and quite independently of it.

It is perhaps owing to Jullien's remarkable views on the subject of Zoological nomenclature that his results have been either ignored or discredited by later observers. The existence of the compensation-sac is denied by Pergens[2] for species of *Schizoporella* in which it is certainly present; and Levinsen[3] expressly controverts the statement that the so-called "median pore" of *Microporella malusii* is open to the exterior.

I have been led to make observations on this subject as the result of the examination of what I believe to be a new species of *Euthyris*[4], kindly sent to me from Port Jackson by Mr T. Whitelegge. The study of this species completely confirms the accuracy of Jullien's statements. I am fully in accord with him in attaching importance to the characters of the "front-wall," or "*paroi frontale*," as a guide to the classification of the Cheilostomata. His statement that the compensation-sac occurs "chez toutes les espèces de Cheilostomiens monodermiés[5]" is a generalisation which does not appear to have been founded on a large number of cases; but it is nevertheless in all probability substantially true.

My own results may be summarised as follows:—

(i) The central group of the Cheilostomata is constituted by such families as the Membraniporidae, Flustridae, Farciminariidae, etc. In these the front wall (= opercular wall) remains entirely or to a large extent membranous, the membranous part being known as the "aperture." The operculum is a moveable part of this membrane, its base-line not being usually strengthened by a basal sclerite, the presence of which would introduce rigidity where flexibility is most wanted. A series of parietal muscles originates from the lateral wall of the zooecium, on each side, and is inserted into the membranous aperture. The contraction of these muscles exerts a pressure on the fluid of the body-cavity, and thereby causes the protrusion of the polypide;—a mechanism described by Nitsche in 1871[6].

(ii) A second group of Cheilostomata is constituted by the

[1] *Challenger Reports*, "Polyzoa Suppl." Part 79, 1888, pp. 17, 18, 3, 10.
[2] "Untersuchungen an Seebryozoen," *Zool. Anz.*, XII. 1889, p. 507.
[3] "Polyzoa, Hauchs Togter," Copenhagen, 1891, p. 285.
[4] I hope to publish an account of this species in a paper dealing more fully with the compensation-sac.
[5] *Mém. Soc. Zool. France*, I. 1888, p. 275. The "monodermiés" correspond for the most part with the Lepralioid or Escharine forms of other authors.
[6] *Zeitschr. wiss. Zool.*, XXI. p. 426.

Microporidae, Steganoporellidae, and other families in which the opercular wall remains membranous, but the body-cavity has been subdivided by a horizontal partition or cryptocyst[1] which is always more or less incomplete distally. The occurrence of the cryptocyst has resulted in a modification of the parietal muscles, most of which have disappeared, leaving a group on each side, near the distal end of the zooecium. Each group may pass through a special foramen ("opesiule") in the cryptocyst, in order to reach the membranous opercular wall (*Micropora*, etc.); or the cryptocyst may be specially modified in this region in relation with the parietal and other muscles (*Steganoporella*).

(iii) The Cribrilinidae, including the genera *Membraniporella* and *Cribrilina*, constitute a group which is transitional from the Membraniporidae to the next main group. In some of these forms (and perhaps in all which are rightly referred to the Cribrilinidae), a process foreshadowed in many species of *Membranipora* (well seen in *M. pyrula*, Hincks), results in the covering of the membranous opercular wall by a roof formed by the overarching of a series of calcareous spines developed round the proximal and lateral parts of the aperture. These spines ultimately meet one another, the intervals between them remaining as slits or series of pores which probably admit water[2] into the subjacent space. The distal pair of spines may appear to articulate with the base-line of the semicircular operculum, but the operculum is really continuous with the original membranous opercular wall, and not with the roof formed by the overarching spines. The parietal muscles are arranged as in *Membranipora* or *Flustra*. The oral spines of certain Cribrilinidae and of many other Cheilostomes are probably serially homologous with the overarching marginal spines. It is significant that many of the Cretaceous Cheilostomes belong to the Cribrilinidae[3].

(iv) The condition found in Lepralioid or Escharine forms, in which the free surface is entirely calcified, is a further development of the Cribrilinidan arrangement.

The calcareous front wall corresponds with the united overarching spines of *Cribrilina*, the membranous opercular wall of which is represented by the floor of a large compensation-sac, which lies beneath the front wall, and usually has walls of great tenuity. This sac opens to the exterior at the proximal border of the operculum, which in many cases possesses a well-marked basal sclerite, strengthening its edge along the line where it comes into contact with the calcareous front wall.

[1] Cf. *Quart. J. Micr. Sci.*, XLIII. 1900, p. 228.
[2] I have not at present complete proof that this statement is correct.
[3] Cf. Canu, "Rev. Bryozoaires Crétacés figurés par d'Orbigny," 2e Partie, *Bull. Soc. Géol. France*, (3) XXVIII. 1900, p. 440.

The operculum is merely in contact with the calcareous wall, and retains its original relations in being continuous with the membranous floor of the compensation-sac. Numerous muscles pass from the vertical calcareous walls of the zooecium to the floor of the sac. It may safely be concluded that the contraction of these muscles will dilate the sac, thereby introducing water into it from the outside and exercising a pressure on the fluid of the body-cavity, resulting in the protrusion of the polypide. The muscles which produce this effect are thus, in their function as in their morphological nature, to be regarded as parietal muscles; and their relation to the compensation-sac is an important link in the chain of evidence tending to show that the calcareous "front wall" is derivable from an arrangement similar to that of Cribrilinidae. The front wall may be developed in two very different ways:—

(*a*) In *Umbonula verrucosa*, it grows over the aperture at a higher level, as a continuous overarching lamina, which commences proximally and laterally; the space which it covers always remaining widely open to the exterior. A *Membranipora*-like opercular wall is present in the young zooecium, and the parietal muscles develop *in situ*. This arrangement is in no way modified during the later development, and the operculum still retains its primitive condition of having no basal sclerite. So far as I can judge from dry material, calcification proceeds in the same manner in certain species of *Porella, Mucronella* and *Escharoides*; which are accordingly to be regarded as related to *Umbonula*.

(*b*) In *Lepralia pallasiana, Schizoporella linearis, Euthyris obtecta, Catenicella cornuta*, and others, a different mode of development is followed. The front wall appears at first sight to result from the direct calcification of a *Membranipora*-like membranous wall which is usually distinctly visible in the young zooecium. The compensation-sac is at first not present, but develops from a mass of cells beneath the proximal edge of the operculum. These cells soon arrange themselves round a cavity, which appears semicircular when seen from above, the diameter of the semicircle coinciding with the proximal edge of the operculum, and the arc curving on the proximal side of the base. Numerous muscle-fibres *radiate* from the walls of this sac to the more proximal parts of the lateral walls of the zooecium. The sac rapidly grows in a proximal direction until it underlies the whole or the greater part of the calcareous front wall. The muscles which at first radiated from it are now arranged as two lateral series of parietal muscles. The identity of arrangement of these muscles and those of other Cheilostomes suggests that the phylogenetic origin of the calcareous wall has here been modified in ontogeny, in such a way that the development of the

Membranipora-like opercular wall is postponed until it is formed as the floor of the compensation-sac. The existence of this sac can be easily demonstrated in the adult zooecia of various species of *Schizoporella* and *Catenicella*, in *Urceolipora nana*, *Catenaria lafontii*, etc.

(v) In certain species provided with a "median pore" (*Calwellia bicornis*, "*Euthyris*" *episcopalis*), development of the compensation-sac occurs as in iv (*b*), except that the sac opens to the exterior by means of the median pore. It opens in the same way in the adult zooecia of *Calwellia* (*Onchopora*) *sinclairii*[1], *Ichthyaria oculata*, *Onchoporella bombycina*, Busk, *Urceolipora dentata*, and probably in *Microporella malusii*. In all these cases except in *O. bombycina* and *U. dentata* (in which suitable preserved material was not available) the parietal muscles inserted into the sac have been demonstrated. The view supported by Gregory[2] that the median pore has been formed by the closure of the calcareous sinus which receives the tongue-like projection of the operculum in *Schizoporella*, *Urceolipora nana* and others is probably correct. There is, however, no essential difference between the anatomy of *Lepralia* and that of *Schizoporella*; and the validity of Gregory's divisions Holothyriata and Schizothyriata, characterised respectively by the absence and the presence of a sinus or median pore, is not supported by the study of the compensation-sac.

(vi) The genera *Scrupocellaria*, *Menipea*, and *Caberea* contain species which are provided with the so-called "scutum" or "fornix," which is a large spine, expanding from its base, which overarches the *Membranipora*-like opercular wall. The remarkable fact that the scutum is present or absent, well-developed or small, in each of these genera suggests that *Scrupocellaria* and its allies are forms in which the membranous opercular wall became protected by a single spine instead of in the manner characteristic of the Cribrilinidae. The scutum is probably to be regarded as a vestigial structure in those species in which it is small or absent. The high development of avicularia or vibracula (or both) in these genera confirms the view that they are not to be regarded as a very primitive group of Cheilostomes.

(vii) The foregoing considerations indicate that a complete rearrangement of the Cheilostomata is required. The division Cellularina in particular is an unnatural one, and consists of Cheilostomes belonging to several distinct groups which have taken on a dendritic habit of growth. The Bicellariidae, for instance, retain a *Membranipora*-like arrangement of their opercular wall and parietal muscles. Species of *Catenicella*, *Catenaria*,

[1] Cf. Waters, *Challenger Rep.*, Part 79, p. 17.
[2] "British Palaeogene Bryozoa," *Trans. Zool. Soc.*, XIII. 1895, p. 222.

and others are to be regarded as branching Lepralioid forms. Other Cellularine genera, *e.g. Calwellia* and *Ichthyaria*, are probably more nearly related to *Microporella*, although it is not impossible that the median pore has been evolved more than once. I agree with Waters in thinking that *Ichthyaria* cannot retain its position in the family Bicellariidae. *Urceolipora dentata* should perhaps be removed to *Calwellia*. "*Euthyris*" *episcopalis* should be placed in a distinct genus, characterised in part by its conspicuous ovicells, and the fact that both layers of their wall are calcareous. *Flustra militaris*, Waters, appears to belong to the same genus, and is probably a Lepralioid form in which most of the calcification of the front wall has been secondarily lost. The Lepralioid genera are probably to be arranged in two divisions (see iv (*a*) and (*b*)), according to the manner in which the calcareous front wall is developed.

(viii) The calcareous matter of Cheilostomata is probably always covered by an "epitheca" limited by a cuticle of chitinous or other organic substance. In certain cases this is so conspicuous that the generic or specific name has been taken from it (*e.g.*, *Calymmophora*, *Lepralia vestita*). The formation of layers of superposed zooecia in *Cellepora*, etc. appears to take place as the result of the separation of the epitheca from the calcareous wall, the subjacent space developing into a new zooecium. The secondary thickening which the calcareous front wall commonly undergoes, and the occlusion of the orifices of the zooecia in the older parts of the colony, in certain cases, are probably due to the existence of this living membrane external to the calcareous matter. The membrane usually passes into the "raised lines" which may form the outlines of the zooecia. In *Euthyris obtecta* this connexion seems to have been lost, and there is a large, continuous, extra-zooecial space on the front and back walls of the frond. On the front surface the space is traversed by the necks of the flask-shaped zooecia, the orifices lying in the same plane as the epitheca. The arrangement closely resembles that described by D'Orbigny in one of the Cretaceous family Steginoporidae[1], but if Jullien is correct in his statements[2], these forms have developed what may be described as a tertiary front wall by the growth of certain branched peristomial spines. It is a noteworthy fact that the cavity beneath this tertiary wall has a floor which resembles the outer surface of a *Cribrilina*.

The continuous epitheca of *Euthyris obtecta* is held at a distance from the zooecia by papillae which are calcareous at

[1] *Disteginopora* (*Thoracophora*) *horrida*, "Pal. Française," "Terrains Crétacés," v. 1850—51, p. 237, Pl. 687 bis, fig. 4.

[2] "Les Costulidées, nouvelle Famille de Bryozoaires," *Bull. Soc. Zool. France*, XI. 1886, p. 609.

their base and are best developed on the back wall. They are also present on the front wall, where, however, the epitheca is mainly held in position by those parts of the zooecia which immediately surround the opercula. Many Cretaceous species were probably provided with an epitheca which was stretched at a distance from the zooecia, as is indicated by the occurrence of similar papillae[1].

[Dec. 13, 1900.—During the correction of my proofs I have received an important memoir, "Contribution à l'histoire naturelle des Bryozoaires Ectoproctes" (*Trav. Inst. Zool. Montpellier*, N.S., Mém. No. 8, 1900), kindly sent to me by the author, M. Louis Calvet. I can do no more, on the present occasion, than to call attention to the fact that M. Calvet, on p. 278, discredits Jullien's account of the compensation-sac, and suggests that the existence of strong parietal muscles indicates that the calcareous front wall is flexible in Lepralioid forms. My own observations, on the contrary, show clearly that the parietal muscles are inserted into the floor of a compensation-sac.]

[1] Cf. D'Orbigny, *tom. cit.*, Pl. 718, figs. 3, 7, 10, 11, 15; Pl. 721, figs. 3, 4, 7, 8; etc.

The Natives of the Maldives[1]. By J. STANLEY GARDINER, M.A., Fellow of Gonville and Caius College, and Balfour Student of the University of Cambridge.

[*Read* 12 November 1900.]

The Maldives are a group of islands in the Indian Ocean to the south-west of the Indian Peninsula. They extend from lat. 8° N. to lat. 1° S. and from long. 72° E. to long. 74° E., and include about 300 inhabited islands. To the north lies the Laccadive Group, distant about 150 miles, and intermediate to the two groups is the atoll of Minikoi.

For administrative purposes the Maldives are divided into thirteen provinces, which are called *atolu*, each with a governor, the *atoluveri*. These provinces are often conterminous with the *atolls*, whence arose this term. Ethnologically the group divides itself into four divisions. First the northern atolls are separated from the central atolls by the Kardiva Channel. This is 35 miles broad, and, as the cross-currents during the monsoons are usually of great force, it forms a natural geographical boundary. The northern atolls, being further distant from the equator, are subject to more violent storms than the southern ones; their reefs too are less perfect, never indeed forming quiet lagoons in the centre. The coast pirates, Mopillahs and others, constantly ravaged them. As a consequence of these conditions they have bred a hardier race of people than the rest. The people annually visited the west coast of India and Ceylon, often concluding treaties on their own account with the rajahs of the coast against the pirates. Many married Indian wives, and the people approximate closely to the Mahommedans of the south-west of the Peninsula.

The central division includes ten atolls, from North Male to Haddumati, situated within comparatively short distances of one another. The people were under the direct rule of the sultan, and, whatsoever affected Male, the capital, influenced all. Formerly trade was carried on with Arabia and Malaysia, both in Maldivan and foreign bottoms. Many Arabs settled in the group, and the

[1] I am indebted to the Government Grant Committee of the Royal Society and the British Association for financial assistance in carrying out this work. I am also under great obligations to Mr C. Forster Cooper, my companion, for his ready assistance at all times.

crews of their vessels, being of the same religion, took temporary wives with the result that many of the present race have a very marked tinge of the Arab in their features and forms. Negro slaves were imported from Zanzibar and Jeddah, being employed in the drawing and manufacture of coconut sugar. They married Maldivan women, and their descendants now form the *Ravare* caste. Caucasian slaves too were introduced for the nobles of Male. Many trace descent from Malay traders, but the latter exercised a greater influence on the race by introducing Burmese women. Their beauty was greatly esteemed, and even now the Maldivans consider a Mongoloid cast of features very comely.

Male has been occupied at times by the Portuguese, French and Dutch. The first garrisoned it, and commenced its fortification, but made no permanent settlement, and during the greater period of their sway contented themselves with receiving tribute at Goa. The Dutch succeeded in the middle of the seventeenth century, their chief factor in Ceylon concluding a treaty of friendship, alliance and mutual defence with the Sultan. At the commencement of the nineteenth century the Maldives passed under the influence of the British, the same treaty being annually renewed with the governor of Ceylon, accompanied by mutual presents. The Dutch at times had a garrison at Male, and made of it a great fort, similar to those of southern India and Ceylon. Further they built a breakwater at the edge of the reef towards the lagoon of the atoll, as an outer line of defence. It also formed a safe harbour, in which to repair their vessels. The French merely had a party of troops in Male for six years during the Dutch period. They gave rise to the term *faranje*, which is now applied to all western peoples. From a variety of causes, however, the European races appear to have made no direct nor permanent mark on the race, so that their presence may be practically neglected.

Suvadiva Atoll is separated by the "One and Half Degree Channel," 60 miles across from the central division, while Addu is cut off from Suvadiva by the "Equatorial Channel" of about the same breadth. South Mulaku is an isolated island—not atoll—lying mid-way between Suvadiva and Addu. Through these channels the equatorial current runs with great force, and even a voyage to Male was considered a momentous enterprise, not to be undertaken except with a favourable monsoon. The people of these atolls rarely intermarry with those of others, and in their appearance present a far greater uniformity. To none can the terms Mongoloid or Negroid be properly applied. Indeed the people very closely resemble village Singhalese. Owing to their isolation these atolls are more self-contained than any of the others. They grow their own grain and food-stuffs, manu-

facture their own cloth and mats, and build their own boats. Magic and witchcraft are believed in and practised. In former times they were always semi-independent of Male, perhaps the stronghold of a rival or deposed sultan, the sultan's last point of retreat when harried by European or Indian pirate fleets, themselves too poor to make an invasion profitable, even if feasible. Probably at first a stronghold against Islam, Addu is now noted for its piety and learning. It usually supplies the *Kazi*, or chief judge for the whole group, and quite 10 per cent. of its population reads Arabic more or less fluently.

Thus it will be seen that there has been comparatively little admixture of races in historical times in the Maldives. Early accounts of south India and Ceylon tell of a great island kingdom to the south-west, which can be the Maldives alone. Unfortunately they say nothing of its condition nor religion, but it would appear to have had an unfavourable reputation for hospitality.

Probably the Maldives were converted to Islam at the beginning of the thirteenth century. In 1343 they were visited by Ibn Batuta, the great Arab traveller, who for some time occupied the office of *Kazi*, chief judge and head of the religion. From his account it is evident that the group must have been organised in practically the same manner as it is to-day, and was in nearly the same condition. From what religion they were converted we know not, but there are indications that Christianity at one time got some slight hold at least in the northern atolls. This is not however a question of any importance, but what was the religion before this? Mounds in Landu and Milhadu in Miladummadulu, in Haddumati, Suvadiva, South Mulaku and Addu Atolls suggest by their form a comparison with the dagobas of Anuradhapura. Ghang in Haddumati was evidently a great religious centre, having ruins of fourteen of these, some with smooth sides of squared stone. One in Landu appears to have had a kind of pit or well in the centre, formed by squared blocks of Porites and covered by two large flat masses. Within this in 1848 were found a number of gold or brass ornamented discs. These were unfortunately all melted or destroyed, the greater number being thrown into the sea, as they were deemed to be the cause of an epidemic, which broke out in the island. At South Mulaku a few oval six-sided beads either of an extremely hard clouded glass or of crystal were found. An indication of Buddhism might indeed be gleaned from the above, but I found no other evidence in its support. The presence of an immense Bo tree (*boi gas*) at Male and another in South Mulaku is of no importance; no traditions cling to them. The tanks resemble those of Anuradhapura, but there is no particular Buddhist type. Sweet smelling flowers near mosques, such as the frangipanni (*semper-beddha*) and jessa-

mine (*huwanduma*), are common throughout the East in any open place where people lave and meet together.

The charms, which might be expected to retain some of their primitive facies longest and to be less affected by Islamism, show no leaning to Buddhism. That of *Kassoderie* Dzhin might be taken to represent the Vishnu lingum; of *Oadee-ver-Risa* Dzhin the Shiva lingum; of *Kudaffoor* a sixteen-legged scarabaeus with a Shiva head. Further those of *Ras* and *Beembi* Dzhin have also distinct phallic possibly Vishnu indications. Other charms are purely Arabic or Islamic. All are covered with quotations from the Koran or Sonna books. It is possible that many of the charms may have been introduced from Brahmin India, but it is more probable that the lower features of Brahminism arose among people, who gave rise to the Maldivans as well.

The Maldivans themselves have no stories of the origin of their race nor islands, and always asseverate the indigenous character of their occupation, quoting as arguments the complete dissimilarity of their cloth, dress, mats, lacquer work and boats from those of the mainland. This however is not strictly true, as I found an outrigged boat (*digu doni*). It is now only used by the children as a toy, but formerly it was in common use for inter-atoll voyages. I saw the hulls of several, in all cases hollowed tree-trunks, and the fully rigged vessel would seem to have been very similar to the regular Singhalese and Polynesian canoes.

As to the language I have no personal knowledge of Singhalese, and so doubtless failed to see the close similarity which is said to exist. From a comparison of words of everyday use the relationship was very clear, especially in the southern atolls. Mr Gray in the *Jour. Asiatic Soc.* points to the great resemblance between Maldivan of the present day and Elu, the pure ancient Singhalese of 2000 years ago, which was later corrupted by Sanskrit and Pali. The close likeness between the two peoples bears this out, and the two races would seem to have been the result of a dichotomous branching of a common stem, one division perhaps travelling along the west coast of Hindustan to Ceylon and the other sailing through the Laccadives to the Maldives.

I hope subsequently to publish a full report on this most interesting people in collaboration with Mr C. Forster Cooper of Trinity College, who has paid especial attention to their manufactures, games, boats, etc.

The Atoll of Minikoi[1]. By J. STANLEY GARDINER, M.A., Fellow of Gonville and Caius College, and Balfour Student of the University of Cambridge.

[*Read* 12 November 1900.]

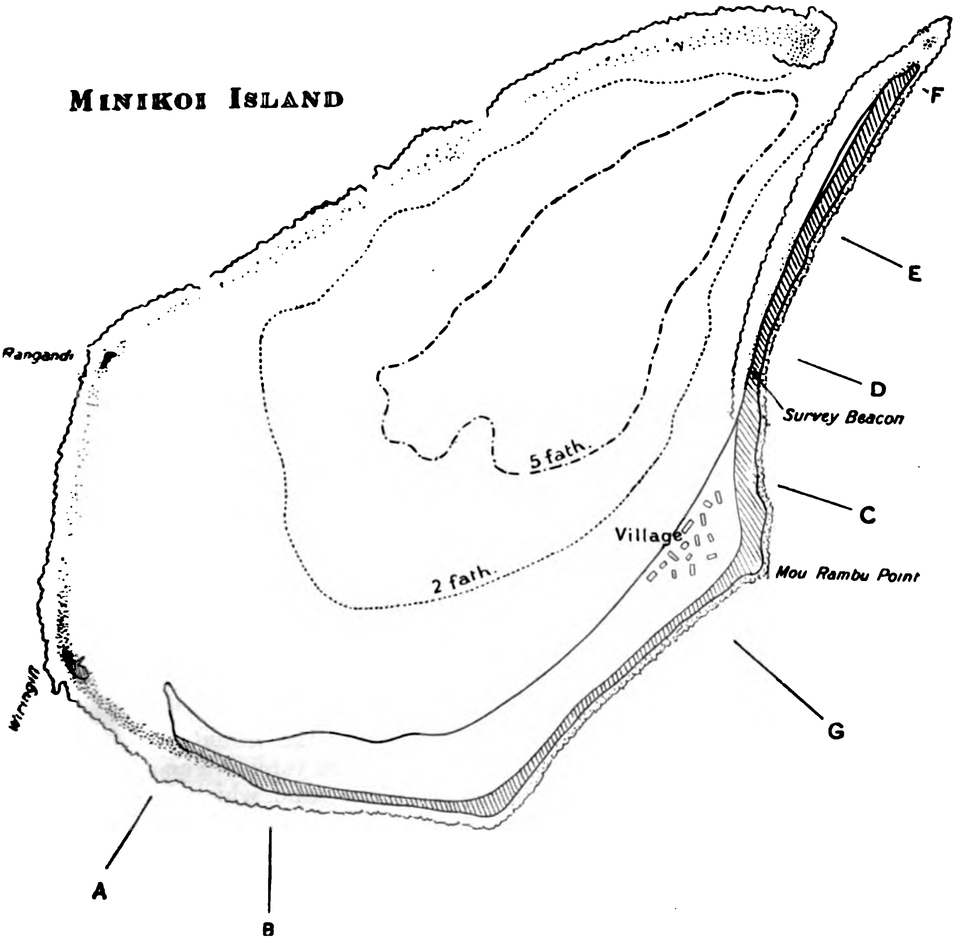
The Atoll of Minikoi is situated in lat. 8° 15′ N. and long. 73° E. For administrative purposes it is called a part of the Laccadive Group, but geographically it rather belongs to the Maldive, if to either. To the north lies the "nine degrees channel" separating Minikoi from the Laccadives, of which Suheli Par and Kalpeni, the nearest, are about 110 miles distant. South is the "eight degrees channel" 71 miles across between Minikoi and Ihavandifolu, the most northern Maldive atoll. From the nearest point on the Indian coast Minikoi is about 215 miles distant.

The Laccadives and Maldives lie on a long narrow bank at a depth of 1200 fathoms with deeps on either side of 2000 to 2400 fathoms. This bank stretches down from the west coast of India to Addu Atoll in lat. 0° 40′ S., but is separated from the Chagos Atoll by a depth of 2500 fathoms.

There is no weather side to the atoll of Minikoi, as it is exposed equally to the north-east and south-west monsoons. Nor indeed can there be said to be two sides principally exposed, since, owing to the proximity of the atoll to the Indian coast, winds from the south-east and north-west constantly occur. During June and July, 1899, the wind varied only four points on each side of N.W. by N., while in August and September it blew from any direction within seven points of S. by W. Hurricanes seldom approach the equator within lat. 8° N. or S. in the Indo-Pacific Oceans, and the atoll is only visited by them about once in every 12 years; even then, as the centre is usually far north, little damage is done. Heavy storms are common at the commencements of the monsoons. The currents in this region depend mainly on the winds, and vary perhaps up to 50 miles per diem.

[1] I am indebted to the Government Grant Committee of the Royal Society and the British Association for financial assistance in carrying out this work. I would thank too Mr L. A. Borradaile for accompanying me to Minikoi; his illness unfortunately prevented the work from being as complete as I had desired.

Owing to the deflection caused by the Indian and Ceylon coasts they obey no known laws in respect to seasons, and impinge equally on all sides of the atoll.



Scale :—1·15 inches to 1 mile.
The distribution of the limestone rock is shown by the shaded area.
The rough (or boulder) zone of the reef is dotted.
A—G show the positions of the sections run across the island.

The tides also are of course at all times largely influenced by the prevailing winds and currents. It is high water, full, and change about $10^{h} 30^{m}$. Springs rise about 6 feet and neaps $3\frac{1}{2}$ feet. The ebb stream is almost invariably longer than the

flood in the proportion of about 7 to 5, but in this it varies greatly in accordance with the strength of the winds, and I have even seen the above reversed. It further alters with the time of high water. The flood always seems to set to the east and the ebb to the west. As the monsoon winds commonly moderate somewhat after the heat of the day and during the night, it will be obvious that the lowest tides are in the morning during the S.W. monsoon and in the evening during the N.E. The difference in rise and fall in heavy weather at springs is often as much as 2 to 2½ feet, and on one occasion I recorded over 3 feet.

The surface temperature of the sea obviously varies greatly from year to year. The Admiralty Charts for Minikoi give February 82°, May 86°, August 81°, November 84°. My observations expressed in round numbers were: June 87°, July 84°, August 82°, September 84°. June was a dry month, but it rained daily for five weeks—often with great force—in July and August. The total rainfall is about 110 inches per annum, the greater part of which falls in the south-west monsoon and at the change of monsoons.

The atoll of Minikoi is of a more or less oval shape, lying in a north-east and south-west direction, length 5 miles by greatest breadth north-west and south-east 2·9 miles. The main island—Minikoi island—lies on the reef to the east and south, and is 6 miles in length. The reef to the west and north is perfect up to a narrow ship's passage at the extreme north point about 2 fathoms deep. It has two islands Wiringili with a few coconut trees about ·5 mile from the west point of Minikoi and Ragandi, a rocky mass, 1·5 miles farther north.

The village is situated on the lagoon side about the centre of Minikoi island, opposite Mou-Rambu, the most south-easterly point of the atoll. South of the village the island varies in breadth up to 800 yards, average about 600 yards. The greater part of this is formed of remarkably fine sand, which has been washed up from the lagoon; it is then blown up the beach, forming land, 2 feet or so above the high tide level. In most parts this action has stopped or is scarcely appreciable now, but in one bay it is still somewhat rapidly proceeding. Its rate can be estimated by (1) the breadth of the beach above the low tide level, (2) the proximity to the beach of trees and shrubs and their size inland from the high tide level, and (3) the amount of green and foliaceous algae washed up on the beach at each tide.

North of the village the island narrows rapidly, and at a distance of about ·75 mile at the survey beacon is only 102 yards between tide marks. It then continues as a mere ridge 70 to 110 yards broad to the north point. The sandy lagoon beach ceases about 6 miles north of the village. It shows a very rapid

washing away as evidenced by (1) its narrowness and steepness, (2) a small sand cliff terminating it above, and (3) numerous fallen coconut trees.

The north part of the island is composed almost entirely of an agglomerate limestone rock, such a rock as is being formed at the present day on any reef in this region freely exposed to the ocean. By the lagoon a little sand is in one place found. The rock further is found as a fringe on the seaward face of the whole of Minikoi island, and also forms the islands of Wiringili and Ragandi. To the north of Minikoi the lagoon face is everywhere rapidly washing away. It has a cliff above, varying up to 10 feet in height, a narrow rocky beach and off this a flat reef, just covered at low tide, about 130 yards broad. On the inner half of this reef and in the beach are a number of rocky masses, or pinnacles, exposed. On these many of the corals are in the position of growth and little broken; and in the cliff the coral masses lie more or less horizontally.

The beach on the seaward face north of Mou-Rambu Point is also very steep with, in places, a cliff above; it is mainly formed of coral masses, which have been part of the rock of the island. They are often much rounded, but even after storms the beach shows no signs of any additions of recently living corals from seaward. There is no reef to seaward as is found ordinarily in such a position, but there is a flat, about 30 yards broad, covered with low green algae, sloping 2 feet and consisting of three ill-defined terraces, strewn with pinnacles. Outside the slope drops to 2 fathoms, and then attains its ordinary character outside atolls. South of Mou-Rambu Point the beach decreases in steepness, and the terrace formation of the flat gives place to a reef, closely resembling the seaward reef of Funafuti island. It reaches a maximum breadth of 120 yards, has a marked reef-flat at the low tide level with little or no coral growth and a broken buttressed edge with masses growing up outside, gradually joining on to the same. Corals are of little importance, nullipores covering the whole edge as off Rotuma and on the most exposed reefs of the Fiji Group.

At the south-west end of Minikoi the fringe of rock continues into a well-marked rough (or boulder) zone, covered completely only at springs. This joins it to Wiringili and thence to Ragandi, continuing round, though less marked, to the deep channel at the north-east corner of the atoll. It is formed largely of loose coral blocks, but among these are found a number of masses, which actually form part of the reef itself. To seaward the reef has the same features as off the south of Minikoi. Against the ships' channel the older natives of the atoll remember the existence of an island with three coconut trees, called Tori-Gandu.

The lagoon, as is commonly the case in small atolls, is shallow and much broken up by shoals. It has a sand flat against the rough zone of the reef, especially broad to the south; this can be waded nearly everywhere at low tide. The central deeper part is evidently enlarging in all directions at the present day. The following give evidence of this:—(1) there is a marked cliff all round from one to about three fathoms; (2) the shoals have all precipitous sides often overhanging; (3) coral growth is absent from the bottom; (4) the depths show a slight general increase as compared with the chart; (5) there are no foraminiferal deposits, the sand being the same everywhere, exceedingly fine, evidently for the most part much broken and triturated coral fragments.

Tracing back the history of the atoll the island would appear to have been formed entirely by either an elevation of the whole atoll, or more probably by a change of level in the surrounding ocean. The highest point of land is 19 feet above the low tide level, and such an height is only found in one position, and may be partly artificial. Allowing amply for denudation, it is fair to suppose a change of about four fathoms. When first the alteration in relative level took place, it is probable that the island extended round the atoll, except perhaps where the present ships' passage exists, the presence of pinnacles at any part being taken as evidence of the existence of former land. The sandy part of Minikoi island was probably mainly an after formation, due to washing up from the lagoon.

At the present day the reef of the atoll is indubitably growing outwards on every side—the curious formation to the north-east being due to special local conditions—and the lagoon is deepening and broadening. There is no evidence of a former central island, such as is commonly found in the Fiji Group, nor is there any indication of subsidence throughout the Laccadives. Indeed it would appear more probable that this atoll has been formed *sui generis*, has perhaps grown up as a flat reef on some mound on the sea floor, subsequently attaining its present ring shape. On this view the numerous deep banks of the Laccadives represent incipient stages in the formation of reefs, while its islands and reefs exemplify the changes which finally produce the perfect atoll. In all this region however there was probably a change of level of the reefs in respect to the sea, which has modified them considerably, but the main features certainly remain.

A full report on the atoll is in preparation and will shortly be published. The foregoing account was read at the recent meeting of the British Association at Bradford.

Note on the Rational space curve of the fourth order. By J. H. GRACE, M.A., Peterhouse.

[*Read* 26 November 1900.]

1. The relations between the geometry of points on a rational quartic and the algebraic theory of a binary quartic have been frequently discussed[1], but the application of similar methods to the line-geometry of the tangents to the curve has not, as far as I know, been previously remarked.

If we supposed the coordinates x, y, z, w of a point on the curve to be expressed as quartic functions of a simple parameter λ so that

$$x = f_1(\lambda), \quad y = f_2(\lambda), \quad z = f_3(\lambda), \quad w = f_4(\lambda),$$

then the six coordinates of the tangent at any point are the six Jacobians of the f's taken in pairs. For convenience I denote these by

$$J_{23}, \ J_{31}, \ J_{12}, \ J_{14}, \ J_{24}, \ J_{34}.$$

2. Each of the four f's is apolar to one and the same quartic f and thus the condition that four points should be coplanar is that the quartic giving their parameters should be apolar to f.

In like manner each of the sextics J is apolar to one and the same sextic ϕ, and the condition that the tangents at six points should belong to the same linear complex is that the sextic giving their parameters should be apolar to ϕ.

3. Let us suppose that $f = a_\lambda^4$, $\phi = c_\lambda^6$, and that $\alpha, \beta, \gamma, \delta, \epsilon, \zeta$ are the parameters of six points on the curve; then it is well known, it can be easily verified, and it is indeed obvious from the simple properties of transvectants that $\alpha, \beta, \gamma, \delta$ are coplanar if

$$a_\alpha a_\beta a_\gamma a_\delta = 0.$$

Further, the tangents at the six points belong to the same linear complex if

$$c_\alpha c_\beta c_\gamma c_\delta c_\epsilon c_\zeta = 0.$$

Hence just as $f \equiv a_x^4$ gives the points at which a plane contains four consecutive points, so $\phi \equiv c_x^6$ gives the points at which a linear complex contains six consecutive tangents to the curve.

[1] See Mr Richmond's recent and comprehensive Memoir (*Camb. Phil. Soc. Trans.* Vol. XIX. Pt. I.), in which full references will be found.

As the points given by ϕ possess a projective property with reference to the curve, it follows from general principles that ϕ is a covariant of f, and being of degree six it must be the sextic covariant.

To verify that this is actually so, we need only prove that ϕ is apolar to J_{12}, say, *i.e.* we have to establish the following algebraical theorem:—

If two quartics are each apolar to a third quartic, then the Jacobian of the former two is apolar to the sextic covariant of the third.

Taking the third in the canonical form $x^4 + 6mx^2y^2 + y^4$ this is easy.

In general terms then we may say that ϕ bears the same relation to the geometry of the tangents as f does to the geometry of the points of the curve.

4. I shall content myself with two deductions from the foregoing principles.

I. The tangents at α, β, γ, δ, ϵ, ζ belong to a linear complex if

$$c_\alpha c_\beta c_\gamma c_\delta c_\epsilon c_\zeta = 0,$$

and given five of the tangents this equation generally determines the sixth uniquely.

If, however, the five be such that any linear complex containing four also contains the fifth, then the above equation must be satisfied by all values of ζ; hence the quintic giving α, β, γ, δ, ϵ must be apolar to $c_\lambda{}^5 c_\zeta$ for all values of ζ, *i.e.* it must be apolar to all first polars of ϕ.

In like manner the tangents at α, β, γ, δ are generators of a hyperboloid if any linear complex containing three also contains the fourth, and thus the above equation must be identical in ϵ and ζ. Hence the quartic giving α, β, γ, δ must be apolar to $c_\lambda{}^4 c_\epsilon c_\zeta$ for all values of ϵ and ζ.

Now taking f in the canonical form $x^4 + 6mx^2y^2 + y^4$, ϕ will be $xy(x^4 - y^4)$ and the quartic has to be apolar to

$$\frac{\partial^2\phi}{\partial x^2}, \quad \frac{\partial^2\phi}{\partial x \partial y}, \quad \frac{\partial^2\phi}{\partial y^2}.$$

It follows at once that the quartic is of the form

$$x^4 + \mu x^2y^2 + y^4,$$

i.e. of the form $H + \lambda f$ when H is the Hessian of f.

Hence four tangents to the curve are generators of a hyperboloid when, and only when, the quartic giving their parameters is of the form $H + \lambda f = 0$.

Among such sets of four are the tangents at the points of superosculation ($f=0$), and the four tangents which meet the curve again ($H=0$). Both these are known results. As a further example we may remark that the tangents at the points of superosculation are each met by one other tangent, and the points of contact of these being obviously given by a rational quartic covariant, the four tangents are generators of a hyperboloid. Other sets of four could be mentioned.

II. If the curve has a stationary tangent then f has a squared factor, for the point of contact of this tangent may be regarded as the coalescence of two points of superosculation. In this case ϕ consists of the sixth power of the same factor, and hence by the acquisition of a stationary tangent the curve has lost six points where a linear complex contains six consecutive tangents.

This is a result depending in the end on only infinitesimal properties of the curve, and hence the same is true of all space curves, viz. the acquisition of a stationary tangent means the loss of six points when a linear complex contains six consecutive tangents. I hope to make use of this fact in a future communication.

Trifolium pratense var. *parviflorum*. By I. H. BURKILL, M.A., Gonville and Caius College.

[*Read* 26 November 1900.]

There are three abnormal states of the common red clover in which the corolla is found unduly shortened. One of these is due to an insect larva which feeds within the bud, stunts its growth, causes it to remain closed and the basal parts to be fleshy: the second occurs when the petals are in part sepaloid: the third is a condition in which the corolla-tube is crumpled and the ovary slightly foliaceous; moreover it generally has peduncles

to the heads and short pedicels to the flowers. This last is *Trifolium pratense* var. *parviflorum*, and has the following synonymy:—

T. pratense var. *parviflorum*, Babingt., Manual Brit. Bot., ed. 1 (1843), p. 72; Lange in Oeder's Flora Danica, t. 2782.

T. brachystylos, Knaf in Lotos, 1854, p. 237.

T. pratense var. *pedicellatum*, Knaf ex Celakovsky, Prod. d. Flora von Boehmen, iii. (1875), p. 669.

T. pratense, forma *T. brachyanthemum β heterophyllum*, Rouy in Rouy et Foucaud, Flore de France, v. (1899), p. 120 (published as Ann. Soc. Sc. Nat. Charente-infér.).

Babington's type-specimens from Elgin, as well as others from Plymouth and Walton-on-Naze, and a type of Lange's figure have been accessible to me in the Herbarium at Cambridge; a type of Knaf's name, collected by Auerswald in Bohemia, has been seen in the Botanical Department of the British Museum of Natural History, South Kensington; at the Royal Gardens, Kew, are specimens collected at Fairmile in Surrey, at St Leonards, at Tonbridge Wells, and at Elgin, from the herbaria of Borrer and H. C. Watson, and from near Bordeaux, collected by C. des Moulins; and I have myself collected it at Hunstanton in Norfolk, Gatton Park in Surrey, Waltham St Lawrence in Berkshire and (in company with Mr G. Nicholson) near Heiligenblut in Carinthia—on each occasion a single root. All these specimens agree very closely.

The first definition of the variety *parviflorum* runs: "heads more or less stalked: calyx-teeth as long as, or longer than, the corolla," and is correct as far as it goes. Celakovsky's description is "Ähren grösstentheils gestielt; Blüthen länger oder kürzer gestielt; Deckblätter theilweise ausgebildet; Griffel kürzer als die Staubgefässe." But the following is fuller and more in accord with the specimens:—plant not robust; heads more or less stalked; bracts sometimes developed; corolla in the mature flower crumpled at the base within the calyx and not exceeding the longest of the calyx-teeth; pistil becoming foliaceous, the ovarial part linear-lanceolate, and often open above; ovules more or less aborted.

Examination of buds not ready to expand reveals no crumpling of the corolla; so that this evidently takes place in the rapid growth of the tube which precedes the expansion of the flower; and it is impossible to resist the assumption that the unusual size of the ovary and the narrowness of the mouth of the calyx are the causes of it.

Phyllody of the ovary to a greater degree than in typical *parviflorum* is not uncommon in *Trifolium pratense*; less modification in this direction I have found in a plant from Glen Clova, Forfarshire, where the peduncle and pedicels were undeveloped, but the corolla crumpled and the ovary elongated,

though seen on microscopic examination to contain two normal seeds.

Nyman[1] correctly called *T. pratense* var. *parviflorum* an abnormal condition; Penzig has given it a place in his *Pflanzen-teratologie*[2]; and Babington[3], until the publication of Lange's incorrect figures of the petals and ovary, doubted if it were more than an accidental state. I have wished here to shew how it is abnormal.

Lange found his specimens at two localities in Denmark; Ascherson[4] records it as occurring near Karlsruhe; and Magnus, who mentions the foliaceous carpels[5], had it from Memel in East Prussia. Others have named additional localities.

Less robust than the common form of *Trifolium pratense*, it resembles superficially the variety of this species called *T. microphyllum* by Lejeune in his *Flore des environs de Spa*[6], a type of which may be seen at Kew. As Lange wrote '*T. pratense* var. *microphyllum*' on the label of his specimen, I believe that he recognised this: but *T. microphyllum* (T. pratense var. microphyllum, *Lejeune and Courtois*) is not an abnormality.

Similar also in habit are plants with prolification of the flower, which I have seen from various places in Britain and have collected near Bagnères-de-Bigorre in the Pyrenees; and superficially similar in the flower-head is *T. pratense* var. *multifidum*, Seringe[7]—another abnormality, of which a type may be seen at Kew. It is abnormal from sepalody of the petals.

[1] *Conspectus Florae Europeae*, Oerebro, 1878, p. 173.
[2] Genoa, 1890, I. p. 386.
[3] *Memorials, Journal and Botanical Correspondence of C. C. Babington*, Cambridge, 1897, p. 421.
[4] *Verhandl. bot. Vereins Brandenburg*, xx. 1878, p. 110.
[5] *Ibid.* xxi. 1879, p. 80.
[6] Liège, 1811, II. p. 115. *T. microphyllum*, Desv. is *T. pratense*, but I do not know for certain in what form or variety.
[7] *in D.C. Prod.* II. (Paris, 1825), p. 195.

On the leakage of Electricity through dust-free air. By C. T. R. Wilson, M.A., Sidney Sussex College.

[*Read* 26 November 1900.]

Elster and Geitel have shown than an electrified body gradually loses its charge when freely exposed in the open air or in a room. Their results are in agreement with previous experiments of Linss. They conclude from their experiments that free ions exist in the atmosphere. The experiments described in this paper prove that ionisation can be detected in a small closed vessel containing dust-free air not exposed to any known ionising agents. To eliminate any uncertainty due to leakage through the insulating supports, the system from which the leakage was measured was fixed by means of a small bead of sulphur to a conducting rod passing through the wall of the vessel and kept at a constant potential equal to the initial potential of the leaking system. To reduce the capacity of the latter to the smallest possible amount the whole system from which the leakage was measured was reduced to a small brass strip with a narrow gold-leaf attached, the deflections of which, read by means of a microscope, served to measure the potential. With a capacity of ·73 centim. there is a nearly constant fall of potential in a vessel containing 163 c.c. of air at atmospheric pressure, amounting to 3 volts per hour, the initial voltage being 220.

The rate of leak is the same in filtered air whether the apparatus be filled and used in the laboratory (where contamination with radio-active substances might be feared) or in the country.

The leakage takes place in the dark at the same rate as in diffuse daylight.

The rate of leak is the same for positive as for negative charges.

The quantity lost per second is the same when the initial potential is 120 volts as when it is 210 volts. Such voltages produce the "saturation" current and the rate of leak may therefore be used to measure the ionisation.

The rate of leak is to a first approximation proportional to the pressure; at a pressure of 43 millims. the leakage is about one-fourteenth of that at atmospheric pressure.

If we take the value found by Prof. J. J. Thomson for the charge carried by each ion, $6{\cdot}5 \times 10^{-10}$ E.U., we can take the experiments as indicating that 20 ions of either sign are produced per second in each c.c. of air at atmospheric pressure.

Observations on the Minute Structure of the Surface Ice of Glaciers. By S. SKINNER, M.A., Christ's College. [With Plate II.]

[*Received* 20 December 1900.]

The structure of glacier ice has been the subject of much discussion, and it seems to be generally admitted that at some distance below the white surface of a glacier the ice is compact, transparent and free from fissures through which any flow of liquid can occur. Experiments in support of this are those of Huxley[1] who showed that there was no infiltration of coloured liquids poured into cups cut in the solid ice. Helmholtz[2] has shown that closed fissures can be formed in solid ice under pressure, and Tyndall[3] has shown that closed cavities can be formed by the action of radiant heat.

If a block of the solid compact ice be exposed to the light and heat of the solar radiation it becomes porous and with sufficient exposure to intense sunlight will break up into prismatic granules of various sizes. Hence above the compact ice the layer which is thus exposed during summer is permeable to coloured fluids which flow in the spaces between the granules. This porous layer may be as much as a foot in thickness depending on the exposure of the ice. The top layer of this granular portion is modified in a further manner, and the experiments described in the following paper had as their primary object the examination of the minute structure of this first layer which forms the brilliant white surface of a clean glacier.

A convenient method of examining the superficial structure of ice is that of Lohmann[4] who pours a mixture of ice-cold water and plaster of Paris on the surface. In this way a cast is formed which shows, in reverse, details which are difficult to make out by direct examination. He has applied the method in the caverns of the Hartz mountains in which ice is found during the whole year, and the casts show a remarkable regularity of structure. Similar casts have been obtained by the present author in the Schafloch[5], a limestone cavern north of Interlaken, and as these are

[1] Huxley, *Phil. Mag.*, 1857, vol. XIV. p. 241.
[2] Helmholtz, *Popular Lectures on Scientific Subjects, Ice and Glaciers.*
[3] Tyndall, *Glaciers of the Alps*, p. 353.
[4] H. Lohmann, *Das Höhleneis unter besonderer Berücksichtigung einiger Eishöhlen des Erzgebirges.* Jena, 1895.
[5] Two and a half hours above Merligen on Lake Thun; 5840 feet above sea level.

simpler than those obtained from glacier ice a description of them will now be given.

In Figure 1 a cast of a portion of a column of ice from the Schafloch is shown. Its greatest length is 19·5 centimetres and the direction of this length was vertical in the cave. As the cast is a negative all the cavities in the cast are really prominences in the ice[1]. The general appearance has given rise to the terms 'prismatic structure' used by Browne[2] and Bonney[3], and 'honey-comb ice' used by Lohmann. The crystals lie side by side with their optic axes parallel and at right angles to the surface exposed to the source of cold, just as in pond-ice the axes are normal to the surface of the water. They, therefore, present to a spectator the appearance of an irregular mosaic, like the tops of the columns in the Giant's Causeway. When crystals are formed in this manner, if the water contain any impurities, these will be collected on the outsides of the crystals where they are in contact with one another. As is well known water which contains salts or gases in solution has a lower apparent freezing-point than pure water. We may imagine the freezing of such water to occur thus: from some point six rays spring, forming a star, and on these secondary rays grow. It is pure ice which is forming, and the solution of impurities, which is becoming more concentrated, is thrust away in front. The impure layers, therefore, will solidify in the interstitial spaces last, and they will on fusion commence to melt first.

If heat be supplied to ice formed in this manner so slowly, that the distribution of temperature is almost uniform, the fusion of the interstitial matter will occur before the purer ice and the individual crystals will be separated sometimes wholly, and at other times incompletely, by furrows. This kind of melting is especially favoured in ice-caverns where the temperature during a great part of the year is only a fraction of a degree above zero. The cast (Fig. 1) is that of a case of this kind. The plaster of Paris has flowed into the interstitial spaces to different depths up to 5 millimetres. The same phenomenon is shown by artificial ice if the block be allowed to warm very slowly. It is also shown by many alloys on fusion.

On the exposed surface of many crystals in the Schafloch a special kind of marking may be seen. The hexagonal base is traversed by a number of lines which are approximately parallel.

[1] A positive may readily be made by pressing putty on the cast. But to get the visual appearance of a positive this is unnecessary, as the figure when looked at in a certain way reverses itself and appears to be a positive. I find this optical reversal especially easy with figure 2 when the figure is inverted. Viewed in a certain way a coin which is actually in relief appears to be incised.

[2] Browne, G. F., *Ice-Caves of France and Switzerland*, 1865.

[3] Bonney, *Proc. Camb. Phil. Soc.*, March 4, 1867.

The lines are known as Forel's 'Streifen' or streaks. Their explanation is, as Emden[1] remarks, the darkest point in our knowledge of ice granules, but I think some light may be thrown upon the process of their formation if a comparison be made between the photographs in his paper and those of Ewing and Rosenhain[2] in their Bakerian Lecture on 'the Crystalline Structure of Metals.' The resemblance is very striking. Compare, for instance, the markings in Emden's Figure 6 and Ewing's Figure 14. Professor Ewing has shown that "the structure of metals is crystalline, and remains crystalline when the form of the metal is altered," and that plastic deformation in metals is due "to slips on cleavage or gliding planes within each individual crystalline grain and partly (in some metals) to the production of twin crystals." There is no true shearing as in viscous deformation. I think it is probable that Forel's streaks are evidence of a similar property in ice crystals, and that they are indications on the crystal surface of slips which have happened in the ice crystal under stress. The relation of the general direction of these slip-bands to the crystal axis has not been made out in the case of metals, but it appears it may vary from crystal to crystal. Emden[3] was unable to find any relation between the axis and the direction of these lines. In my own observations I have seen them only on the hexagonal bases of the crystals, but it must be remembered that the exposed surface of the cave ice consists principally of the irregular hexagonal bases.

The ice cavern contains air saturated with moisture at a temperature very near 0° C. If such slips were formed these conditions would be suitable for their preservation.

In Figure 2 a cast is shown of the crystalline structure of a block of compact glacier ice after half an hour's exposure to the sun. This fragment of ice was cut from a portion of a blue band on the upper Grindelwald glacier opposite the place called 'Im Schlupf[4].' The block at first was quite compact, transparent with a continuous surface. After the exposure the cast shows that fusion has commenced at the joints of the crystalline granules and so the surface is traversed by furrows more or less straight. The general appearance of the cast is like a bird's-eye view of a hilly country divided by hedges into fields of varying sizes. The spaces enclosed by the furrows are the exposed faces of the granules, and it will be seen how very much they vary in size. The horizontal width of the cast is 12 centimetres. Between the

[1] R. Emden, *Ueber das Gletscherkorn, Neue Denkschriften der allgemeinen Schw. Gesell.*, xxxiii. i. p. 21.
[2] Ewing and Rosenhain, *Phil. Trans.*, 1899, A. 353.
[3] Emden, *loc. cit.*, p. 22.
[4] 1700 metres above sea level.

furrows very little can be made out in the cast, and the faces appear to be quite continuous. However, the optic axes are in all directions, and not parallel to one another as in the cave ice.

It might be expected that the actual surface of a glacier would present a similar weathering as in Figure 2 with the exception that the furrows were deeper and broader. It is, however, much more complex and a cast of the surface is represented in Figure 3. The cast is 10 centimetres high. The light has been arranged so that the ridges stand out prominently on the left-hand portion of the cast, and especially towards the upper part the granules are evident as small spaces surrounded by walls. But in the faces of these granules there are other cavities into which the plaster has flowed. These cavities have rounded edges, and appear either as cylindrical drill holes, or as elliptically-shaped recesses. The former are probably due to fragments of dust which have absorbed heat from the sun and so melted a passage for themselves into the ice granule. This action of small particles of dust can often be seen on a glacier. The latter are possibly the closed cavities of Tyndall[1] which have arisen as flat star-shaped vacuoles, which when the surface of the glacier melts are opened to the air. The drill holes and the cavities may occur anywhere on the surface of the glacier granule. They are absent in the cave ice where fusion by radiation is not possible. This forms a point by which the fusion in the two cases may be immediately distinguished. In cave ice the fusion is confined to the interstitial spaces between the crystals, whilst in the case of the glacier surface fusion occurs both in the interstitial spaces and at points over the surface of the crystal wherever dust has settled.

Two days after this cast was made the same spot on the glacier was visited and a very irregular honey-comb structure of plaster was found on the ice. This consisted of the plaster which had run down into the deeper cavities, and had not been removed with the cast on account of its frailty. The general surface of the glacier had melted and left the plaster net-work standing up. It was about an inch high, and showed the form of the glacier granules and cavities, but no further peculiarities were noticed than those already described.

We may sum the results of this paper in two conclusions:—

(1) Forel's Streaks are probably indications of slip-bands or twinning in ice crystals under stress.

(2) The porous nature of the surface ice of a glacier is due to fusion in the interstitial spaces between the granules, to the formation of dust drill holes, and to the opening to the surface of Tyndall's closed fusion cavities.

[1] Tyndall, *loc. cit.*

FIG. 1.

FIG. 2.

FIG. 3

To face page 36.

On a solar Calorimeter used in Egypt at the total solar Eclipse in 1882[1]. By J. Y. BUCHANAN, F.R.S.

[*Received* 15 December 1900.]

While engaged in discussing questions connected with the physics of the ocean, I found the want of definite knowledge of the amount of solar heat which really reaches the surface of the land or sea in a form which can be collected, measured and utilised. There was no lack of actinometrical observations, but I found it impossible from them to obtain the data that I sought. The aim of most observers has been to arrive by more or less direct means at what is known as the solar constant, that is, the quantity of heat which is received in unit time by unit surface when exposed perpendicularly to the sun's rays outside of the limits of the earth's atmosphere. For my purpose the radiation arriving at the outside of the earth's atmosphere was of no importance. What I did want to know and to measure was the amount of solar radiation which strikes the earth at the sea level and is there revealed as heat. It is the energy of this radiation which maintains the terrestrial economy. I was not satisfied with the values of it which could be deduced from the experiments which had been made with the view of ascertaining the value of the solar constant, and I determined to utilise the opportunity of a visit to Egypt in company with the expedition for observing the total eclipse of the sun in 1882 to make observations for myself on the amount of heat which could actually be collected from the solar radiation in these favourable circumstances. I determined to use a calorimeter which should depend for its indications on change of state and not on change of temperature. The ice calorimeter naturally suggested itself; but apart from the fact that in 1882 ice was not so universally procurable as it is now, the indications of the ice calorimeter are apt to be seriously modified by the condensation of moisture from the air. I therefore determined to make a steam calorimeter in which the sun's rays should be collected by a conical reflector of definite area and thrown on an axial tube which should represent the boiler.

Locality. The astronomers had fixed on a spot on the banks of the Nile close to the town of Sohag and in latitude 26° 37′ N. for the observation of the eclipse, and experience showed that it had been

[1] See *Proceedings of the Royal Society of Edinburgh* (1882), XI. 827.

very well chosen. The eclipse was total at 8.34 a.m. on the 17th May, 1882, civil reckoning. The maximum duration of totality that was expected was 70 seconds, and in fact it lasted longer than 65 seconds.

The expedition arrived on the 8th May, and I was able to begin work with the calorimeter on the 11th. As the instrument was new in every way the work of the first few days was mainly directed towards learning the manipulations and finding out and rectifying defects. The instrument worked at once much more satisfactorily than I could have expected, and the only important alteration which had to be made was to replace the original metal dome as steam space by a glass tube. This performed the functions of a gauge-glass, a steam space and a guarantee against priming. It is of course essential that nothing but condensed steam should arrive in the receiver, and with the glass steam dome this can be assured. Improvements of one kind or another were made every day up to the 15th. On the 16th, 17th and 18th experiments were carried out with the apparatus in best working order and under very favourable circumstances. They are collected in Table III. The observations made on the morning of the 17th immediately after the total phase of the eclipse are given separately in Table IV. The instrument was constructed and was taken out to Egypt for use under ordinary conditions. Its exposure during the later phases of the eclipse was not originally contemplated, yet the results are full of interest.

The observations were made on the 16th, 17th and 18th May. The sun's declination at apparent noon was 19° 8′, 19° 22′ and 19° 35′ on these days respectively. We take the mean declination for the period as 19° 22′ N.

The latitude of the station being 26° 37′ N. the mean meridian altitude of the sun was 82° 45′ = 82·75°. Table I. gives the sun's altitude and azimuth at noon and at every half-hour on each side of noon until sunset. These data were obtained graphically by measurements on the globe. It will be seen that when the sun is more than one hour from the meridian its altitude changes at the rate of about 6·5° in half an hour. When the altitude has fallen to 45° about 3¼ hours from noon, the water of the boiler has begun to invade the glass steam dome owing to the inclination which it is necessary to give the instrument in order to keep it pointed towards the sun. This does not prevent the instrument acting perfectly well, as will be seen, in Table III., from the observations made on the afternoons of the 17th and 18th, but it is necessary to watch the operation very closely. Moreover, the principal object of the observations is to find the maximum distilling effect of the sun, and this is not likely to occur when it is more than three hours either before or after noon. The period during which, if

possible, continuous observations should be made is from 9 a.m. to 3 p.m.

TABLE I.

Hour from Noon	Sun's Azimuth	Sun's Altitude	Hour from Noon	Sun's Azimuth	Sun's Altitude
0	0°	82°·75	4	95°	34°·5
0·5	46	79·5	4·5	98·5	28
1	65	74	5	102	21·5
1·5	73·5	67·5	5·5	105	15
2	81	61	6	108	9
2·5	85·5	54	6·5	111	2
3	89	48	6·65	112·5	0
3·5	93	41			

During the three days, the 16th, 17th and 18th May, the conditions were very favourable, and particularly so on the forenoon

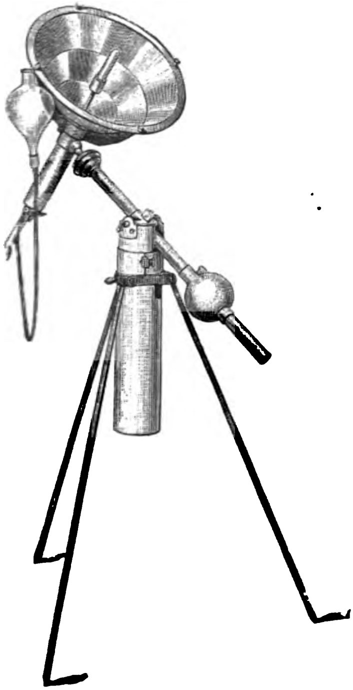


FIG. 1.

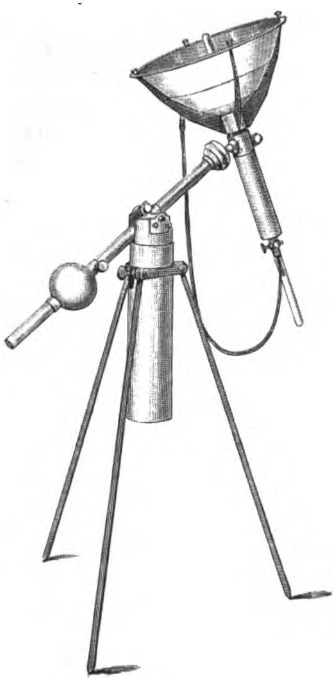


FIG. 2.

of the 18th when the sun was intensely hot and the atmosphere motionless. On none of these three days was the sun at any time obscured by cloud. The only interference with the sun's rays was by dust, with which the desert air is always charged. It made itself evident by settling upon the mirror surfaces of the reflectors, where it formed a dust of infinite fineness which could only be seen when regarded edgewise. The results obtained on the forenoon of the 18th are to be taken as the best.

Figs. 1 and 2 are photographic representations of the instrument showing its general appearance and arrangement. Fig. 3 gives a section of the calorimeter, and Fig. 4 a perspective view.

Construction of the Calorimeter. Fig. 3 is a principal section of the instrument by the plane which contains its axis OP and that of the earth QS. The dimensions of the parts are most easily specified by their projections on the axis OP and on a line at right angles to it; but as the sections at right angles to OP are all circular it will be sufficient specification of the section of the instrument at any point, say B, to give the distance LB on the axis from one extremity L of the steam tube to B, and the radii of which the projections on the axis are the point B. These radii would be, in order from the axis outwards, $\frac{1}{8}$, $\frac{1}{4}$, 3 and 4 inches.

In the following table the first line contains the points on the axis, the second contains their distance from L, the lower extremity of the steam tube, the third contains the radius of the innermost circle, the section of the steam tube, and the following lines contain the radii of the other circles in ascending order.

Points on the Axis ...	L	K	C	B	A	E	F	G	H	
Distance of points from L	0	4	16	17	19	$19\frac{1}{2}$	$20\frac{1}{2}$	22	$22\frac{1}{2}$	inches
Radii of the circular sections of which each point is the centre and the projection	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$		„
		$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{15}{32}$	$\frac{15}{32}$		„
		1	1	3	5	$\frac{1}{2}$	$\frac{1}{2}$	$6\frac{3}{4}$		„
			$2\frac{3}{4}$	4	$5\frac{1}{2}$			7		„

The measurements are given in British units, because these were used in its construction. The construction of the reflector will be described later. The mirrors are carried on arms of sheet brass which spring from a piece of brass tube which fits telescopically over the condenser tube. Their outer extremities are kept in position by being fixed to a flat ring of sheet brass, $\frac{1}{4}$-inch wide. The mirrors are made each of a band of sheet copper properly shaped, and bent round until the edges abut, when they are soldered. The internal conical surface is then electroplated with silver, using such technical precautions as shall insure the production of a bright mirror surface.

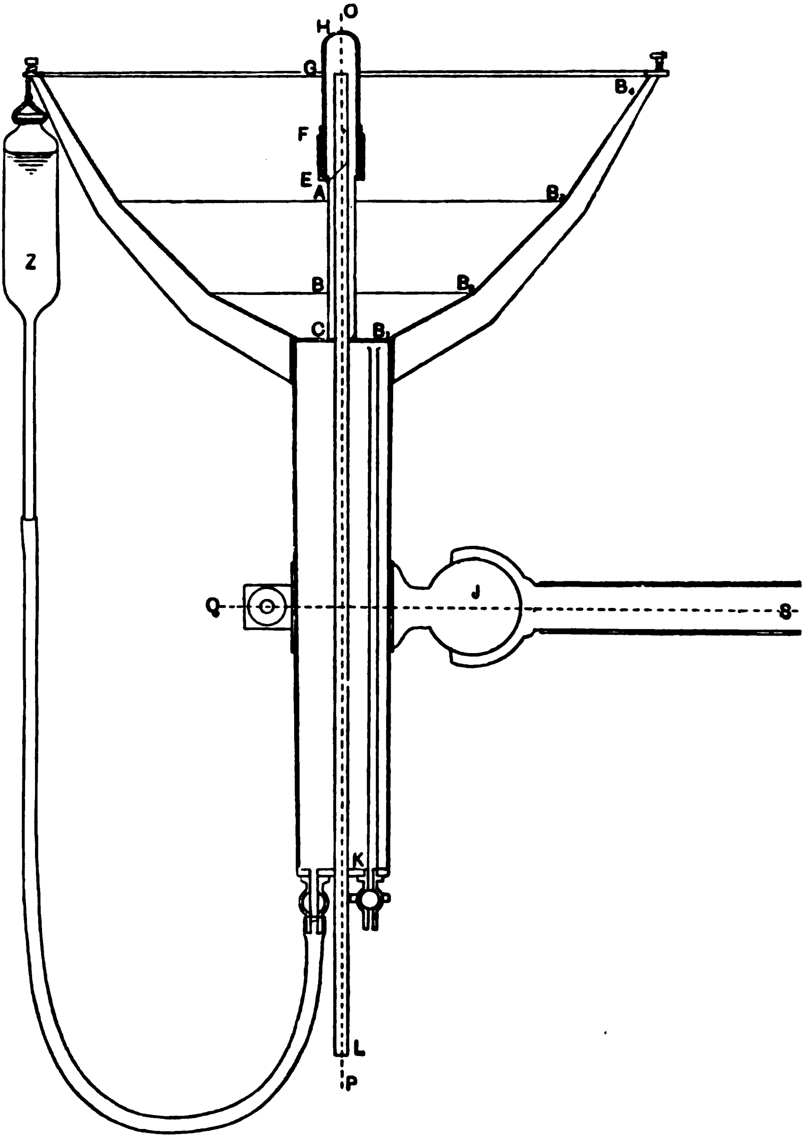


Fig. 3. Calorimeter. Principal Section.

The main condenser is the tube KC, 12 inches long and 2 inches in diameter. Out of it at the top springs the boiler tube C, B, A, E, of silver and $\frac{1}{2}$-inch in diameter. At E, and from E to F its diameter is 1 inch, and this carries the steam dome, which is a glass tube closed at one end and inserted into the part EF, where it is fixed with a screw collar and washer. The steam tube passes axially through the whole instrument, terminating just inside of the glass dome. The steam condensed in it runs out at the lower extremity L and is received in a graduated tube in which it is measured or weighed. The glass reservoir Z, which is shown hanging from the outside rim of the reflector, is connected by an india-rubber tube with the bottom of the condenser and the instrument becomes a U-tube, of which the reservoir and india-rubber connection are one limb and the condenser and boiler the other. The instrument is thus easily filled with water and the height at which it stands in the space EF is regulated by means of Z.

When the instrument is going to be set in action it is pointed axially to the sun. When in this position, the tube EF throws a strong circular shadow on the top of the main condenser CB, and concentric with it. With the rotation of the earth the axis moves away from the direction of the sun and the shadow becomes eccentric. The appearance of eccentricity strikes the eye at once, and it is rectified by a slight motion of the instrument round its polar axis. The instrument requires adjustment every two or three minutes.

When pointing truly to the sun all the rays which strike the reflector are reflected on the length AB of the axis. But the boiler tube having a radius of $\frac{1}{4}$-inch intervenes and receives these rays on its blackened surface. The rays reflected from the inner extremity of the inner mirror are reflected on a part of the boiler tube a little below the line BB_2, and those reflected from the outer extremity of the outer mirror are reflected on a part of the boiler a little above the line AB_2. This is due in both cases to parallax.

When the sun's rays strike the surface of the boiler, those that are not thrown back again are absorbed by its blackened surface and passed by conduction through the metal to the water which occupies the space round the steam tube. When everything was at the temperature of the air, and the instrument was pointed to the sun at 2 p.m., the water boiled in 40 seconds, and it continued to boil so long as the instrument truly followed the sun and as the sun was not obscured. This operation had to be stopped when, in order to follow the sun, the instrument had to be inclined at such an angle that the water of the boiler began to

trespass too far into the glass dome. The greatest meridian altitude of the sun was 83°, and it was found inconvenient to follow the sun to altitudes less than 45°, so that the instrument was never used in a truly vertical position. This has an advantage which will be appreciated by the chemist, who always inclines a test-tube when he is going to boil a liquid in it. The boiling proceeded with perfect regularity even when the sun was at its hottest, as on the forenoon of the 18th May; and with the glass dome as steam space everything could be followed minutely. The steam developed in the boiler rises into the dome, from which it finds exit through the inside steam tube *GL*. In it the steam passes at least as far as *B* uncondensed, because the temperature of the water boiling outside is slightly higher than its own. But immediately it passes *B* it is surrounded by water which at first is colder than itself and it is condensed. In this process the steam gives out its latent heat and raises the temperature of the water outside in the condenser correspondingly, and the water produced from the steam runs down the tube and is caught in the receiver. When steam is in presence of water there is no delay in condensation so soon as the temperature of the water is the smallest fraction of a degree below the temperature of saturation. Therefore so soon as the water which moistens the inside of the steam tube has been cooled at all, it instantly condenses steam sufficient in amount to raise its temperature to that of saturation. The result is that the actual condensation of the steam takes place at the upper part of the condenser and immediately below the boiling space. As the instrument is to all intents and purposes motionless and no circulation of water is maintained in it, the hot water remains at the top of the condenser and from it hot feed is supplied to the boiler. While there comes to be a layer of considerable thickness of water at or very near the boiling-point at the top of the main condenser, that part of this water which finds itself forced into the annular space *CB*, if it is not actually at the boiling-point when it enters at *C*, as its inner surface is heated by the full supply of steam as it leaves the boiling water, it cannot fail to attain the boiling temperature before it reaches *B*. Therefore *when the instrument has settled down into steady working, the whole of the heat which reaches the water from the sun is used in transforming water at its boiling-point into steam of the same temperature.* It is essential that the distillation be kept running continuously and the water produced in successive intervals of time weighed or measured. If the meteorological conditions are such that the boiling is interrupted, then it is of no use attempting to make observations, as they would have no value. The reason why I thought it so important to have the apparatus for use with the expedition was that the climate of Egypt in

the month of May is very dry and hot and the sky usually cloudless, while the sun also attains a very considerable meridian altitude. Further, of all the results obtained, the one of greatest importance is the maximum. It is necessarily lower than the possible maximum with a perfect instrument under perfect meteorological conditions. But in order to know that we have the maximum we must make many observations, because the conditions that are apparently the most advantageous are not always so in reality.

It will be seen that besides the tap at the bottom of the condenser which communicates by the india-rubber tube with the reservoir there is one which communicates with the top of the condenser; it was intended for the removal of the hot water as it was replaced by colder water at the bottom. It was found better to allow the hot water to accumulate at the top, as has been described, and, when the heat threatened to pass too far down, to change the whole of the water and start afresh. As the temperature of the water for an inch or two at the top is at, or nearly at boiling temperature it loses heat by radiation and convection at a much greater rate than if the water of the condenser were thoroughly mixed and assumed an average temperature. This is an important feature. It is however better with a reflector having the condensing power of the one used to have a larger condenser not only in order to hold more water and so render less attention necessary, but for the mere mechanical purpose of balancing the weight of the reflector. In the instrument used the two were much too nearly of a weight. In designing another, I should make the diameter of the condenser 3 inches, and its length 18 inches. The other dimensions seemed to be in every way suitable. The condensing power of the reflector depends on the ratio of its effective area to the focal surface of the boiler tube. For the same length of focal line the heating surface varies with the circumference or the diameter of the tube, so that if the diameter of the boiler tube were increased from half-an-inch to three-quarters of an inch the condensation of rays would be 32 instead of 48 fold. From experience during the latter part of the eclipse this intensity would be insufficient and it would be necessary to increase the area of the reflector in about the same ratio, we should then be able to collect water at the rate of over 2 grammes per minute, which would be an advantage. The steam tube is large enough for a much greater rate of distillation. But as we can at will alter the diameter of the boiler tube, or its length, or the collecting area of the reflector, the variations that we can make are endless and it is probable that amongst the number of them a combination will be found which is more efficient than the first one that was tried.

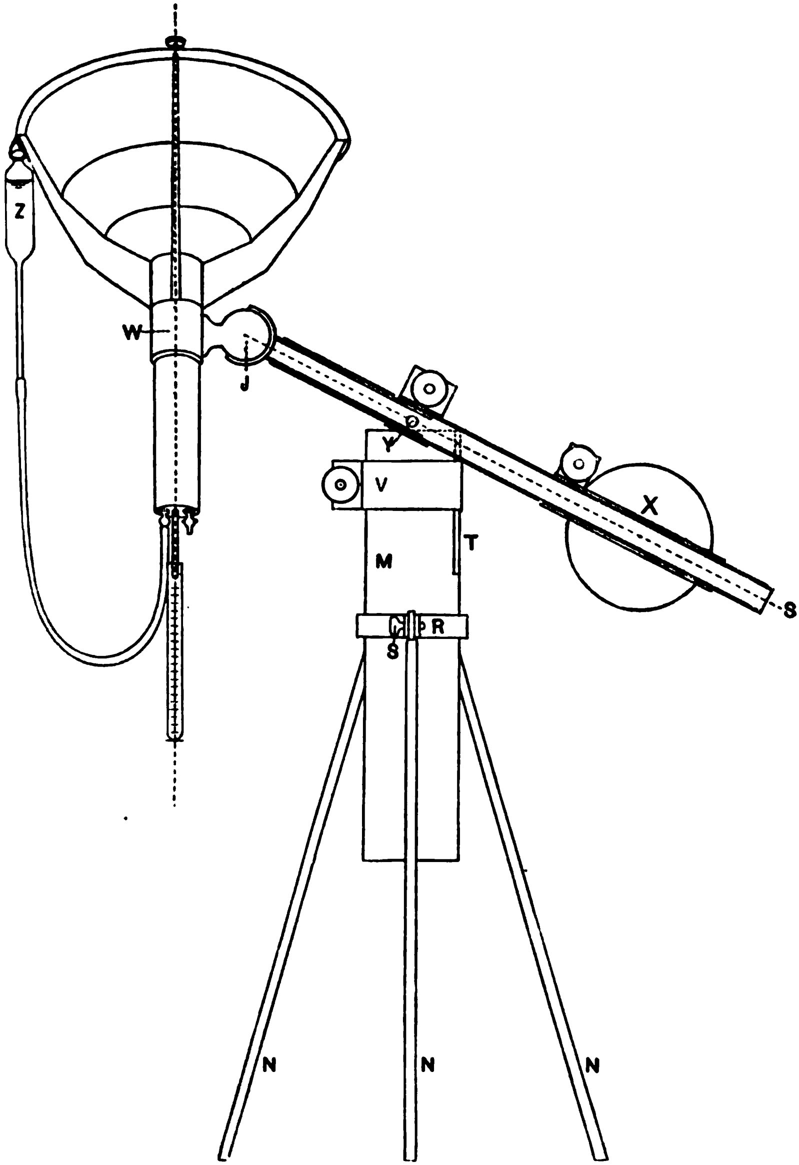


FIG. 4. CALORIMETER ON EQUATORIAL MOUNTING. PERSPECTIVE VIEW.

The *Equatorial Mounting* is shown in Fig. 4. The main or central part is a piece of stout brass tube M, 4 inches in diameter and 18 inches long. It is supported on an iron tripod NN by an iron ring R in three segments which are pinched together by screws SS. From the top edge of the tube for six inches downwards, a slot T is cut of rather over an inch clearance, and a collar V consisting of a length of 2 inches of tube that telescopes over the central tube, slips up and down it and can be clamped in any position. The slot is intended to receive the polar axis JS when adjusted for latitude, the collar being clamped in the right place and the weight X, which is much heavier than the calorimeter, keeps the polar axis resting firmly on the collar. The polar axis which is a tube of 1 inch diameter is pivoted round a horizontal axis Y which is also a piece of brass tube, working in bearings on the top of the central tube When the tripod has been set up so that the central tube is vertical the polar axis is adjusted for latitude by a quadrant or protractor, or if the pole-star is available, it is brought to be visible through the tube which forms the polar axis. The calorimeter is held by a collar W which surrounds it and is clamped on the condenser tube. This collar is attached to a ball and socket joint J which is carried by a piece of tube which fits telescopically into the polar axis. The ball and socket joint was found to be the simplest means of giving a motion for adjusting the calorimeter for declination. We have thus a form of equatorial mounting which is simple, effective, and cheap, as it is almost entirely made out of brass tubes.

Construction of the Reflector. In designing the reflector actually used, the following determining conditions were adopted.—Length of focal line to be 2 inches; angle of the middle mirror to be 45° and its upper rim to have a radius of 5 inches. With these data the 45° mirror can be completed at once and the specifications of the outer and inner mirrors follow by a simple graphical construction. The physical principle involved is that the angles at which rays strike and are reflected from a mirror surface are equal. The construction is shown in the diagram Fig. 5. On the straight line OP which represents the axis of the instrument, lay off the length $AB = 2$ inches; this is the focal line. Through A and at right angles to AB draw Ax, and on it make $AB_3 = 5$ inches. Through B draw BB_2 parallel to AB_3 and make $BB_2 = 3$ inches. Join B_2B_3. B_2B_3 is obviously the line representing the principal section of the mirror inclined at 45° to the axis. Its length $B_2B_3 = \sqrt{8} = 2{\cdot}83$ inches. If the line B_2B_3 be continued until it cuts AB produced in B_2', then B_3B_2' is the generating line of the complete cone of which the 45° mirror is a portion. Its length is obviously $\sqrt{50} = 7{\cdot}07$ inches. The flat band which, when bent

round until its edges abut, forms the reflecting surface of the 45° mirror is specified by the following construction.

On a sheet of paper, or on the silvered sheet of metal, describe from the same centre two circles with radii of 7·07 and

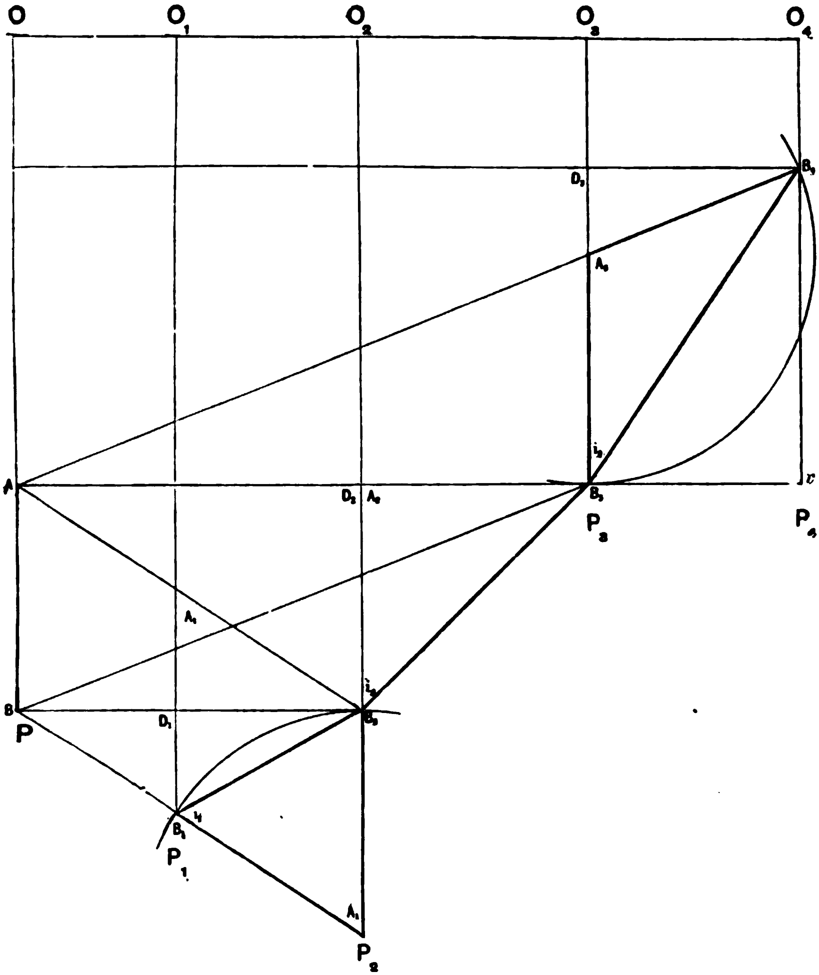


Fig. 5. Geometrical construction of Reflector used.

$7{\cdot}07 - 2{\cdot}83 = 4{\cdot}24$ inches respectively. The circumference of the greater circle is 44·44 inches. The upper rim of the 45° reflector has a radius of 5 inches, therefore its circumference

is 31·42 inches. The difference between these two circumferences is 13·02 inches. If 44·44 represent 360° of an arc then 13·02 represents 105·5°. From the common centre of the two circles draw to the outer circumference two radii inclined to one another at an angle of 105·5°. The construction is then complete. If it has been carried out on the sheet of metal from which the actual mirror is to be constructed, we first cut out the disc of 7·07 inches radius; we then apply the shears to the point where one of the radii cuts the circumference; we cut along it until we reach the inner circumference, we then cut round this circumference along an arc of 254·5°, when we arrive at its inner section with the second radius, which is then followed until the outer circumference is reached. The annular disc, less the sector of 105·5° amplitude, which remains, is the metal band which, when bent round until its edges abut, forms the 45° mirror.

Outer Mirror. Through B_3 draw O_3P_3 parallel to OP and on it lay off $B_3A_3 = BA = 2$ inches. From A_3 as centre, at the distance A_3B_3 describe a circular arc. Join AA_3 and produce the line AA_3 till it cuts the arc in B_4. Join B_3B_4. B_3B_4 is the line of section of the outer mirror. For, having in view the properties of triangles and of parallel lines, it is clear that the lines O_4B_4 and B_4A make equal angles with the line B_3B_4. But O_4B_4 is the direction of the incident ray at B_4; therefore B_4A is the direction of the reflected ray, and the ray which strikes the outer rim of the mirror is reflected upon the upper extremity of the focal line. In the same way it is evident that the incident ray O_3B_3, which strikes the inner rim of the mirror, is reflected along the line B_3B and falls upon B, the lower extremity of the focal line. Consequently parallel rays which strike the mirror in points between B_4 and B_3 are reflected on AB and strike it in points between A and B which are homologous as regards position with the points in the mirror between B_4 and B_3 which are struck by the primary rays.

In the isosceles triangle $B_3A_3B_4$ the angle A_3 is equal to the angle A_3AB, therefore

$$\tan A_3 = \frac{B_3A_3}{AB_3} = -\frac{AB}{AB_3} = -\tfrac{2}{5} = -0{\cdot}4;$$

therefore $$A_3 = 111°\,48';$$

and the angle at the base

$$i = \tfrac{1}{2}(180° - A_3) = 34°\,6'.$$

Further, the base

$$B_3B_4 = 2 \times AB \cos i = 3{\cdot}31 \text{ inches}.$$

Therefore the width (m_3) of the outer mirror is 3·31 inches, and it is inclined to the axis at an angle of 34° 6′.

The radius of the upper rim of the outer mirror is

$$AB_3 + D_3B_4 = AB_3 + B_3B_4 \sin i = 5 + 3{\cdot}31 \sin 34^\circ 6' = 6{\cdot}85 \text{ inches.}$$

Further, if the line B_4B_3 be continued till it cuts the axis in B_3', then B_4B_3' is the generating line of the complete cone of which the outer mirror is a portion.

Its length is obviously

$$\frac{6{\cdot}85}{\sin 34^\circ 6'} = 12{\cdot}23 \text{ inches.}$$

The width of the mirror is 3·31 inches. Therefore the annular band of silvered metal has an outer radius of 12·23 inches and an inner radius of 8·92 inches, and the sector to be removed from it has an amplitude of

$$360^\circ \frac{12{\cdot}23 - 6{\cdot}85}{12{\cdot}23} = 158{\cdot}5^\circ.$$

Inner Mirror. Through B_2 draw O_2P_2, and on it lay off, below the line BB_2, the length $B_2A_1 = BA = 2$ inches. From A_1 at distance A_1B_2 describe a circular arc. Join A_1B. This line cuts the arc in B_1. Join B_1B_2. B_1B_2 is evidently the line of section of the inner mirror. The demonstration is the same as in the case of the outer mirror.

Also $$\tan A_1 = \frac{BB_2}{AB} = \tfrac{3}{2} = 1{\cdot}5\,;$$

$$\therefore\ A_1 = 56^\circ\, 19',$$

whence $$i_1 = 61^\circ\, 51',$$

and $$m_1 = 2\,.\,AB \cos i_1 = 1{\cdot}88 \text{ inches.}$$

Therefore the inner mirror has a width of 1·88 inches and is inclined to the axis at an angle of 61° 51′.

Further, if B_2B_1 be continued to cut the axis in B_1', B_2B_1' is the length of the generating line of the complete cone of which the mirror forms a part, and it is the greater radius of the annular disc of silvered metal out of which the band is to be cut. Its length is

$$\frac{BB_2}{\sin 61^\circ\, 51'} = 3{\cdot}40 \text{ inches.}$$

The inner radius of the annular disc is 1·52 inches, and the amplitude of the sector to be removed is

$$360° \frac{3·4-3}{3·4} = 42·3°.$$

The numerical data just worked out and relating to the reflector used are collected in the following table:

No. of Mirror		1	2	3
Description of Mirror		Inner	Middle	Outer
Inclination to the Axis ...	i	61° 51′	45°	34° 6′
Inner Radius ...... inches		1·34	3·0	5·0
Outer Radius ...... ,,	R	3·0	5·0	6·85
Width of Mirror... ,,	m	1·88	2·83	3·31
Outer Radius of flat band ...... ,,	M	3·40	7·07	12·23
Inner do. .... ,,	$M-m$	1·52	4·24	8·92
Amplitude of Sector to be removed	$360° \frac{M-R}{M}$	42·3°	105·5°	158·5°

The condition that one of the mirrors should be inclined at an angle of 45° to the axis was suggested by the fact that this is the angle of greatest efficiency and by the consideration that it is an angle which is familiar in mechanical workshops, and is on that account perhaps more likely to be laid off accurately than another. There is, however, no particular advantage in making this restriction, because in designing a series of mirrors for a reflector one of them is sure to be inclined at an angle of nearly 45° and to have an efficiency which is sensibly the same as if the angle were 45°.

The general problem, to construct the principal section of a reflector consisting of a series of conical mirrors when the direction and length of the common focal line are given and the position relatively to a point on this line, of a point occupying a definite position in the line of section of one of the mirrors is simple. The diagram, Fig. 6, shows a construction of this kind.

The line OP is the axis of the reflector, it is also the direction of the incident rays when the instrument is in use. Make it the

axis y of rectangular coordinates with the upper extremity of the focal line, A, as origin: ordinates measured in the direction AB

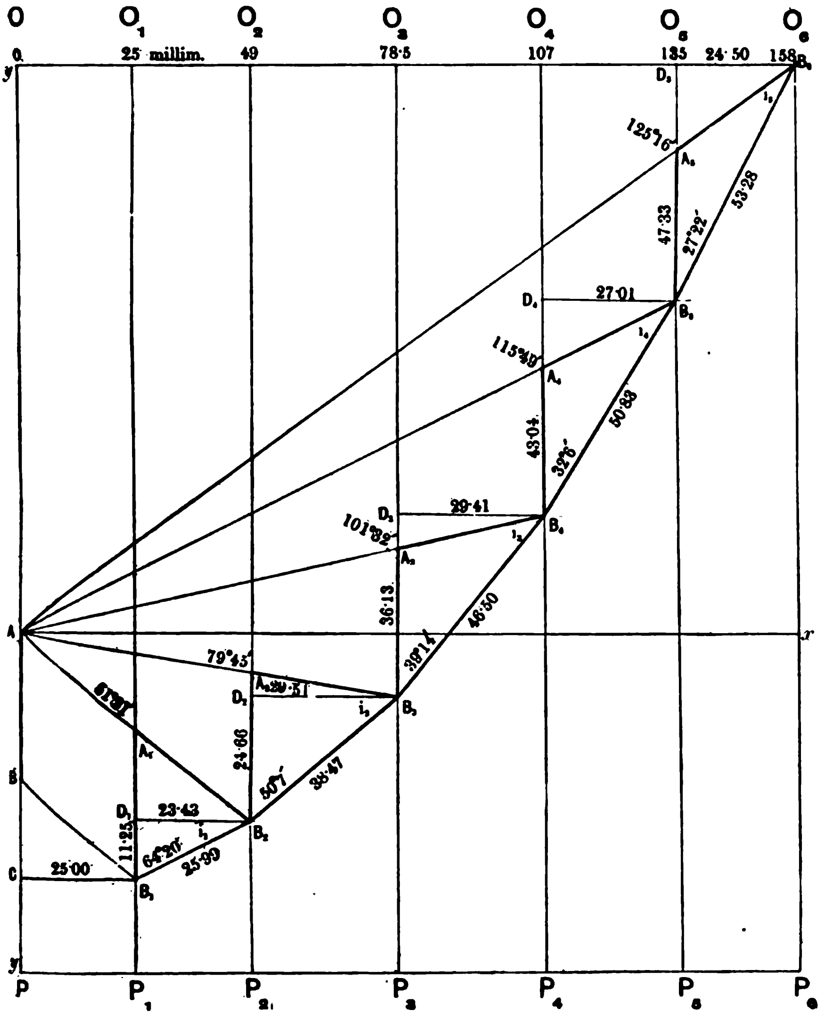


FIG. 6. GEOMETRICAL CONSTRUCTION OF REFLECTOR, when the position of a point on one of the mirrors, and the position and length of the focal line, are given.

are to be reckoned positive, those measured in the reverse direction are to be reckoned negative.

Abscissae are to be measured on a line at right angles to OP, and they are positive when measured to the right.

Join BB_1; and through B_1 draw a line O_1P_1 parallel to OP, and on it lay off $B_1A_1 = BA$.

Join AA_1 and produce it to a point B_2, so that $A_1B_2 = AB$.

Join B_1B_2; then B_1B_2 is the line which represents in section the innermost mirror.

Through B_2 draw a straight line O_2P_2 parallel to OP, and on it lay off the length $B_2A_2 = BA$.

Join AA_2 and produce it to a point B_3, so that $A_2B_3 = AB$.

Join B_2B_3; then B_2B_3 is the line which represents in section the second mirror of the series.

Through B_3 draw O_3P_3 parallel to OP, and on it lay off $B_3A_3 = BA$.

Join AA_3 and produce it to a point B_4, so that $A_3B_4 = AB$.

Join B_3B_4; then B_3B_4 is the line which represents in section the third mirror in the series; and so on.

It is evident from the properties of parallel lines that the angle which the incident ray makes with the outer extremity of any one of these lines is equal to the angle made with it by the line connecting that point with the upper extremity of the focal line. Therefore all the rays parallel to the axis which strike the outer extremity of a line of section are reflected upon A, the upper extremity of the focal line. In the same way all the rays parallel to the axis which strike the inner extremity of a line of section are reflected upon B the lower extremity of the focal line. Consequently all the rays parallel to the axis which fall upon intermediate points in the line of section are reflected upon the corresponding points between A and B on the focal line. Therefore all the rays parallel to the axis which strike the reflector are reflected and condensed on the focal line AB.

If the graphic construction is effected on the natural scale, all the measurements, both linear and angular, can be taken from it directly with sufficient exactness to enable the reflector to be constructed. On the other hand, the geometrical construction is so simple that there is no difficulty in arriving at all the values by calculation. It will be apparent from the diagram (Fig. 6) that most of the elements of each section are contained in an isosceles triangle $A_nB_nB_{n+1}$. In it the angle A at the apex of one triangle is derived from the data of the previous triangle. The angle of inclination to the axis of the mirror is

$$i = \tfrac{1}{2}(180^\circ - A),$$

and the length of the base or width of the mirror is

$$m = 2AB\cos i.$$

To find the succeeding values of A let us take A_1 and A_2. A_1 is evidently equal to CBB_1 and to CAA_1; therefore

$$\tan A_1 = \frac{B_1C}{CB},$$

and these are known from the coordinates of the given point B_1, therefore A_1 is known and the values of i_1 and m_1 follow as above. Through B_2 draw B_2D_1 at right angles to A_1B_1 and cutting it at D_1.

Then $$D_1B_2 = m_1 \sin i_1,$$

and $$D_1B_1 = m_1 \cos i_1.$$

Then $$\tan A_2 = \frac{CB_1 + D_1B_2}{AC - (D_1B_1 + A_2B_2)}$$

$$= \frac{CB_1 + m_1 \sin i_1}{AC - AB - m_1 \cos i_1},$$

whence A_2 is found.

The values of i_2 and m_2 follow as before, and we have

$$\tan A_3 = \frac{CB_1 + m_1 \sin i_1 + m_2 \sin i_2}{AC - AB - m_1 \cos i_1 - m_2 \cos i_2},$$

whence A_3 is found; and by thus proceeding step by step the elements of all the mirrors in the series are easily obtained.

The diagram (Fig. 6) was actually constructed on the following numerical data:—

$$AB = 30 \text{ millimetres}, \ BC = 20 \text{ millimetres},$$

and $$CB_1 = 25 \text{ millimetres};$$

and the values of the different elements as calculated are collected in the following table. It is to be remembered that $\tan A$ is always the quotient $\frac{x}{y}$ of the coordinates of the point A, and that the values of y change sign at the origin. Thus for A_1,

$$y_1 = 50 - 30 = 20;$$

for A_2, $$y_2 = 50 - 11{\cdot}25 - 30 = 8{\cdot}75;$$

for A_3, $$y_3 = 50 - 11{\cdot}25 - 24{\cdot}66 - 30 = -15{\cdot}91;$$

and so on.

In order to make the table complete the specifications of the metal bands for the mirrors are added.

SECTIONS OF MIRRORS.

No. of Mirror	n	0	1	2	3	4	5
Angle at apex of isosceles triangle	A		51° 21′	79° 45′	101° 32′	115° 49′	125° 16′
Inclination of mirror to axis	i		64° 20′	50° 7′	39° 14′	32° 6′	27° 22′
Width of mirror (millim.)	m		25·99	38·47	46·50	50·83	53·28
Projection of mirror on axis of x (mm.)	a	25·00	23·43	29·51	29·41	27·01	24·50
	$a_0 + a_1 + \ldots a_n = \Sigma a = x$	25·00	48·43	77·94	107·35	134·36	158·86
Projection of mirror on axis of y (mm.)	b	0	11·25	24·66	36·13	43·04	47·33
	Σb	0	11·25	35·91	72·04	115·08	162·41
	$20 - \Sigma b = y$	20·00	8·75	−15·91	−52·04	−95·08	−142·41
	$\frac{x}{y} = \tan A_{n+1}$	1·250	5·535	− 4·902	− 2·065	− 1·414	− 1·115
	A_{n+1}	51° 21′	79° 45′	101° 32′	115° 49′	125° 16′	131° 53′

ANNULAR BANDS FOR MIRRORS.

Outer radius of band	$\frac{x}{\sin i} = M$		53·80	101·56	169·72	252·84	345·33
Inner radius of band	$M - m$		25·99	38·47	46·50	50·83	53·28
	$M - x$		5·37	23·62	62·37	118·48	186·48
Amplitude of Sector to be removed	$\frac{360°(M-x)}{M} =$		35° 56′	83° 39′	125° 55′	168° 43′	194° 22′

TABLE II. METEOROLOGICAL OBSERVATIONS ON 16TH, 17TH, AND 18TH MAY, 1882, AT SOHAG IN LAT. 26° 37′ N.

	Time	Temperature of the air		Evaporation		
		Dry bulb, ° Fahr.	Wet bulb, ° Fahr.	Temp. of water, ° Fahr.	Volume of water, c.c.	Evaporation, millimetres
1882 May 15	6 0 P.	82·5	°	80·0	400	
„ 16	5 30 A.	63·0		51·0	345	2·26
	5 45 „	63·0		58·0	400	
	8 45 „	87·0		75·5		
	10 0 „	90·0		81·0		
	11 0 „	92·0	63·5	81·0		
	12 0 „	91·5	63·2	82·0		
	2 0 P.	95·0	62·5	82·5		
	3 30 „	95·0	62·5	79·0		
	4 0 „	91·5	60·2	76·5		
	4 30 „	90·0	59·5	73·5		
	5 0 „	89·0	58·2	67·0		
	5 30 „	87·0	58·0			
	6 0 „	85·3	57·3	63·5	145	11·09
	6 0 „	85·3	57·3	80·0	400	
„ 17	5 40 A.	65·0	50·0	51·5	346	2·25
	6 10 „	67·0	52·0	60·0	400	
	8 0 „	74·0	56·5	69·0		
	8 10 „	73·0	56·0	68·0		
	8 16 „	73·0	57·1			
	8 21 „	72·5	56·0			
	8 26 „	71·2	55·5			
	8 30 „	71·0	55·0			
	9 0 „	77·0	58·5	68·5		
	10 0 „	86·5	64·5	81·0		
	11 0 „	91·0	66·0	83·5		
	2 30 P.	94·0	66·2	81·0		
	3 30 „	94·0	64·0	78·0		
	4 30 „	91·0	60·2	73·5		
	6 0 „	86·0	56·5	62·0	158	10·52
	6 20 „			80·0	400	
„ 18	6 20 A.	60·5	49·5	55·0	333	2·75
	6 30 „	60·5	49·5		400	
	7 20 „	73·5	54·0	69·5		
	9 0 „	85·0	61·5	82·0		
	10 0 „	92·0	65·5	87·0		
	11 0 „	98·5	64·0	89·0		
	12 0 „	95·0	66·1	88·5		
	2 0 P.	105·0	65·0	85·0		
	3 0 „	103·5	64·5	81·5		
	4 0 „	97·6	64·0	78·0		
	4 35 „			77·0		
	6 15 „	96·5	60·2	68·0	140	11·30

Meteorological Observations and Notes. The climate at Sohag is a desert climate tempered by the influence of the Nile. This influence extends only a very short distance from the banks of the river. As the population is confined to the banks of the river its benefits are enjoyed by the whole population. During the few days in May that the expedition sojourned at Sohag the sun attained a meridian altitude of roughly 83°, so that its power differed very little from that of a vertical sun. The prevailing wind is from the North which gives a freshness to the atmosphere while it also enables the countless sailing craft on the Nile to navigate its waters against its not insignificant current.

While occupied with the calorimeter I made observations on the temperature of the air using both the wet and dry bulb thermometers, and I also measured the evaporation by night and by day of water exposed freely in a plate raised about 6 inches above the ground. A glance at Table II. or Fig. 7 which contain these results will indicate better than any description the nature of the climate in that part of Egypt in May.

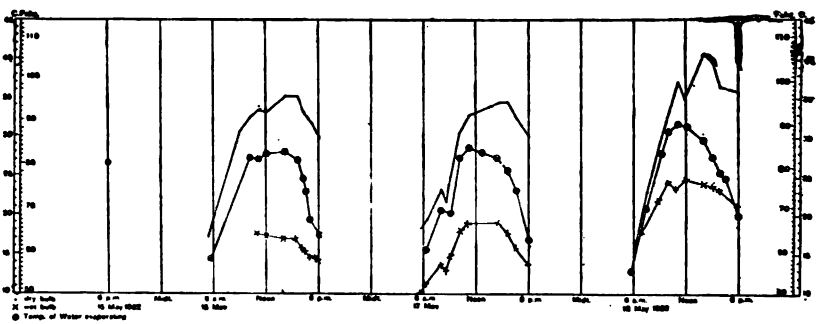


FIG. 7. METEOROLOGICAL OBSERVATIONS ON THE 16TH, 17TH, AND 18TH MAY, 1882.

The latitude of the station was 26° 37′ N. so that, on the ocean, it would be in the heart of the Trade Wind. In fact the Trade Wind regions of the ocean are the desert regions of the sea. The water at the surface is there drier than anywhere else, that is, a given volume of it contains more salt and less water than is to be found either on the equatorial or on the polar side of the region. The northerly wind on the Nile is the Trade Wind blowing from colder to hotter latitudes and always increasing its evaporative power. Thanks to this power the temperature of the Nile is lower than it would otherwise be. I took its temperature frequently at all hours of the day, it varied only between 74·5° and 76° Fahr., while the temperature of the air above it varied from 50° to 105° Fahr.; the variations of the wet

bulb thermometer are much less, namely from 49·5° to 66·2° Fahr. The temperature of the water of the Nile is very nearly the mean of the maximum and minimum temperatures of the air above it. These means were on the

14th,	15th,	16th,	17th,	18th,	mean
77·5°,	74·5°,	79·0°,	79·5°,	82·75°,	78·65°.

The temperature of the Nile is a little below this mean. The large range of temperature in the air is partly due to the cooling effect of the evaporation from the surface of the water.

The evaporation experiments were made on water contained in a deep plate. It contained 400 cubic centimetres and exposed a free surface of 243 sq. centimetres when full, and 230 sq. centimetres when nearly empty. It was set upon a tin cone which raised it about 6 inches above the ground. The difference of the effect of the sun upon the water and upon the sand close to it was well shown on the 16th at 2 p.m. when the water in the plate had a temperature of 82·5° F. while the temperature of the sand was 134° F., making a difference of more than 50° F.

The following notes of the weather were made at the time and are of use when taken in connection with the observations of the calorimeter.

16th May.—The sun rose in a cloudless sky and there was a very light wind from the west. As the morning wore on, it came round more and more to the north and freshened slightly. At 10 a.m. the wind seemed freshening and came in gusts, retarding distillation. In the afternoon it was calm.

17th May.—The day of the eclipse, the sun rose in a cloudless sky and the inhabitants of Sohag had already begun to collect on the banks of the Nile where they remained until the eclipse was over. The temperature recorded as at 8.30 a.m. was really observed 50 seconds after totality began, and I had no difficulty in reading the thermometer although all the principal stars were shining brightly, along with an unsuspected comet which appeared with totality, between two and three sun's diameters from the darkened disc and with a slightly curved tail quite as long as the sun's diameter. This comet had not been detected before and I understand that it was never seen afterwards. It was a very striking feature of the eclipse to those whose occupations enabled them to look at it. I made a sketch immediately totality was over. Perhaps the most impressive period of the eclipse is during the two or three minutes that precede the total phase. Until a very large proportion of the sun's disc has been obscured the decrease of light causes no remark, especially from people who are accustomed to climates where clouds are more common than sunshine. But when the time comes that every minute and indeed every second alters by many per cent. the

visible radiating area of the sun and when at last this area is halved in one second and becomes nothing in the next, the effect which this sudden extinction of the sun produces, is a very profound one. And this not on man only, but also on the beasts. There were some turkeys in the camp, and they went about as usual until the final phase above indicated began, when they showed every symptom of alarm. When the sun reappears his light increases as rapidly as it disappeared, and five minutes after totality all interest in the eclipse has gone. What struck me most besides the comet, were two so-called protuberances. I say *so-called* because to the naked eye they look much more like indentations or notches in the moon's disc and coloured red. This is a subjective effect and due to the same cause as the "black drop" in the case of the transit of Venus.

Of all the natural phenomena which I have had the opportunity of witnessing there is none which produces so powerful an impression as a total eclipse of the sun. In connection with this it may be recalled that the eclipse of 17th May, 1882, repeats itself after 19 years on the 17th May, 1901, with this important advantage, that in place of seventy seconds the maximum duration of totality will be six minutes and a half, and it will occur very nearly at noon at stations in Sumatra and Borneo.

The 18th May was the hottest day experienced. Perfect calm reigned until 2 p.m. when a breeze began to blow up the Nile and continued throughout the afternoon although it was never very strong. During this forenoon the maximum results were obtained with the calorimeter and the temperature of the air reached its maximum 105° F. at 2 p.m. It will be noticed that the temperature of the wet bulb thermometer was only 65°, or 40° F. below the dry bulb; the air was of extraordinary dryness. One effect of a climate such as this where great dryness is associated with very high temperature, is that, although perspiration is abundant, the skin is never moist, indeed it is so dry that it has a tendency to crack. Another remarkable subjective effect of high air temperatures such as those of the afternoon of the 18th is the notice given when the temperature of the air passes from below that of the human body to above it. It is a matter of common experience that in preparing a warm bath, very slight differences of temperature can be appreciated by the hand when the water is at, or about the temperature of the human body. With air the conditions are different; the capacity for heat of all the air that can at any moment touch exposed portions of the body is very small and produces no noticeable effect. But although it cannot do so directly it can do so vicariously for instance through the metal rim of a pair of spectacles. The calorimetric work described in this paper necessitated continuous exposure to

the rays of the sun which were being collected and measured, and in order to protect the eyes from the intense glare of the sun it was prudent to use neutral tinted spectacles. The moment the temperature of the air passed upwards through the temperature of the skin was signalised by the spectacles feeling hot. Although the temperature rose to 105° F. the capacity for heat of the rim of a pair of spectacles is too insignificant to cause any inconvenience.

Discussion of Observations.

The observations made with the Calorimeter on the 16th, 17th and 18th May are given in detail in Table III. and they are represented graphically in Fig. 8. On the 16th the distillate was received in a cylinder capable of holding over 100 c.c. and graduated into single cubic centimetres. In the two columns for rates on this day, the one is the rate per minute while 10 c.c. were collected, and the other the rate per minute while 20 c.c. were collected. On the 17th and 18th a tube graduated into half cubic centimetres and holding 20 c.c. was used. Time was taken as every 5 c.c. were collected, and the tube was emptied when 20 c.c. had been collected. The readings for every 5 c.c. were made without removing the receiver from the distilling tube. The portion of 20 c.c. was measured in a truly vertical position and is more exact than the measurement of its constituent portions of 5 c.c., although every care was taken to note the time when exactly 5 c.c. had run without running the risk of losing any of the distillate.

The most important manipulation is attending to the equatorial motion of the instrument. The observed rate of distillation agrees the more closely with the true rate the more carefully the axis of the instrument is kept pointed towards the sun. This was controlled by observing the shadow of the steam space on the top of the condenser with which it is concentric.

The position was adjusted every two or three minutes when it was usually put a shade in advance of the true position so as to give it a position correct for the middle of the interval.

The calorimeter ought always to be fed with pure distilled water. Unfortunately this was not available, and Nile water had to be used. It contains a considerable amount of earthy carbonates and is apt, after prolonged use, to froth. With the glass dome, however, this was at once detected, and if it was serious the water was changed.

TABLE III. GIVING RATES OF DISTILLATION OBSERVED ON THE 16TH, 17TH AND 18TH MAY, 1882, AT SOHAG, LAT. 26° 37′ N.

	Apparent Solar Time	Volume of Distillate	Rate for Distillation of 10 c.c.	Rate for Distillation of 20 c.c.	
1882	hr. min. sec.	c.c.	c.c. per min.	c.c. per min.	
May 16 A.M.	9 26 0	0			Cloudless sky, wind very slight from the West, drawing to North on the Nile.
	36 55	10	0·916		
	46 55	20	1·000	0·956	
	56 40	30	1·026		
	10 6 50	40	0·984	1·002	
	17 5	50	0·976		Wind seems to be freshening and comes in gusts, retarding distillation.
	29 15	60	0·821	0·893	
	39 20	70	0·992		
	49 50	80	0·952	0·972	
	59 20	90	1·053		
	11 8 5	100	1·144	1·096	
	12 0	0			
	21 10	10	1·091		
	29 45	20	1·166	1·127	
	38 25	30	1·155		
	46 50	40	1·189	1·171	
	55 35	50	1·144		
P.M.	12 3 50	60	1·212	1·177	
	12 5	70	1·213		
	19 0	80	1·447	1·330	
	27 10	90	1·225		
	34 10	100	1·429	1·327	
	1 27 0	0			
	35 0	10	1·250		
	42 35	20	1·320	1·285	
	50 0	30	1·349		
	2 15 30	0			
	23 15	10	1·292		
	31 30	20	1·213	1·253	
	40 0	30	1·177		
	48 5	40	1·237	1·207	
	56 35	50	1·177		
	3 4 40	60	1·237	1·207	
	16 55	73			
	22 50	80		1·101	
May 17 A.M.	8 34 0	0			Eclipse total.
	51 0				Exposed calorimeter.
	58 0				Water begins to "sing."
	9 1 0				Water begins to boil.
	3 0				Water boils briskly.
	17 0	0			Water begins to distil and to be collected.
	19 30	1	0·400		
	21 0	0			
	29 30	5	0·588		Sky quite cloudless.

TABLE III.—*continued.*

	Apparent Solar Time	Volume of Distillate	Rate for Distillation of 5 c.c.	Rate for Distillation of 20 c.c.	
1882	hr. min. sec.	c.c.	c.c. per min.	c.c. per min.	
May	9 36 5	10	0·760		
17	40 55	15	1·035		
A.M.	45 45	20	1·035	0·806	
	47 0	0			
	51 15	5	1·177		
	56 0	10	1·053		
	59 50	15	1·306		
	10 4 5	20	1·177	1·171	
	5 0	0			
	8 52	5	1·292		
	14 35	11	1·101		
	18 40	16	1·225		
	22 20	20	1·091	1·155	
	23 30	0			
	27 50	5	1·155		
	31 55	10	1·225		
	36 5	15	1·201		
	40 30	20	1·133	1·177	
	41 30	0			
	45 35	5	1·225		
	49 30	10	1·278		
	53 30	15	1·250		
	57 45	20	1·177	1·232	
	58 30	0			
	11 2 45	5	1·177		
	7 0	10	1·177		
	11 15	15	1·177		
	15 35	20	1·155	1·171	
	16 30	0			
	20 25	5	1·278		
	24 30	10	1·225		
	28 25	15	1·278		
	33 0	20	1·091	1·212	
	34 30	0			
	38 50	5	1·155		
	42 55	10	1·225		
	47 0	15	1·225		
	51 50	20	1·035	1·155	
	54 0	0			
	58 25	5	1·133		
	12 2 30	10	1·225		
	6 30	15	1·250		
	10 40	20	1·201	1·202	**The apparatus was emptied and filled with fresh water for the afternoon's work.**

TABLE III.—*continued.*

	Apparent Solar Time	Volume of Distillate	Rate for Distillation of 5 c.c.	Rate for Distillation of 20 c.c.	
	hr. min. sec.	c.c.	c.c. per min.	c.c. per min.	
1882 May 17 P.M.	2 1 0	0			On exposing to the sun the water began to "sing" in 5 seconds and boiled in 40 seconds.
	5 45	5	1·053		
	10 55	11	1·177		
	14 35	15	1·091		
	19 5	20	1·111	1·106	From 2 p.m. to 2.30 p.m. rather more wind than before. Sky all day quite cloudless. Wind North, but not much of it. The weather altogether very much like the 16th only it feels somewhat warmer.
	24 30	0			
	28 50	5	1·155		
	33 15	10	1·133		
	37 55	15	1·071		
	42 50	20	1·071	1·091	
	44 0	0			
	49 5	5	0·984		
	53 40	10	1·091		
	57 55	15	1·177		
	3 2 45	20	1·035	1·066	
	10 0	0			
	14 40	5	1·071		
	19 50	10	0·968		
	24 30	15	1·071		
	29 40	20	0·968	1·015	
	31 30	0			
	37 10	5	0·883		
	42 10	10	1·000		
	47 10	15	1·000		
	52 50	20	0·883	0·943	
	55 30	0			
	4 1 0	5	0·909		
	6 20	10	0·937		
	11 55	15	0·896		
	17 20	20	0·923	0·916	
	19 30	0			
	25 25	5	0·845		
	31 5	10	0·883		
	37 0	15	0·845		
	43 55	20	0·720	0·819	
May 18 A.M.	8 47 30	0			Cloudless sky, absolutely no wind.
	51 25	5	1·278		
	55 20	10	1·278		
	59 20	15	1·250		
	9 3 20	20	1·250	1·264	
	5 0	0			
	8 55	5	1·278		
	12 35	10	1·365		
	16 20	15	1·333		

TABLE III.—*continued.*

	Apparent Solar Time	Volume of Distillate	Rate for Distillation of 5 c.c.	Rate for Distillation of 20 c.c.	
	hr. min. sec.	c.c.	c.c. per min.	c.c. per min.	
1882 May 18 A.M.	9 20 20	20	1·250	1·306	
	22 0	0			
	25 45	5	1·333		
	29 20	10	1·397		
	33 50	16	1·333		
	37 5	20	1·225	1·326	
	39 15	0			
	43 0	5	1·333		
	46 50	10	1·306		
	50 40	15	1·306		
	54 25	20	1·278	1·306	
	57 0	0			
	10 0 50	5	1·306		
	4 35	10	1·333		
	8 25	15	1·306		
	12 35	20	1·201	1·285	
	15 0	0			
	18 45	5	1·333		
	22 40	10	1·278		
	26 15	15	1·397		
	30 15	20	1·250	1·313	
	32 0	0			
	35 40	5	1·365		
	39 0	10	1·501		
	42 25	15	1·465		
	46 15	20	1·306	1·405	
	47 30	0			
	51 5	5	1·397		
	54 35	10	1·429		
	58 5	15	1·429		
	11 2 0	20	1·278	1·381	The calm was broken by a light northerly breeze lasting about 10 minutes.
	4 0	0			
	7 35	5	1·397		
	11 0	10	1·465		
	15 20	16	1·381		
	18 25	20	1·301	1·389	
	21 30	0			
	24 50	5	1·501		
	28 15	10	1·465		
	31 40	15	1·465		
	35 20	20	1·365	1·447	
	37 30	0			
	41 5	5	1·397		
	44 40	10	1·397		

TABLE III.—*continued.*

	Apparent Solar Time	Volume of Distillate	Rate for Distillation of 5 c.c.	Rate for Distillation of 20 c.c.	
	hr. min. sec.	c.c.	c.c. per min.	c.c. per min.	
1882 May 18 A.M.	11 48 5	15	1·397		
	51 55	20	1·306	1·389	
	53 30	0			
	57 0	5	1·429		
	12 0 40	10	1·365		
	4 10	15	1·429		
	8 35	20	1·133	1·326	Perfectly calm.
P.M.	1 55 30	0			Light northerly breeze.
	59 20	5	1·306		
	2 3 15	10	1·278		
	7 10	15	1·278		
	11 10	20	1·250	1·274	
	12 30	0			
	17 25	6	1·225		
	20 40	10	1·225		
	25 35	16	1·225		
	29 0	20	1·177	1·214	
	31 0	0			
	35 25	5	1·133		Northerly wind freshening.
	40 35	11	1·166		
	44 5	15	1·144		
	48 45	20	1·071	1·128	
	51 0	0			
	55 10	5	1·201		North wind quite fresh.
	59 45	10	1·091		
	3 4 25	15	1·071		
	9 0	20	1·091	1·112	
	11 0	0			
	15 25	5	1·133		
	20 10	10	1·091		
	24 40	15	1·111		
	29 10	20	1·111	1·111	
	31 0	0			
	35 15	5	1·333		
	39 45	10	1·111		
	44 25	15	1·071		Wind freshening still.
	49 35	20	0·968	1·076	
	52 30	0			
	57 10	5	0·937		
	4 1 30	10	1·155		Water beginning to froth.
	6 0	15	1·111		
	11 20	20	0·937	1·062	

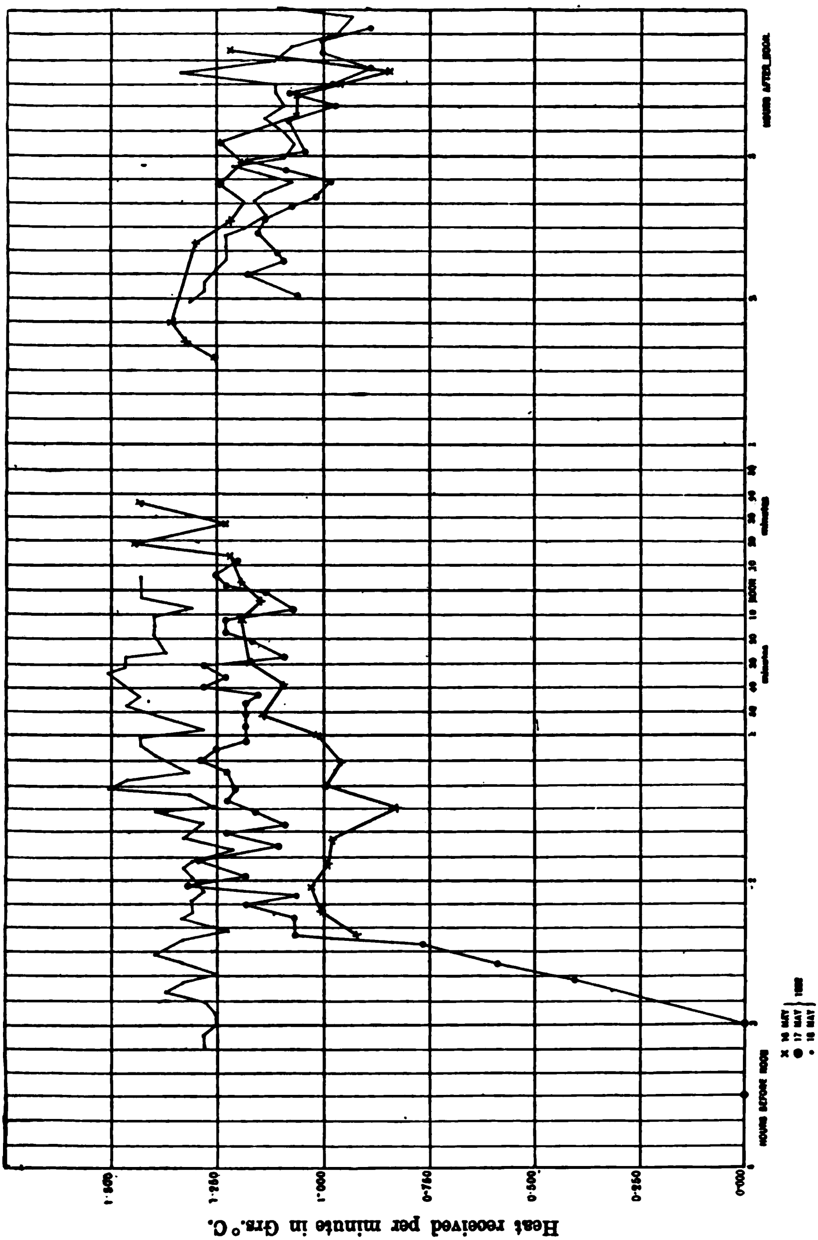

0·000
0·250
0·500
0·750
1·000
1·250
1·500
Heat received per minute in Grs. °C.

Fig. 8.

By far the most important agent in altering the true rate, as due to the sun alone, is the wind. During the three days we were fortunate in having both calm and wind, so that an idea can be formed of the cooling effect of wind. On the 16th with a calm afternoon the mean rate between 2 and 3 p.m. was 1·221, and on the 17th, when it was breezy, the rate was 1·087 or about 10 per cent. less.

The breezes which occur on the Nile are usually cool and from the north. They did not at any time exceed force 3 of Beaufort's scale. They were never steady, but came in puffs or gusts, so that one 20 c.c. or even 5 c.c. interval would be affected and the subsequent one not.

On the 13th some satisfactory observations were made when the sun though behind a cirrus cloud was still able to keep distillation going. The rate was 0·752 at 10 a.m. When the sun had cleared the cloud the rate rose to 0·91 at 10.30 a.m.

If we look over the list of figures in Table III. or their graphical representation in Fig. 8, we notice that there is considerable variability in the results whether the interval which we consider be that required for the distillation of 5 c.c. or 20 c.c. Further, this variability from one interval to another is more remarkable than the change of rate due to change of the sun's altitude. Yet the sun's altitude which is 83° at noon is only 48° at 9 a.m. or 3 p.m. If we express it in zenith distance, the zenith distance is at noon 7° and increases to 42° at 9 a.m. and 3 p.m. We conclude that *the energy of the radiation received by a surface held perpendicularly to the sun's rays is, within considerable limits, very little dependent on the sun's zenith distance.*

The weather on each of the three days was very fine and each of them taken by itself would have been held to be very favourable for this kind of experiment. Yet amongst the three very good days the forenoon of the 18th was incomparably the best; the sun shone its strongest and the air was motionless; moreover, instrumentally everything was in best working order. Therefore to ascertain the greatest amount of heat that can be obtained from the sun's rays we examine the results obtained in the forenoon of the 18th, and we find that at about half-past ten, 5 c.c. were distilled in three minutes and twenty seconds, being at the rate of 1·501 c.c. per minute. Nearly an hour later the same time is registered for the distillation of 5 c.c., but owing to the greater zenith distance of the sun the former must be held to be the higher rate. The correction to be applied to either of these rates in order to reduce it to its value for a vertical sun is evidently insignificant and we take 1·5 c.c. per minute as the highest rate observed.

In attempting to form an estimate of the extent to which this may fall short of the true rate under perfect conditions, we have

to consider the rates observed at other times during the three days and the following table, which gives the amount of water distilled in each hour in the different days, may be used.

If the experiments were to be repeated, I do not know in what particular the conditions of weather, as they were on the forenoon of the 18th, could be made better. Yet if we admit that they might be improved in the proportion that the conditions between 10 and 11 a.m. on the 18th are better than those obtained between the same hours on the 16th, when they were the most unfavourable, the rate would have to be increased in the proportion 58·8 : 82·4 and it would become $1·5 \times 1·4 = 2·1$ c.c. per minute.

Time		Cubic centims. water distilled		
from	to	16th	17th	18th
A.M.				
9	10			78·1
10	11	58·8	70·25	82·4
11	12	69·5	69·7	82·8
P.M.				
2	3	73·2	65·5	71·6
3	4		57·9	64·8

This correction is certainly too great when considered as an allowance for faulty weather, and even if held to cover all instrumental deficiencies, such as imperfect equatorial adjustment and others, I believe it will be still much in excess of the truth; moreover, I am convinced that if the calorimeter furnished steam at this rate it would be in such conditions that it would be impossible to stand by it and attend to it on account of the excessive heat.

Having thus indicated the maximum correction which can be applicable to our observations we return to the consideration of the observations themselves, where we are on the sure ground of experiment.

In the circumstances we may, without sensible error, take the cubic centimetre of water to weigh one gramme. In specifying quantities of heat we do so in gramme-degrees (Celsius) (gr.° C.), or kilogramme-degrees (kg.° C.), as the case may be. Similarly, quantities of work are expressed in kilogramme-metres (kgm.). We take the latent heat of one gramme of steam as 535 gr.° C., and the specific heat of water as unity, and the mechanical equivalent of heat as 0·425 kilogramme-metres per gramme-degree. Therefore the heat required to transform 1·5 grs.

of water at 100° C. into steam of the same temperature is 803 gr.° C., and this is the greatest amount of heat which the calorimeter has recorded in one minute. On careful measurement of the calorimeter, especially the reflector, I find that its actual collecting diameter is 34·3 centimetres, less that of the condenser tube, 5·1 centimetres. So that its collecting area is

$$924 - 20{\cdot}5 = 903{\cdot}5 \text{ square centimetres (cm.}^2\text{)}.$$

Therefore the rays of the sun falling perpendicularly on a surface of 903·5 cm.^{2} supplied it with heat at the rate of 803 gr.° C. per minute. This is equivalent to 8888 gr.° C. per square metre; and 8888 gr.° C. suffice for the generation of 16·6 grs. of steam at 100° C. *Therefore by the use of ordinary mechanical appliances it is possible under favourable geographical and meteorological conditions to collect on a square metre of surface exposed perpendicularly to the sun's rays the energy of generation of* 16·6 *grs. of steam per minute.* But 8888 gr.° C. of heat are equivalent to 3777 kgm. of work; and this work is done in one minute, therefore the agent is working at the rate of at least 0·84 horse-power.

The agent is the energy of the sun's rays which fall upon a surface of one square metre, exposed perpendicularly to them at the distance of the earth. If the sun throws so much radiant energy that it can be collected and utilised at the earth's surface at the rate of 0·84 horse-power per sq. metre, then, as the area of a great circle on the earth's surface is $129{\cdot}9 \times 10^{12}$ sq. metres, the useful energy received by the whole earth is at the rate of

$$109 \times 10^{12} \text{ horse-power.}$$

Taking the radius of the earth's orbit to be 212 times the radius of the sun, the radiation of one sq. metre of the sun's surface is spread over 45,000 sq. metres of the earth's surface; therefore *the sun must radiate energy at the rate of at least* 37,000 *horse-power per sq. metre of its surface.*

Observations during the Eclipse.

The calorimeter was directed to the sun as soon after totality as possible. At 8 hr. 34 min. the sun was totally eclipsed; at 8.51 the calorimeter was directed to the sun but no boiling took place. At 8.58 the water began to "sing"; at 9.1 it boiled; at 9.3 it was boiling briskly, but it was not till 9.17 that the first drop of distillate fell into the receiver. By 9.19·5 1 c.c. had passed, and between 9.21 and 9.29·5 5 c.c. passed.

The observations made at this time are collected in Table IV. —In the first column is the apparent solar time of each observation, in the second column is the volume of distillate collected at that time, in the third column is the mean time of collecting

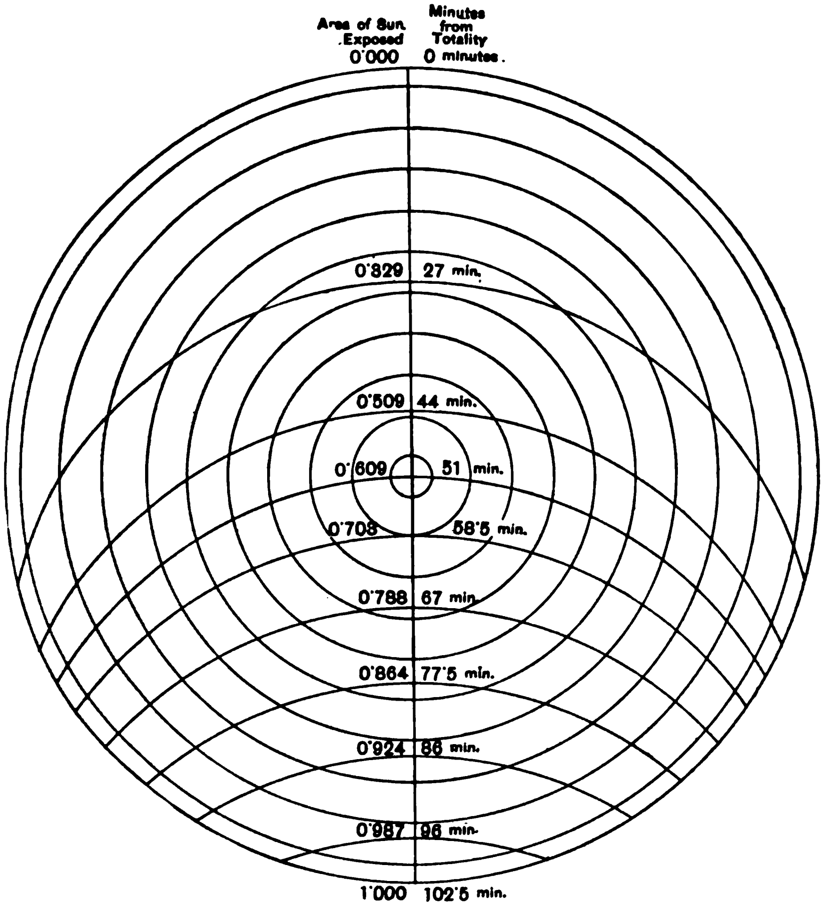


FIG. 9. DIAGRAM OF EXPOSED SURFACE OF THE SUN AT SUCCESSIVE EPOCHS AFTER TOTALITY.

each portion, in the fourth column is the date stated in minutes after totality, in the fifth column is the average rate of distillation in c.c.'s per minute during the interval, and in the sixth column is the percentage of the sun's disc exposed.

The diagram Fig. 9 shows the progress of the eclipse and the portions of the sun successively uncovered.

From Table IV. we see that when distillation has begun, it increases at a much greater rate than does the exposed sun's surface. This must be so in the early stages, because we see that it is not till 26 minutes after totality and when already 33 per cent. of the sun's surface has been uncovered that the water in the boiler boils, and it takes 16 minutes more before any distillate is collected. Even when 50 per cent. of the sun is exposed the rate of distillation is only 0·4 c.c. per minute. After this more weight may be attached to the observations, but their numerical significance is not great. The experiment was not originally contemplated. The instrument was constructed for use with the strongest uneclipsed sun that could be found. Still it shows that useful information could be obtained by arranging for making trustworthy observations during the progress of an eclipse. The provisions which it would be necessary to make are instructive, because they indicate some of the capabilities and defects of the instrument.

TABLE IV.

Apparent Solar Time, A.M.	Cubic centims. collected	Mean date and Interval, A.M.	Minutes from Totality	Rate of Distillation	Amount of Sun's surface exposed
hr. min. sec.		hr. min. sec.		c.c. per min.	
8 34 0	0		0		0·000
9 1 0	0		27		0·329
9 17 0	0				
9 19 30	1	9 18 15	44	0·400	0·509
9 21 0	0				
9 29 30	5	9 25 15	51	0·589	0·609
9 36 5	10	9 32 47	58·5	0·759	0·703
9 40 55	15				
9 45 45	20	9 40 55	67	1·034	0·788
9 47 0	0				
9 51 15	5				
9 56 0	10	9 51 30	77·5	1·111	0·864
9 59 50	15				
10 4 5	20	10 0 0	86	1·237	0·924
10 5 0	0				
10 8 52	5				
10 14 35	11	10 9 45	96	1·146	0·987
10 18 40	16				
10 22 20	20	10 18 30	102·5	1·161	1·000

First of all it must be remembered that the calorimeter is efficient only when it is running continuously and at or nearly at its full load. In the case of a total eclipse there must be an interval during which the sun cannot keep steam however large the reflector may be and however great its condensing power may be. We have seen that when exposed cold as soon as possible after the total phase of the eclipse, it was 27 minutes after totality before the water boiled. One third of the sun was then uncovered. It is therefore reasonable to suppose that, if the eclipse had happened at noon so that the first half of it could have been utilised as well as the second half, the sun would have kept steam in the calorimeter and it would have continued to distil until two thirds of the sun's surface had been obscured. Then distillation, if it did not cease, would become so slow that its rate would have no value, and fifty-four minutes would elapse before one third of the sun would again be uncovered during which the calorimeter would get cold. *During this interval steam must be kept artificially.* This is very easy. The glass tube which forms the steam dome is attached to a metal collar which screws down on a washer. It can therefore be easily detached. If then the steam tube of the calorimeter be connected by means of an india-rubber tube with a flask in which water is kept boiling, steam can be passed through the calorimeter at the normal rate until it is judged suitable to expose it again to the sun. There is no difficulty about this. It might however be well for use during an eclipse to provide increased reflector power. But it would be necessary to shade it with a diaphragm when used with the uneclipsed sun, and the comparison of the heat of the eclipsed sun with that of the uneclipsed sun would be defective. Fortunately the power of varying the constants of the instrument is so great that one or two trials would suffice to fit it for use during an eclipse.

Although quite insignificant as a natural phenomenon an annular eclipse is better for calorimetric experiments than a total one. Next year on 11th November there will be an annular eclipse visible in Ceylon. The annular phase will last over ten minutes and at its greatest 0·875 of the sun's disc will be covered. It is pretty certain that the calorimeter used in 1882 would not keep steam through this phase, but a larger reflector might be used. It would be worth while to have a reflector of such a size that steam would certainly be kept through the whole eclipse, especially during the annular phase when all the radiation is from the peripheral region.

Conclusion.

It is usual for writers on this subject to express the heating effect of the sun's rays in gramme-degrees received by one square centimetre exposed perpendicularly to them for one minute outside the limits of the earth's atmosphere. This is termed the *solar constant.* Expressed thus our maximum rate is 0·89 gr.° C. per sq. centimetre per minute at the base of the earth's atmosphere. If we add 11 per cent. for deficiencies from all sources we have 1 gr.° C. heat received at the sea level on a surface of 1 sq. centimetre exposed perpendicularly to the sun's rays per minute; and from the conditions under which the maximum rate was observed on the 18th May, I believe that this figure is as likely to be above the truth as below it. If however it is thought that the allowance should be more liberal, we have seen that our maximum rate corresponds to 0·84 horse-power per sq. metre; if we make this one horse-power per sq. metre we have certainly got as much radiant energy as it is possible to collect at the level of the sea. Further, in speculations connected with physical meteorology we are not entitled to postulate a more abundant supply. From this supply falls to be deducted the energy of evaporation which however is returned on precipitation, also the energy of storms which to a large extent are secondary features attending the changing hygrometric state of the atmosphere. Notwithstanding the apparently perfect transparency of the atmosphere on the morning of the 18th we must admit that some of the energy was lost by absorption in the passage through the earth's atmosphere; but the small effect produced by great variations of the zenith distance of the sun on the rates observed shows that this effect cannot be great, in fact it is entirely masked by very slight motion of the air. Most recent writers put the value of the solar constant at from 3 to 5 gr.° C. per $cm.^2$ per min., the greater part of which is added to the observed value in order to compensate for the supposed absorption by the air. Thus Scheiner[1] in a recent work writes:—"From what precedes it is apparent that the values which have been found for the solar constant do not differ so very much from each other. The older determinations have without doubt given too small values, the later ones point with great certainty in the direction that the solar constant is included between the amounts of 3·5 and 4·0 gr. cal." Now even the lower of these values can be true only

[1] *Strahlung und Temperatur der Sonne*, von Dr J. Scheiner, Leipzig, Engelmann, 1899; see p. 38.

if we admit that the atmosphere absorbs at least as much heat as it transmits. But we know by every-day experience the far-reaching effects produced by what it transmits; what does it do with the heat that it absorbs? Do we see any evidence of work being done at the rate of two or even of one horse-power per sq. metre, remembering that the energy of storms is already accounted for? If it is absorbing heat at the rate of 2 gr.° C. per cm.^{2} per minute, how does it come that the atmosphere is so cool? Again, looking to the length of time that the present state of things has existed, how has the atmosphere not long ago arrived at the state in which it emits as much energy as it absorbs, so that its effective power of absorption would be *nil*?

I do not ask these questions lightly. The subject has occupied my attention off and on for the last eighteen years, and I believe that the only answer to them is that *the value of the solar constant which is now accepted is very much exaggerated.* This view is, I think, supported by the following consideration.

Taking the length of the sun's radius as unity we have in the accompanying table the distance (d) from its centre to the sun's surface, and to the three inner planets, Mercury, Venus, and the Earth, and the squares of these distances (d^2). The

Name of Body	Sun's surface	Mercury	Venus	Earth
Distance from Sun's centre, d	1·0000	82·0646	153·3466	212·0000
Square of Distance, d^2.........	1	6,735·0	23,515·0	44,944·0

squares of the distances represent the area on each planet over which the radiation per unit area of the sun's surface is spread. It will be seen that the area on the earth's surface covered by the radiation from a given area of the sun's surface is almost double that covered by the same radiation on the surface of Venus, and therefore the intensity of radiation on Venus is almost exactly double that on Earth. In other words, the true value of the solar constant at a point on the orbit of Venus is almost exactly double its true value at a point on the earth's orbit. It is impossible to believe that the cloudless atmosphere of the Earth, the whole mass of which is only 1 kg. per sq. centimetre, can produce an absorbing effect equal or superior to the dissipating effect of such a distance as that separating the orbits of Venus and the Earth.

To conclude, it has been shown that under favourable meteorological and geographical conditions, by the use of ordinary and necessarily imperfect mechanical appliances it is possible to

collect from a square metre of surface exposed perpendicularly to the sun's rays 8888 gr.° C. of heat which are equivalent to 3777 kgm. of work per minute or 0·84 horse-power. If we allow 16 per cent. for losses from all causes the result is one horse-power received from the Sun by every square metre of the area included in a great circle of the Earth which is roughly 130×10^{12} sq. metres and *this figure in horse-power represents the working value of the Sun in its relations to the Earth.* Accepting the value of one horse-power per sq. metre at the distance of the Earth we find by simple arithmetic that *the working value of* 1 *sq. metre of the sun's surface must be* 45000 *horse-power.* It follows that the area of the sun's surface which we may regard as hypothecated to the earth's heat service is no more than 2900 square kilometres which would be contained in a circle of 60 kilometres diameter and would subtend an angle of at most one tenth of a second. As over five hundred millions of such areas are included in a great circle of the Sun, it is clear that the maintenance of the Earth's heat is well assured.

Theorems on Matrices and Bilinear Forms. By T. J. I'A. BROMWICH, M.A., St John's College.

[*Received* August 1900.]

This paper consists of three parts; in part 1 is a discussion of Sylvester's rule for the biorthogonal reduction of a bilinear form, and a short account of former papers on the same subject. Part 2 contains an account and comparison of various formulae used by different writers to evaluate functions of a matrix or bilinear form. Part 3 contains an investigation of the invariant-factors of any function of a matrix.

1. *Reduction of a bilinear form by biorthogonal substitutions.*

This problem has been considered by the following authors:

Beltrami, *Giornale di Matem.*, 1873, t. 11, p. 98.
Jordan, *Liouville's Journal*, 1874, t. 19 (2me série), p. 35.
Kronecker, *Berliner Monatsberichte*, 1874 (16 May) = *Ges. Werke*, Bd. 1, p. 410.
Cosserat, *Annales de Toulouse*, 1889, t. 3, pp. 1—12.
Sylvester, *Comptes Rendus*, 1889, t. 108, pp. 651—653.
" *Messenger of Math.*, vol. 19, pp. 1, 42.

Jordan's method depends on finding stationary values of the given form, when the variables are subject to the two conditions $\Sigma x^2 = 1 = \Sigma y^2$; his process is finally a step-by-step method.

Kronecker's paper consists in the main of various criticisms on Jordan's and a sketch of an alternative method.

Cosserat considers some special cases of Jordan's method, with particular consideration of the alternate form when the coefficients of $x_r y_s$ and of $x_s y_r$ are equal and opposite in sign; while the coefficient of $x_r y_r$ is zero.

Sylvester's proof (given in the *Messenger*) depends on an infinite repetition of two infinitesimal orthogonal substitutions. By this means he proves the *possibility* of reducing the given bilinear form with two orthogonal substitutions. Sylvester also gives without proof a rule (*Comptes Rendus*) for finding the two reducing substitutions of a given form. Here we give a short proof of this rule, using Frobenius's[1] method of combining symbolically bilinear forms; or Cayley's[2] for combining matrices.

[1] *Crelle's Journal* (1878), Bd. 84, p. 1.
[2] *Phil. Trans.* (1858), vol. 148, p. 17; *Coll. Works*, vol. 2, p. 475.

In Frobenius's notation the symbolical product of two bilinear forms A, B is given by

$$AB = \Sigma \frac{\partial A}{\partial y_r} \frac{\partial B}{\partial x_r}, \qquad (r = 1, 2, \ldots, n)$$

and it is to be remarked that the product has the same effect as making a linear substitution on the x's in B or on the y's in A. The accented letter A' denotes the *conjugate* form of A, obtained by interchanging the x's and y's in A.

Suppose that we have

$$B = RAS,$$

where A is the given bilinear form, and R, S are orthogonal forms. Then, taking the conjugate forms (i.e. changing x_r to y_r and *vice versa*) we have

$$B' = S'A'R'.$$

Hence

$$BB' = R(AA')R',$$

$$B'B = S'(A'A)S,$$

for we have $R'R = SS' = E$ (by definition of orthogonal forms), where E is the unit-form (*Einheitsform*), i.e. the identical or unit-matrix in Cayley's and Sylvester's terminology.

We now see that the problem of reducing A to a canonical form by two orthogonal substitutions depends on reducing (AA') by one orthogonal cogredient substitution and $(A'A)$ by another. We observe that both of the forms (AA') and $(A'A)$ are symmetrical, consequently the problem is much simpler than the corresponding one of reducing any bilinear form.

To find R we have to consider the determinant

$$| AA' - \lambda E |;$$

and we observe that if the coefficients of A be all real, AA' and E are real positive definite forms; hence all the roots of $| AA' - \lambda E | = 0$ are real and positive; and all the invariant-factors are linear[1].

Sylvester's method of reduction is now seen to be equivalent to that of Weierstrass[2] or Darboux[3] for reducing symmetrical forms with linear invariant-factors. Sylvester assumes that all the roots of $| AA' - \lambda E | = 0$ are distinct, but the necessary modification is not very great. In fact, if we remember that the

[1] Weierstrass, *Berliner Monatsberichte*, 1858, p. 207 et seq.; *Ges. Werke*, Bd. 1, p. 233 et seq.

[2] l.c. supra; and *Berl. Monatsber.*, 1868, p. 310 = *Ges. Werke*, Bd. 2, p. 19.

[3] *Liouville's Journal*, 1874, t. 19, 2me série, pp. 347—397.

invariant-factors are all linear, Darboux's rule may be at once reduced to the following:—

Let $(\lambda - c)$ be an α-times repeated factor of $| AA' - \lambda E |$ and write

$$\Delta_m = \underset{t=0}{\text{Lt.}} t^{-(\alpha-m)} \begin{vmatrix} c_{rs}, & p_r^{(k)} \\ p_s^{(k)}, & 0 \end{vmatrix} \quad \begin{pmatrix} r, s = 1, 2, \ldots, n \\ k = 1, 2, \ldots, m \end{pmatrix},$$

this representing a determinant of $(n+m)$ rows and columns (according to a notation suggested by Frobenius[1] and Nanson[2]) where $\Sigma c_{rs} x_r y_s = AA' - (c+t) E$ and the p's are arbitraries. Also write ξ_m for the determinant obtained from Δ_m by replacing one line $p_1^{(m)}, \ldots, p_n^{(m)}$ with $x_1, \ldots, x_n$; η_m is found similarly with $y_1, \ldots, y_n$.

Finally put
$$X_m = \xi_m / (\Delta_{m-1}\Delta_m)^{\frac{1}{2}},$$
$$Y_m = \eta_m / (\Delta_{m-1}\Delta_m)^{\frac{1}{2}},$$

and then we shall have

$$AA' - \lambda E = \Sigma (c - \lambda)(X_1 Y_1 + \ldots + X_\alpha Y_\alpha),$$

where the summation extends to all roots of $| AA' - \lambda E | = 0$, and the products on the right are ordinary, not symbolical.

We treat $(A'A - \lambda E)$ in the same way; we should note that the invariant-factors of $|A'A - \lambda E|$ and of $|AA' - \lambda E|$ are the same. For we have

$$A(A'A - \lambda E) = (AA' - \lambda E) A,$$

and thus the invariant-factors must be the same by a known theorem. Thus we shall find

$$A'A - \lambda E = \Sigma (c - \lambda)(\Xi_1 \mathrm{H}_1 + \ldots + \Xi_\alpha \mathrm{H}_\alpha),$$

where c, α are the same as in the last expression.

Hence we have two substitutions R, S such that

$$R(AA' - \lambda E) R' = \Sigma c_r x_r y_r - \lambda E,$$

$$S'(A'A - \lambda E) S = \Sigma c_r x_r y_r - \lambda E,$$

or
$$RR' = E = SS',$$

and
$$(RA)(RA)' = \Sigma c_r x_r y_r = (AS)'(AS).$$

Thus if we take the form

$$B = \Sigma c_r^{\frac{1}{2}} x_r y_r$$

[1] *Berliner Sitzungsberichte*, 1894, "Ueber das Trägheitsgesetz, etc."
[2] *Phil. Mag.*, vol. 44, 5th Series (1897), p. 396.

we have $$B = B' \text{ and } BB' = B^2 = \Sigma c_r x_r y_r,$$

and on substitution

$$(B^{-1}RA)(B^{-1}RA)' = E = (ASB^{-1})'(ASB^{-1})$$

or the two forms $B^{-1}RA$, ASB^{-1} are orthogonal. Now the product of two orthogonal forms is itself orthogonal, so as R, S are orthogonal we have

$$P = B^{-1}(RAS),\ Q = (RAS)B^{-1}$$

as two orthogonal forms.

Hence $$RA(SP^{-1}) = B = (Q^{-1}R)AS,$$

and SP^{-1}, $Q^{-1}R$ are both orthogonal, so the original problem has been solved. But in one case it is found that we do not need to calculate P, Q. To prove this we have that

$$Q = BPB^{-1}$$

is an orthogonal form and thus $Q = (Q')^{-1}$.

Or, since $B' = B$, we have

$$BPB^{-1} = (B^{-1}P'B)^{-1} = B^{-1}PB,$$

for P is orthogonal and so $(P')^{-1} = P$.

Hence $$B^2P = PB^2 \text{ and } B^2 = \Sigma c_r x_r y_r,$$

and if *all* the coefficients c_r are different, this equation can only hold if P is of the form $\Sigma d_r x_r y_r$, where the d's may be any arbitraries. Now P is orthogonal and so we must have

$$d_r^2 = 1. \qquad (r = 1, 2, \ldots, n)$$

In this case

$$RAS = BP = \Sigma(\pm c_r^{\frac{1}{2}})x_r y_r,$$

and the form has been again reduced by the two orthogonal substitutions R, S. Obviously a *sufficient* condition (though not necessary) is that all the roots of $|AA' - \lambda E| = 0$ should be different. In this case we have say

$$AA' = \Sigma c_r X_r Y_r, \quad A'A = \Sigma c_r \Xi_r \mathrm{H}_r,$$

and then $$A = \Sigma(\pm c_r^{\frac{1}{2}})X_r \mathrm{H}_r$$

which is Sylvester's rule.

But if the roots of $|AA' - \lambda E| = 0$ are not all different it may be necessary to modify one or other set of variables[1] by a further

[1] That is, either the X's or the H's (it is immaterial which); the fact that the two substitutions R, S do not necessarily reduce A seems not to have been noticed before.

orthogonal substitution in order to complete the reduction. The further substitution is always easily calculated by forming RAS and comparing it with B.

Sylvester's numerical example (*Comptes Rendus*, t. 108, p. 653) is

$$A = 8x_1y_1 - x_1y_2 - 4y_1x_2 + 7x_2y_2.$$

Then $$AA' = 65x_1y_1 - 39(x_1y_2 + x_2y_1) + 65x_2y_2,$$

$$A'A = 80x_1y_1 - 36(x_1y_2 + x_2y_1) + 50x_2y_2,$$

the roots of our determinantal equation are

$$\lambda = 26 \text{ or } 104.$$

Then on calculation

$$X_1 = (x_1 + x_2)/\sqrt{2}, \qquad \Xi_1 = (2x_1 + 3x_2)/\sqrt{13},$$

$$X_2 = (-x_1 + x_2)/\sqrt{2}, \qquad \Xi_2 = (-3x_1 + 2x_2)/\sqrt{13}.$$

Thus, as the roots of the determinantal equation are different, we have

$$A = \sqrt{26}\,X_1\mathrm{H}_1 + \sqrt{104}\,X_2\mathrm{H}_2.$$

2. *Expressions for functions of a bilinear form.*

Let A be the given form and let $\phi(r) = |rE - A|$ denote the fundamental determinant of the form; further, let $\psi(r)$ be the quotient of $\phi(r)$ by the H.C.F. of *all* the first minors of $\phi(r)$. Then we know[1] that $\psi(A) = 0$; and that $\psi(r)$ is the expression of lowest dimensions in r which vanishes when r is replaced by A.

We shall use the notation

$$\psi(r) = (r-a)^\alpha (r-b)^\beta (r-c)^\gamma \ldots,$$

and it should be remembered that every factor of $\phi(r)$ appears in $\psi(r)$; but possibly to a lower power in case the factor is repeated in $\phi(r)$.

We start with considering a rational function of the form; there is of course no theoretical difficulty in calculating such functions directly. It is, however, worthy of notice that the following method is practically easier, whenever the roots of $\phi(r)$

[1] Frobenius, *Crelle*, Bd. 84 (1878), p. 12; *Berliner Sitzungsber.*, 1896, p. 601. Ed. Weyr, *Monatshefte für Math. und Phys.* Bd. 1 (1889), p. 187. Muth, *Elementartheiler* (Leipzig, 1899), p. 34. H. F. Baker, *Proc. Lond. Math. Soc.*, vol. 31, 1899, p. 195.

can be determined; the method is, also, capable of being extended to functions, which are not rational and algebraic.

Suppose $f(r)$ any rational function of r and then consider the value of the sum of Cauchy's residues for the function of r

$$\frac{f(r)}{\psi(r)} \frac{\psi(r) - \psi(s)}{r - s}$$

at the points $r = a, b, c, \ldots$. We see at once that this sum is equal to

$$f(s) - \left(\begin{array}{c}\text{sum of residues of } \dfrac{f(r)\psi(s)}{\psi(r)(r-s)} \text{ for} \\ \text{the points } r = s,\ r = a, b, c, \ldots\end{array}\right).$$

Now considering *all* the residues of $f(r)/[\psi(r)(r-s)]$ we know by a theorem of Cauchy's that their complete sum is zero; and in addition to the points $r = s$, $r = a, b, c, \ldots$ there are the poles of $f(r)$ and possibly $r = \infty$. Thus the original sum of residues is

$$f(s) + \psi(s)\left(\begin{array}{c}\text{sum of residues of } \dfrac{f(r)}{\psi(r)(r-s)} \text{ for} \\ \text{the poles of } f(r) \text{ and } r = \infty\end{array}\right)$$

$$= f(s) + \psi(s)\, g(s),$$

where $g(s)$ has the property of not being infinite for any of the values $s = a, b, c, \ldots$, for we suppose that $r = a, b, c, \ldots$ are not poles of $f(r)$ otherwise the function of the given bilinear form would have no meaning. It will be recognized that $g(s)$ is the sum of those partial fractions in $\dfrac{f(s)}{\psi(s)}$ which are not infinite for $s = a, b, c, \ldots$; so that $g(A)$ has a meaning.

Now write $s = A$ in the result last obtained (which we may do, for the equation is rational on each side) and then, since $\psi(A) = 0$ and since $g(A)$ has a meaning,

$$f(A) = \text{sum of residues of } (rE - A)^{-1} f(r) \text{ for the points } r = a, b, c, \ldots.$$

We can write this in a different form, for we have an equation of the type

$$\begin{aligned}(rE - A)^{-1} = & \frac{A_1}{r-a} + \frac{A_2}{(r-a)^2} + \ldots + \frac{A_\alpha}{(r-a)^\alpha} \\ & + \frac{B_1}{r-b} + \frac{B_2}{(r-b)^2} + \ldots + \frac{B_\beta}{(r-b)^\beta} \\ & + \ldots,\end{aligned}$$

where $A_1, \ldots, A_\alpha, B_1, \ldots, B_\beta, \ldots$ are certain bilinear forms which can be calculated without any great difficulty when $(rE - A)^{-1}$ is written out as the quotient of two determinants. It follows that

$$f(A) = \Sigma \left[f(a) A_1 + f'(a) A_2 + \ldots + \frac{f^{\alpha-1}(a)}{(\alpha-1)!} A_\alpha \right],$$

the summation extending to all the points $a, b, c, \ldots$.

This definition is readily extended to such transcendental functions of A as are defined by power-series, convergent for the values $a, b, c, \ldots$; for our theorem will hold up to any finite number of terms (n) of the series; and the right-hand side of the equation has a definite limit as n tends to infinity, provided that $f(a), f'(a), \ldots, f(b), f'(b), \ldots$ have finite limits. Making these assumptions it is natural to define $f(A)$ as given by the limit of the right-hand side for n infinite.

Such transcendental functions have been given by Schur[1], Mettzler[2], and Taber[3]; as far as I know, the exponential function is the only one that has been of use in any investigations of importance; this function has been used by Schur in the theory of continuous groups and by Taber in certain researches on the linear automorphic substitutions (of a bilinear form) which can be generated by an infinitesimal substitution belonging to the same group.

It may be useful to point out that with the above definition for the function exp A, we have

$$\exp(mA) = (\exp A)^m,$$

if m be an integer. The proof of this follows without any difficulty from the expression given below (foot of p. 85). But $\exp(A+B)$ is only equal to the product $(\exp A)(\exp B)$ if A and B are commutative (*i.e.* if $AB = BA$).

If $f(r)$ be an algebraic function of r, *i.e.* the root of an algebraic equation whose coefficients are rational functions of r, we can readily extend our definition so as to obtain $f(A)$ by a rational process. Thus, let $y = f(r)$ be an algebraic function of r, defined by the equation

$$\theta(y, r) = p_0 y^k + p_1 y^{k-1} + \ldots + p_k = 0$$

in which $p_0, p_1, \ldots, p_k$ are rational functions of r, which may be assumed to be integral functions without loss of generality. We assume also that the values $r = a, b, c, \ldots$ (which are given as before by $\psi(r) = 0$) are *not* such as to make the y-discriminant of θ vanish; for, if the discriminant vanished at say $r = a$, the expansion of y in the neighbourhood of $r = a$ would contain

[1] *Math. Annalen*, Bd. 38 (1890), p. 271.
[2] *American Journal of Math.*, vol. 14 (1892), p. 326.
[3] *Math. Annalen*, Bd. 46 (1895), p. 561; and several other papers.

fractional powers of $(r-a)$, contrary to what is assumed below. At each of the points $r=a, b, c, \ldots$ select any one of the k power-series which represent y in the neighbourhood of the point considered, and expand the quotient $y/\psi(r)$ in ascending powers of $(r-a)$ near $r=a$, of $(r-b)$ near $r=b$, etc. Finally, keep only the negative powers in each of the series so obtained and take their sum, which can be put in the form $h(r)/\psi(r)$, where $h(r)$ is an integral function of r, in general of degree one less than that of $\psi(r)$.

Then let $$g=[y-h(r)]/\psi(r),$$
so that $$h=y-g\psi;$$
thus
$$\begin{aligned}\theta(h, r) &= \theta(y-g\psi, r)\\ &= \theta(y, r)-g\psi\frac{\partial\theta}{\partial y}+\ldots+(-1)^k p_0(g\psi)^k\\ &= R\psi,\end{aligned}$$
where R is plainly an integral function of y, r, g, ψ and so is not infinite at any of the points $r=a, b, c, \ldots$. Further, since h, ψ are both rational functions of r, R must also be rational in r. It follows from these properties of R that we may substitute A for r in R, and then also in the last equation, which gives
$$\theta(h(A), A)=R(A)\psi(A)=0,$$
for we have $$\psi(A)=0.$$

Hence $h(A)$ satisfies an algebraic equation of the same form as that which defines $f(r)$; and we may write accordingly
$$f(A)=h(A),$$
and take this equation as the definition of $f(A)$. This method of proof is slightly amplified from that given by Frobenius for the case $A^{\frac{1}{2}}$ (*Berliner Sitzungsberichte*, 1896, p. 7).

It is easy to see that $h(A)$ is really equal to the sum of the residues of $(rE-A)^{-1}f(r)$ for the points $r=a, b, c, \ldots$; for as proved above $h(A)$ is the sum of the residues of $(rE-A)^{-1}h(r)$ at these points. Now we constructed $h(r)$ so that $[h(r)-f(r)]$, when expanded near $r=a$, should start with a term in $(r-a)^{\alpha}$; it follows that the residues of
$$(rE-A)^{-1}f(r) \text{ and of } (rE-A)^{-1}h(r)$$
are equal for the point $r=a$; the same holds for $r=b, r=c, \ldots$ by a similar proof. It will follow that
$$f(A)=h(A)=\Sigma\left[f(a)A_1+f'(a)A_2+\ldots+\frac{f^{\alpha-1}(a)}{(\alpha-1)!}A_{\alpha}\right],$$
just as before.

With regard to the priority of discovery it may be sufficient to

quote Frobenius (*Berliner Sitzungsberichte*, 1896, p. 11), who, after defining $f(A)$ in the way just described, says—"In dieser Weise hat Stickelberger in seiner akademischen Antrittsschrift *Zur Theorie der linearen Differentialgleichungen* (Leipzig, 1881) die allgemeine Potenz definirt und...benutzt. Eine weniger genaue Definition giebt Sylvester *Sur les puissances et les racines des substitutions linéaires* (*Comptes Rendus*, t. 94, 1882, p. 55)."

It has not been possible for me to consult this work of Stickelberger's, but from Frobenius's statement it seems clear that Stickelberger was the first author to publish a general definition of any power of a bilinear form or matrix[1]. Sylvester's definition is less exact, because it does not allow for the possibility that $|rE-A|$ may have repeated factors; and the same objection applies to his definition of any function of a matrix[2], which is

$$f(A)=\Sigma f(a)\,\frac{(A-bE)(A-cE)\ldots(A-lE)}{(a-b)(a-c)\ldots(a-l)},$$

where $|rE-A|=(r-a)(r-b)(r-c)\ldots(r-l)$. It is easy to see that this definition is included in Frobenius's as a special case; but the latter seems easier for purposes of actual calculation, even when Sylvester's can be applied. Sylvester has used his formula to find the square-root of a quaternion[3].

Buchheim and Taber were led independently to extensions of Sylvester's formula to the case of repeated factors in $|rE-A|$. We shall now indicate how their extensions are related to Frobenius's form already given. Buchheim's result[4] seems to have been the first definition of any function of a matrix in the case when $|rE-A|$ has repeated factors; it is not very different from one of the definitions already obtained, though he determines the function which we have called $\psi(r)$ by the fact that $\psi(r)=0$ is the equation of lowest degree satisfied by $r=A$; no method being given for finding $\psi(r)$ when A is known. If now we write

$$\theta(r)=(r-a)^{\alpha}\frac{f(r)}{\psi(r)},\quad \psi_a(r)=\frac{\psi(r)}{(r-a)^{\alpha}},$$

Buchheim's formula is equivalent to

$$f(A)=\Sigma\psi_a(A)\left[\theta(a)E+\theta'(a)(A-aE)+\ldots\right.$$
$$\left.+\frac{\theta^{\alpha-1}(a)}{(\alpha-1)!}(A-aE)^{\alpha-1}\right].$$

[1] Cayley (1858) had obtained the expression for any power of a 2-rowed matrix; but his method seems quite impracticable in general.

[2] *Johns Hopkins Univ. Circulars*, 3 (1882), pp. 9 and 210.

[3] *Phil. Mag.*, 5th Series, vol. 16, pp. 267 and 394; vol. 17, p. 392; vol. 18, p. 454 (1883—84).

[4] *Phil. Mag.*, 5th Series, vol. 22 (1886), p. 173.

To show that this is the same as Frobenius's formula, let us consider the value of

$$H(s) = \Sigma \psi_a(s) \left[\theta(a) + \theta'(a)(s-a) + \ldots + \frac{\theta^{\alpha-1}(a)}{(\alpha-1)!}(s-a)^{\alpha-1} \right],$$

where s is any quantity.

But

$$\psi_a(s) \left[\theta(a) + \theta'(a)(s-a) + \ldots + \frac{\theta^{\alpha-1}(a)}{(\alpha-1)!}(s-a)^{\alpha-1} \right]$$

is the residue at $r = a$ of the function

$$\psi_a(s) \frac{\theta(r)(s-a)^{\alpha}}{(s-r)(r-a)^{\alpha}} = \frac{f(r)\,\psi(s)}{(s-r)\,\psi(r)}.$$

We have accordingly to consider the sum of the residues of this function at $r = a, b, c, \ldots$; now the residue of $f(r)/(s-r)$ is zero for each of these points (assuming as before that $r = a, b, c, \ldots$ are not singular points for $f(r)$) and thus we may replace $H(s)$ by the sum of the residues of

$$\frac{f(r)}{\psi(r)} \frac{\psi(s) - \psi(r)}{s - r} = \frac{f(r)}{\psi(r)} \chi(r, s),$$

at $r = a, b, c, \ldots$. Here $\chi(r, s)$ is a symmetrical polynomial in r, s and has the property that.

$$\chi(r, A) = (rE - A)^{-1} \psi(r).$$

Now $H(s)$ is a polynomial in s, and $H(A)$ is therefore found by writing A for s; we have just obtained a value for $H(s)$ which has a meaning when s is replaced by A, so we can write

$$H(A) = \text{sum of residues of } f(r)(rE - A)^{-1} \text{ for } r = a, b, c, \ldots.$$

Hence Buchheim's definition of $f(A)$ is equivalent to Frobenius's; though it may be remarked that the actual calculation of this formula is certainly much more tedious than that of Frobenius's.

In explaining Taber's extension of Sylvester's formula[1] it will be convenient to consider first some forms derived from A and given by Frobenius[2]. Let us denote by $A(s)$, $B(s)$, $C(s)$, ... the residues for $r = a, b, c, \ldots$ of the function

$$\frac{\psi(r) - \psi(s)}{\psi(r)(r-s)}.$$

[1] *American Journal of Math.*, vol. 16 (1893), p. 123; *Math. Annalen*, Bd. 46 (1895), p. 561.

[2] *Crelle's Journal*, Bd. 84 (1878), p. 54 (§ 13); the investigation given above is taken from the *Berliner Sitzungsberichte*, 1896, p. 604.

Then by Cauchy's theorem, as there are no poles of the function other than $r = a, b, c, \ldots$ in the finite part of the r-plane, we have

$$A(s) + B(s) + C(s) + \ldots = 1.$$

But, considering the mode of formation of $A(s)$, it is clear that $A(s)$ is divisible by all the factors of $\psi(s)$ except $(s-a)^\alpha$; or $A(s)$ is divisible by $(s-b)^\beta (s-c)^\gamma \ldots$. In the same way $B(s)$ will be divisible by $(s-a)^\alpha (s-c)^\gamma \ldots$ and so on.

It follows that $[A(s) \,.\, B(s)]$ and every similar product must be divisible by $\psi(s)$; hence by the relation between $A(s)$, $B(s)$, $C(s)$, ... we see that $[A(s)]^2 - A(s)$ is also divisible by $\psi(s)$.

Since $A(s)$, $B(s)$, $C(s)$, ... are rational algebraic functions of s, we may put A for s in each of them; let the resulting forms be denoted by $A_a, A_b, A_c, \ldots$[1].

Then since $\psi(A) = 0$ we have

$$A_a^2 = A_a, \quad A_a A_b = 0,$$

and

$$A_a + A_b + A_c + \ldots = E.$$

Further, we have that $(s-a)^\alpha A(s)$ is divisible by $\psi(s)$ and so on replacing s by A

$$(A - aE)^\alpha A_a = 0.$$

But we have

$$\frac{(r-a)^\alpha - (s-a)^\alpha}{(r-a)^\alpha (r-s)} = \frac{1}{r-a} + \frac{s-a}{(r-a)^2} + \ldots + \frac{(s-a)^{\alpha-1}}{(r-a)^\alpha},$$

and combining this with our last result we see that

$$(rE - A)^{-1} A_a = \left[\frac{E}{r-a} + \frac{A-aE}{(r-a)^2} + \ldots + \frac{(A-aE)^{\alpha-1}}{(r-a)^\alpha}\right] A_a.$$

Remembering that $\Sigma A_a = E$, we now see that

$$(rE - A)^{-1} = \Sigma A_a \left[\frac{E}{r-a} + \frac{A-aE}{(r-a)^2} + \ldots + \frac{(A-aE)^{\alpha-1}}{(r-a)^\alpha}\right],$$

for A_a and A are commutative, as A_a is a function of A only.

Now taking the sum of the residues of $(rE - A)^{-1} f(r)$ we see that

$$f(A) = \Sigma A_a \left[E f(a) + (A - aE) f'(a) + \ldots + \frac{(A-aE)^{\alpha-1}}{(\alpha-1)!} f^{\alpha-1}(a)\right].$$

[1] It is easy to see that $A_a = A_1$, $A_b = B_1, \ldots$ where $A_1, B_1, C_1, \ldots$ are defined as before (p. 80). In fact Frobenius's proof in *Crelle* starts from the functions defined as $A_1, B_1, C_1, \ldots$ have been.

This expression is of the same form as Taber's; but his definitions of the functions corresponding to A_a, A_b, A_c, ... are much longer than those given above and are superficially, at any rate, very different. Taber states[1] that his functions have the same characteristic properties, namely

$$A_a^2 = A_a, \quad A_b^2 = A_b, \ldots$$
$$A_a A_b = 0, \quad A_b A_c = 0, \ldots$$
$$A_a + A_b + A_c + \ldots = E,$$

and this seems to justify the assumption that the two sets of functions are really the same. It should, however, be observed that, in Taber's final result, the indices are not the same as our $\alpha, \beta, \gamma, \ldots$; but are the indices of the factors $(r-a)$, $(r-b)$, $(r-c)$, ... in $\phi(r) = |rE - A|$. These indices will be in general greater than $\alpha, \beta, \gamma, \ldots$ and their use may lead to the retention of various terms in the value for $f(A)$ which actually vanish; for, we have seen that $A_a(A - aE)^l = 0$ whenever $l > (\alpha - 1)$. This modification of Taber's form arises from the fact that, although $\phi(A) = 0$, yet in general $\phi(r) = 0$ is not the equation of lowest degree satisfied by $r = A$.

3. *The invariant-factors of any function of a bilinear form.*

Frobenius has proved (*Crelle*, Bd. 84, pp. 24, 25) that if $f'(r) \neq 0$ at $r = a, b, c, \ldots$, then $f(A)$ has invariant-factors[2] $[r - f(a)]^\alpha$, $[r - f(b)]^\beta$, ... corresponding to the ones $(r-a)^\alpha$, $(r-b)^\beta$, ... of A. I have had, however, occasion to calculate the invariant-factors for some functions which do not satisfy these conditions; and it seems as if a note on the general theory of such a case might be of interest.

Suppose then that for $r = a$ we have

$$f(a) = 0, \; f'(a) = 0, \; \ldots, \; f^{k-1}(a) = 0,$$

but
$$f^k(a) \neq 0;$$

we shall investigate the invariant-factors of $f(A)$ which correspond to the single one $(r-a)^\alpha$ of A.

[1] For the proofs of these properties, Taber refers to a paper of his, which I have not been able to find. An account of Taber's other papers may be found in the *Clark University Decennial Volume* (Worcester, Mass. 1899) where there is, however, no reference to Stickelberger's or Buchheim's papers.

[2] Here by the invariant-factors of a bilinear form is meant those of the characteristic determinant of the form; e.g. the invariant-factors of A imply those of $|rE - A|$.

It is known that a substitution P can be found[1] such that

$$A = P(A_1 + A_2) P^{-1},$$

where, corresponding to the invariant-factor $(r-a)^\alpha$, we have

$$A_1 = a(x_1y_1 + \ldots + x_\alpha y_\alpha) + x_2y_1 + x_3y_2 + \ldots + x_\alpha y_{\alpha-1}$$
$$= aE_1 + C_1 \text{ say,}$$

and A_2 does not contain any of the variables $x_1, \ldots, x_\alpha, y_1, \ldots, y_\alpha$. Now we have

$$f(A) = f[P(A_1 + A_2)P^{-1}] = Pf(A_1 + A_2)P^{-1},$$

so that $f(A)$ has the same invariant-factors as $f(A_1 + A_2)$. Again, since A_1, A_2 have no common variables, we have

$$f(A_1 + A_2) = f(A_1) + f(A_2) = f(A_2) + f(aE_1 + C_1)$$
$$= f(A_2) + E_1 f(a) + C_1 f'(a) + \frac{C_1^2}{2!} f''(a) + \ldots + \frac{C_1^{\alpha-1}}{(\alpha-1)!} f^{\alpha-1}(a).$$

To see that the last line is correct we have only to calculate $(rE_1 - A_1)^{-1}$ and expand in powers of $(r-a)$. We find that

$$(rE_1 - A_1)^{-1} = \frac{E_1}{r-a} + \frac{C_1}{(r-a)^2} + \frac{C_1^2}{(r-a)^3} + \ldots + \frac{C_1^{\alpha-1}}{(r-a)^\alpha},$$

and so our expression for $f(A_1)$ follows by what has been proved before.

But in the present case we have

$$f'(a) = 0,\ f''(a) = 0,\ \ldots,\ f^{k-1}(a) = 0,$$

and thus here

$$f(A_1 + A_2) = f(A_2) + E_1 f(a) + \frac{C_1^k}{k!} f^k(a) + \ldots + \frac{C_1^{\alpha-1}}{(\alpha-1)!} f^{\alpha-1}(a).$$

So we have to calculate the invariant-factors of the determinant

$$|rE - f(A_1 + A_2)| = |\{rE_1 - f(A_1)\} + \{rE_2 - f(A_2)\}|;$$

since $\{rE_1 - f(A_1)\}$ and $\{rE_2 - f(A_2)\}$ have no common variables we may calculate their invariant-factors separately. So consider

[1] For the actual determination of P we may consult Jordan, *Cours d'Analyse*, t. 3, p. 175 et seq.; also a series of papers by Burnside, Baker, Dixon and the author in volumes 30—32 of the *Proceedings of the London Math. Society* (1899—1900).

the expansion of $[rE_1 - f(A_1)]^{-1}$ in powers of $p = r - f(a)$; it is readily seen to be

$$(pE_1 - F_1)^{-1} = \frac{1}{p} E_1 + \frac{1}{p^2} F_1 + \frac{1}{p^3} F_1^2 + \ldots + \frac{1}{p^q} F_1^{q-1},$$

where $$F_1 = \frac{C_1^k}{k!} f^k(a) + \frac{C_1^{k+1}}{(k+1)!} f^{k+1}(a) + \ldots + \frac{C_1^{\alpha-1}}{(\alpha-1)!} f^{\alpha-1}(a),$$

and q will be the index of the first invariant-factor to base p. In order to determine q, we note that $F_1^q = 0$ and $F_1^{q-1} \neq 0$. Hence we must have $qk \geqq \alpha$, while $(q-1)k < \alpha$, for $F_1^q = 0$ provided $C_1^{qk} = 0$ and C_1^α is known to be zero. If then we write $\alpha = kl + m$, where $m = 1, 2, \ldots, k$, we have $q = l + 1$. Clearly l is the quotient and $(m-1)$ the remainder in dividing $(\alpha - 1)$ by k.

To determine the number of invariant-factors which take the form p^q, we have a theorem of Stickelberger's[1] connecting this number with the rank of the form multiplied by $1/p^q$ in the expansion of $[rE_1 - f(A_1)]^{-1}$. This form is here $F_1^{q-1} = F_1^l$; its rank is thus the same as that of C_1^{kl}, or is $\alpha - kl = m$. Thus we have m invariant-factors p^q. To determine the remaining invariant-factors we have to apply an extension of Stickelberger's theorem[2] and calculate the rank of the coefficient of $1/p^{q-1}$ in our expansion; this rank will be ($2m$ + the number of invariant-factors p^{q-1}). Now the rank in question is that of $F_1^{q-2} = F_1^{l-1}$ which is that of $C_1^{k(l-1)}$; or is $\alpha - k(l-1) = m + k$. Thus we have $(k-m)$ invariant-factors each with index $q - 1 = l$.

Further $$m(l+1) + (k-m)l = kl + m = \alpha,$$

and so these k invariant-factors are all that correspond to the one $(r-a)^\alpha$, of the original form; which is verified by calculating the rank of the coefficient of $1/p^{s+1}$ in $(pE_1 - F_1)^{-1}$.

We have then the theorem—

If A be a bilinear form, one of whose invariant-factors is $(r-a)^\alpha$, and if $f(A)$ be a function of the form such that

$$f'(a) = 0,\ f''(a) = 0,\ \ldots,\ f^{k-1}(a) = 0,\ f^k(a) \neq 0$$

(while $f(a)$ may or may not be zero), then $f(A)$ will have k invariant-factors[3] corresponding to the one $(r-a)^\alpha$; of these m

[1] *Crelle's Journal*, Bd. 86 (1879), p. 42, Satz VII; see also Muth's *Elementartheiler*, p. 135, Satz XVI; and also the paper quoted next.

[2] The proof given by the author, *Proc. Lond. Math. Soc.*, vol. 32 (1900), p. 86, can be used to show that if

$(\lambda A - B)^{-1} = Z_1(\lambda - c)^{-e} + Z_2(\lambda - c)^{-e+1} + \ldots + Z_e(\lambda - c)^{-1} +$ positive powers of $(\lambda - c)$,

then the rank of Z_r is $[rm_1 + (r-1)m_2 + \ldots + m_r]$, where m_k denotes the number of invariant-factors of $|\lambda A - B|$ of the form $(\lambda - c)^{e-k+1}$.

[3] It ought, perhaps, to be pointed out, that these invariant-factors do not necessarily form *consecutive* sets; as there may be others $[r - f(a)]^{l+1}$ from other invariant-factors of the original form.

will be of the form $[r-f(a)]^{l+1}$ and $(k-m)$ of the form $[r-f(a)]^{l}$; where l is the quotient and $(m-1)$ the remainder when $(\alpha-1)$ is divided by k.

In particular, if $\alpha=1$, or the invariant-factors of A to base $(r-a)$ are *linear*, we have (as remarked by Frobenius) linear invariant-factors $[r-f(a)]$ of $f(A)$, whatever may be the value of k. Another special case is when $\alpha \gtreqless k$, and we have α linear invariant-factors $[r-f(a)]$. Of this Frobenius has given an example[1]; in his results we have

$$f(a)=1,\ f'(a)=0=f''(a)=\ldots=f^{\alpha-1}(a),$$
$$f(b)=0=f'(b)=\ldots=f^{\beta-1}(b),$$
$$f(c)=0=f'(c)=\ldots=f^{\gamma-1}(c).$$

Hence we have here $(\alpha+\alpha'+\alpha''+\ldots)$ linear invariant-factors $(r-1)$; all the others being simply r. Thus the equation satisfied by $f(A)$ is

$$[f(A)]^2-f(A)=0,$$

as proved by Frobenius.

It will be observed that in case we have

$$f'(a)=0=f''(a)=\ldots=f^{k-1}(a),$$

it will in general be impossible to find a function g, such that $g(f(A))=A$. For, by what has been proved, corresponding to the single invariant-factor $(r-a)^{\alpha}$ of A, we have k invariant-factors of $f(A)$; and thus at least k of $g(f(A))$; so, unless either $\alpha=1$ or $k=1$, there will be more invariant-factors of $g(f(A))$ than of A, and the equality is impossible[2].

In conclusion it may be well to point out that there is no difficulty in verifying my results on the invariant-factors of $f(A)$ by direct calculation of the H.C.F. of successive minors. In fact my results were originally worked out in this way; but it appeared very long to give a satisfactory description of the process, while the method given above is comparatively short although apparently a less direct way of attacking the problem.

[1] *Berliner Sitzungsberichte*, 1896, p. 601; the functions are those defined at the end of part 2 above.

[2] Frobenius, *Crelle*, Bd. 84, p. 14 (Satz IV, V).

On the question as to whether or not there are any free charged ions produced during the combination of hydrogen and chlorine; and on the effect produced on the rate of the combination by the presence of such ions. By J. J. THOMSON, M.A., F.R.S.

[*Received* 16 January 1901.]

The experiments described below were made to see whether or not free ions charged with electricity are produced when hydrogen and chlorine are combining under the influence of light.

The electrical conductivity of a gas affords a very delicate test for the presence of these ions, as the rate of leak of electricity from a gold-leaf electroscope will be affected to an appreciable extent by the presence of a few thousand ions per cubic centimetre; and as the number of molecules of a gas at atmospheric pressure in the same volume is about 10^{20}, it will be evident that the electrical test is one of exceptional delicacy.

The electrical conductivity of the mixture of hydrogen and chlorine was measured by observing the rate at which the separation of the leaves of a gold-leaf electroscope immersed in the gas diminished; (in some experiments the leaves of the electroscope were made of platinum-foil). Special care was taken with the insulation of the electroscope, the same arrangement as that used by Mr C. T. R. Wilson[1] was employed; in this form of electroscope the leaves are fastened to an ebonite rod which passes through the sides of the vessel containing the mixed gases, a metal ring is placed round this rod between the gold-leaves and the junction of the rod with the vessel, the gold-leaves are charged up to a high potential by means of a battery of small storage cells, and the ring is *kept* in connection with this battery so that it is maintained permanently at the same potential as the original potential of the leaves; it is evident that with this arrangement any diminution in the divergence of the leaves must be due to a leak through the gas and not along the insulating support.

Such an electroscope was placed in a vessel which was filled with a mixture of equal volumes of hydrogen and chlorine prepared by electrolysis. The arrangements for the preparation of the gases and the measurement of the amount of hydrochloric acid gas formed being the same as those used by Bunsen and Roscoe in their researches on the combination of these gases.

The combination of these gases may be divided into two stages, for it has been shown by Draper and later by Pringsheim

[1] C. T. R. Wilson, *Proc. Camb. Phil. Soc.*, Vol. XI. p. 32.

that the first effect of exposing the mixed gases to light is a considerable increase in volume without any production of HCl, then after an interval which depends on the intensity of the light the formation of HCl begins, and when once started persists.

The conductivity of the mixed gases was tested in both these stages, the gases being exposed to diffuse and not very bright daylight; many experiments were made but in no case was any increase observed in that small rate of leak which exists in these as well as in other gases, even when not exposed to light. It thus appears that both during the combination of hydrogen and chlorine as well as in the preliminary stage marked by an increase in volume there are no free charged ions.

In the next place an experiment was tried to see whether the artificial production of free charged ions in the mixed gases would affect their rate of combination. In this experiment the mixed gases were exposed to Röntgen rays which produced free ions in such numbers that the leaves of the electroscope collapsed in a few seconds, but no effect could be detected either in promoting combination when the gases were kept in the dark or in altering the rate of leak when the gases were exposed to diffuse daylight. The radiation from thorium was also tried, but it was without effect.

PROCEEDINGS

OF THE

Cambridge Philosophical Society.

Notes on some of the Rarer or more Interesting Fungi collected during the past year. By Professor H. MARSHALL WARD, Sc.D., F.R.S.

[*Read* 4 February 1901.]

The following notes are offered in the hope that, if they do not add much to our knowledge of the life history of any particular forms in detail, they are useful as records of species and facts—some of them new—which have come under my observation in the field and in the laboratory. They are by no means to be regarded as the results of attempts to find or to found new species, but simply as facts which have turned up during work—chips from a workshop, as it were, and the only merit that can be claimed for them is that they are fresh from the hands of the workers.

Synchytrium Succisæ, De By.

During one of our botanical excursions in the Fens in the summer, I found the young leaf-rosettes of *Scabiosa Succisa* covered with the small glistening yellow pimples produced by De Bary's *Synchytrium Succisæ*. Examination revealed the presence of the zoosporangia in the hypertrophied cells, and uni-flagellate zoospores escaped in water. All the details so far observed confirm the accuracy of Schröter's figures and description[1]. The record of finding *S. Succisæ* in Cambridgeshire is interesting, since it is I believe the first time this fungus has been discovered in England, and it has only once been found in Scotland, by Prof. Trail of Aberdeen[2].

[1] Cohn's *Beitrage zur Biol.* Bd. I. 1870.
[2] *Scott. Nat.* 1899, No. XXIV. p. 58.

Thamnidium elegans, Link., *Chaetocladium Jonesii*, Fres., *Syncephalis cordata*, Van Tiegh, *Piptocephalis Freseniana*, De By., and *Sporodinia grandis*, Link., have been brought in and determined by Mr Biffen, and it is possible that a form now under investigation may be *Syncephalastrum*, Schröt. To these I can add *Phycomyces nitens*, Kunze and Schmidt, a remarkable large Mucor-like form which, owing to Dr Plowright's kindness, I saw in immense quantity on the débris of burnt oil and cake after a fire at a mill at Lynn some time ago. It is interesting as a fat-splitting fungus. *Chaetocladium* and *Piptocephalis*, it will be noted, are parasitic on another fungus, *Mucor*, and there is an interesting field for further research here awaiting some investigator with the necessary patience and enthusiasm. According to a statement in Massee's *British Fungi*[1], *Piptocephalis* was still unrecorded for Britain, and Mr Biffen's find appears to be new.

Protomyces macrosporus, Unger., a species rendered classical from De Bary's researches and Brefeld's theoretical views as to its morphology, is probably commoner than is usually supposed. I find it in abundance on *Ægopodium podagraria* growing by the ditch in West Road.

Doassansia Sagittariæ, Fischer.

I have found this interesting member of the Ustilagineæ on *Sagittaria Sagittifolia* near Barton. Very similar patches are often produced on the leaves by quite a different fungus. However, I am not now concerned specially with the fungi of Cambridgeshire, of which I hope an account may be given at a later date.

Endomyces Magnusii, Ludwig.

In 1886, Ludwig[2] drew particular attention to this curious fungus, occurring in the slime of trees affected with the disease known as slime-flux. A few weeks ago Mr Biffen brought in a species of *Endomyces*, found in the slime from an Elm in Cambridge, which agrees in important points with Ludwig's form, though there are other points which raise doubts that it may be *E. decipiens*, Tul., or possibly even a new species. I believe this is the first recorded discovery of this fungus in England.

The fungus is particularly interesting theoretically in connection with Brefeld's attempt to found a new system of classification of the Ascomycetes.

Whether new or not, the fungus is undoubtedly interesting, and its further behaviour is being studied.

[1] Reeve & Co. 1891, p. 106.
[2] *Ber. d. d. bot. Ges.* 1886.

Saccharomyces anomalus, Harss., described in the *Annals of Botany*[1] in June last, and several other yeasts now under observation are the nearest allies we have to these *Endomyces*. Mr Barker is at present working on another yeast which appears to be quite new and is remarkable in several important respects.

From these lower Ascomycetes we may pass to the following, *Ctenomyces crispatus*, Eid., and *Gymnoascus Reessii*, Bar., both brought in by Mr Biffen; *Onygena equina*, Willd., a fungus which digests horn, and the life-history of which I worked out in 1899.

Cordyceps ophioglossoides, Ehrp.

I found this in Rothiemurchus Forest last autumn, having nearly stepped on a patch of the dark olive club-shaped ascophores.

On digging, the fungus was easily traced to a brilliant yellow mycelium running in the tuber-like fructifications of *Elaphomyces variegatus*, one of the false-truffles.

An interesting point, shown in the museum specimen, is that the mycelium of the *Elaphomyces* is traceable into the roots of the Pine, on which it forms mycorrhiza. Hence, as the specimen exhibited shows, we have the *Cordyceps* parasitic on the *Elaphomyces*, and the latter on the Pine roots, all linked up by their respective mycelial strands.

Most species of *Cordyceps* occur on Insects, which they kill below ground, and some authorities remove *C. ophioglossoides* to another genus—*Cordylia*. It has also been known as *Torrubia* in the past.

The fungus is now being investigated further.

Leotia lubrica, Pers., seems at least worth mentioning. I gathered it in Yorkshire this autumn. The beautiful green alcoholic extract obtained from the olive-green ascophores would appear to be worth examination.

Of Discomycetes we have had several interesting species. Apart from species of *Exoascus* and *Taphrina*, those curious forms which cause the deformed "pocket plums" in *Prunus*, and the queer "Witches Brooms" of Birches, Alders, &c., I may note the frequent recurrence of *Peziza omphalodes*, Bull, rendered classical by Kihlman's work[2] and Harper's recent confirmation of it[3], on the pots of sand, sterilised by heat, employed in Miss Dawson's cultures. Frank also pointed out how apt this species is to occur on soil which has been heated. There are several such fungi which affect burnt substrata, and interesting problems arise in connection therewith.

[1] Barker, "A Fragrant Yeast," *Annals of Botany*, June 1900, p. 215.
[2] Kihlman. See De Bary, *Morph. and Biol. of Fungi*, p. 206.
[3] Harper, *Ann. of Bot.* Sept. 1900.

Sclerotinia Duriæana, Tul., Quél.[1]

This interesting peziza is comparatively abundant in some of the Fens, on *Carex stricta*, as was shown me by Dr Plowright. The sclerotia fit into the triangular leaves and stems, and when freed have a curious resemblance to Ergot. They germinate freely, and I found the peziza form fully developed in April, in a Fen in Norfolk.

The ascospores germinated readily in water, and gave rise to the curious flask-shaped bodies emitting minute conidia, which have been described by Woronin[2] for other species of *Sclerotinia*.

In richer food-materials, the ascospores gave rise to large mycelia, from which however no other spores were obtained: this matter was not followed up further.

On living leaves of *Carex* the ascospores germinated freely, but although the small mycelia developed flask-shaped condiophores, no infection was observed.

It still remains to be decided whether the fungus is truly parasitic.

Among the Uredineæ may be mentioned *Puccinia Andersoni*, B. and Br.[3], which I found on *Carduus heterophyllus*—itself an interesting plant, by the bye—in Yorkshire, forming dark violet or black patches on the leaves.

Coleosporium Senecionis.

This Uredine was collected in some quantity in its æcidial stage (*Peridermium Pini*) in the early summer in Norfolk on the needles of *Pinus Laricio* and *Austriaca*, and in its Uredo stage later in the summer, on *Senecio sylvaticus* growing beneath the same pines. Miss Dawson has successfully infected plants of *Senecio sylvaticus* and *S. vulgaris* with the æcidiospores, and we are awaiting the results of the reciprocal infections of pine-seedlings. Infection experiments have also been made on other species of *Senecio*, the results of which will appear in due course: *S. Jacobæa*, *S. erucifolius* and *S. aquaticus* apparently refuse infection.

Næmatelia encephala, Fr.

Though by no means uncommon in the Scotch Pine-forests, this fungus is often overlooked and is by no means well known. I found it in abundance this last autumn on dead Pine-twigs, and cultivated it with some success. The basidia are divided by vertical walls, and my results fully bear out Brefeld's[4] confirmation

[1] See Tulasne, *Carpologia.*
[2] Woronin, *Mém. de l'Acad. de St Pétersb.* T. xxxvi. 1888.
[3] *Ann. Nat. Hist.* 1875. Vol. xv. 4th ser. p. 35. No. 1464.
[4] *Unters. aus d. Ges. Geb.* H. vii. p. 107.

of Tulasne's[1] suspicion that *Næmatelia* is nothing but a partially arrested condition of a *Tremella*. In cultures the basidiospores developed numerous yeast-conidia, which go on budding indefinitely in suitable media.

Another yeast-form which I obtained from pine-twigs at Brandon, was investigated at length, and found to develop into a form of *Næmatelia*—or an allied fungus—with orange-coloured, beautifully veined, gelatinous thallus, on which pyriform bodies resembling chlamydospores arose later. These could not be made to germinate, and I regard them as arrested *basidia*. Since the details are being published, with figures, in the forthcoming volume of the *Transactions of the British Mycological Society*, it is unnecessary to go further into the matter here.

Passing now to the Basidiomycetes proper, I found *Exobasidium vaccinii*, Wor., very common on *Vaccinium uliginosum* in Scotland in September, distorting both leaves and stems, and in many cases the deformed pink and white organs were spotted black with a *Dematium*-like fungus which attacks the diseased tissues.

Hydnum imbricatum, L.

Apart from its rarity, this fungus is interesting on account of its size and beauty. We found it in great quantity near Aviemore, and the larger specimens were from eight inches to a foot in diameter, though authorities—except Hennings[2]—put the size much smaller. Hennings also puts this species into a new genus *Phæodon* on account of its brown spores. I was much impressed by the large number of species of *Hydnum* gathered on Speyside this autumn, having myself seen nine, among which this rare *H. compactum*, Pers., is noteworthy if only for the excellence of the specimen: I gathered it near Aviemore.

The rare *H. erinaceum*, Bull, was sent us from the New Forest.

Boletus sulphureus, Fr.

This was found by Mr Plowright of St John's, during one of our botanical excursions in 1899, in a heap of sawdust, and I was again with him when he gathered it in the same spot this last autumn.

Strobilomyces strobilaceus, Berk.

I have twice seen this rare Boletus during the past summer: once in Scotland, where Mr Rea showed me a specimen he had gathered, and once in Yorkshire, where Mr Crossland showed it me *in situ*.

[1] Tulasne, *Ann. Sc. Nat.* T. xix. 3rd ser. p. 203.
[2] *Pflanzenfam.* I. Th. 1 Abt. p. 149.

So far as I am able to judge, this species appears to bear a similar relation to *Boletus* to that which *Lepiota rachodes* bears to *L. procera*: that is to say the pileus becomes so shaggy that it breaks up into scale-like groups of hairs. I have seen a specimen of the common mushroom similarly strobilaceous, and scaly examples of *Boletus scaber* suggest how simple the change might be. Winter[1] keeps it in the genus *Boletus*, and most authors seem to hesitate about placing it separately, though they still do so. Massee, however[2], points out that the spores are warted.

Trametes Pini, Fr.

This destructive tree-killing parasite was seen at Aviemore on several of the Scotch pines, and specimens were collected. Hartig[3] worked out the peculiarities of its action on the wood, the tracheids of which are isolated by the solution of the middle lamella, and their walls delignified by enzymes secreted by the hyphæ, and are transformed into soft cellulose and finally dissolved: since this occurs in patches and streaks, the injured wood exhibits very characteristic marks, from which the presence of the fungus can be inferred even in the absence of fructifications. Much damage accrues from the ravages of this parasite in some parts of Germany, but it appears to be little known with us.

Passing to the Agaricini, *Amanitopsis vaginata*, Roze., may be mentioned, since, though common, it is now made the type of a new genus—an *Amanita* without the ring: I gathered the specimen at Aviemore.

Nyctalis asterophora, Fr.

This was obtained, during an excursion in Norfolk with Dr Plowright, growing on *Russula nigricans*, and exhibits the quaint hymenophores powdered with the stellate spores, regarding which so much controversy was maintained. They are now known to be *chlamydospores*[4] which are formed in addition to ordinary oidia and basidiospores in this remarkable fungus.

Galera tenera, Schaeff.

This is not an uncommon fungus in grassy spots, but our modern English floras do not record it as a coprophilous form. It sprang up on horse-dung in the laboratory, and on cultivation in gelatine media I obtained the characteristically curved series of *oidia* described by Brefeld[5], confirming the accuracy of his figures for this species in all respects.

[1] *Kryptogamen Flora*, vol. I. p. 463.
[2] *Brit. Fungus Flora*, vol. I. p. 257.
[3] *Zersetzungserscheinungen des Holzes.*
[4] See Bref., *Untersuchungen*, H. VIII. Pl. V.
[5] Brefeld, *l.c.* p. 51.

Hypholoma appendiculatus, Bull.

This again cannot be regarded as rare, but I obtained specimens in October of a form regarded by an excellent judge as a *Bolbitius*. This is interesting because *Hypholoma hydrophilus*, Bull, a form very close to *H. appendiculatus*, Bull, was named *Bolbitius* by Fries.

I have germinated the spores, and find their behaviour agrees in all essential respects with that of the species of *Hypholoma* investigated by Brefeld[1], and in a fortnight or so I obtained excellent cultures of the characteristic tufts of *oidia*, thus adding one more to the list of Basidiomycetes which produce secondary asexual spores of this description.

Stropharia albocyanea, Desm.

Recorded in Stevenson[2] as "uncommon," is by no means rare in Cambridgeshire and Norfolk. *S. æruginosa*, Curt., also common, sprang from mycelium on Asparagus roots brought in by Mr Hill, and which I cultivated in the laboratory this summer: it appeared to be parasitic. *Geaster mammosus*, Chev., has been found at Brandon by Mr Biffen, and we find *G. fimbriatus*, Fr., common at Wimpole and elsewhere in the County. *Sphærobolus stellatus*, Tode, is also abundant, as well as *Crucibulum vulgare*, Tul. More remarkable are *Nidularia confluens*, Fr., and *N. pisiformis*, Tul., both from Shouldham in Norfolk, where I have seen the former in great abundance in a cart-rut. *N. pisiformis* is instructive as showing how a *Nidularia* may be derived from such a form as *Octaviana* if the sporangial chambers of the latter become more individualised. *Mutinus caninus*, Fr., was sent us by Dr Plowright, to whom I owe much for specimens and information.

The beautiful exotic *Dictyophora phalloidea* was brought by Mr Yapp from the Malay Peninsula.

Among the rarer or more noteworthy species of larger Fungi which have come under my observation during the past year may be mentioned the following, gathered by various members of the British Mycological Society during a few days' collecting in company with Dr Keith, Dr Stevenson, Dr Plowright and others in Rothiemurchus Forest and neighbourhood last September.

Lepiota cinnabarina, A. and S.; *L. amianthina*, Scop.; *Armillaria robusta*, A. and S.; *Tricholoma equestre*, L.; *T. spermaticum*, Fr.; *T. pessundatum*, Fr.; *T. melaleucum*, Pers.; *Clitocybe fumosa*, Pers.; *Mycena rosella*, Fr.; *Pleurotus acerosus*, Fr.; *Entoloma Bloxami*, B and Br.; *E. jubatum*, Fr.; *Claudopus depluens*, Batsch.;

[1] Brefeld, *Unters. aus dem Gesammt Geb. d. Mykol.* 1889, H. VIII. p. 44.
[2] *Hymenomycetes Britannici*, vol. I. p. 310.

Inocybe hirsuta, Lasch.; *I. fastigiata*, Schaeff; *Flammula sapinea*, Fr.; *F. scamba*, Fr.; *Tubaria paludosa*, Fr.; *Stropharia albocyanea*, Desm.; *Hypholoma capnoides*, Fr.; *H. lachrymabundus*, Fr.; *Cortinarius varius*, Fr.; *C. multiformis*, Fr.; *Paxillus atrotomentosus*, Fr.; *Hygrophorus agathosmus*, Fr.; *Lactarius hysginus*, Fr.; *L. glyciosmus*, Fr.

Boletus bovinus, L.; *B. impolitus*, Fr.; *Polyporus Schweinitzii*, Fr.; *P. fragilis*, Fr.; *P. amorphus*, Fr.; *Trametes Pini*, Fr.; *Merulius pallens*, Berk.

Hydnum fragile, Fr.; *H. aurantiacum*, A. and S.; *H. ferrugineum*, Fr.; *H. scrobiculatum*, Fr.; *H. zonatum*, Batsch.; *H. melaleucum*, Fr.; *Sistotrema confluens*, Pers.; *Corticium sanguineum*, Fr.

Clavaria amethystina, Bull; *C. stricta*, Pers.; *Rhizopogon rubescens*, Tul.; *Helvella lacunosa*, Afzel, and *H. elastica*, Bull; *Peziza violacea*, Pers.; and the very pretty Myxomycete *Tubulina cylindrica*, Bull.

A more complete list of the fungi gathered on Speyside will appear in the forthcoming volume of the *Transactions of the British Mycological Society*.

The following are among the more remarkable species collected last autumn in the neighbourhood of Halifax, Yorks., where Mr Crossland kindly took me over some ground well known to mycologists.

Boletus porphyrosporus, Fr.; *Fomes variegatus*, Secr.; *Hygrophorus Colemannianus*, Blox.; *H. nitratus*, Pers.; *Inocybe plumosa*, Bolt.; *I. asterospora*, Quél.; *Russula ochracea*, Fr.; *R. sanguinea*, Fr.; and *Cortinarius decolorans*, Fr.

Notes on Artificial Cultures of Xylaria. By Miss E. DALE (communicated by Professor Marshall Ward).

[*Read* 4 February 1901.]

Two species, *Xylaria polymorpha* and *X. Hypoxylon*, have been cultivated from the ascospores, which germinated in various nutritive media. The fungus was then grown upon sterilized wood—beech, oak, and silver fir, on which it formed a dense flocculent mycelium, at first white and later grey and then still darker. After 3 or 4 months cylindrical conidiophores arose, whose length varied from about 1 to 3 cms. In *X. polymorpha* each bore numerous conidia on its upper end. In *X. Hypoxylon* the conidiophores have so far been sterile. In the artificial cultures these stromata have not yet developed further. Those

which have produced conidia ultimately shrink and become overrun by the flocculent parts of the mycelium. The sterile stromata are still growing. Free conidiophores were also produced in *X. polymorpha*, while in *X. Hypoxylon* conidia were developed on small tufts of hyphae united into coremium-like bodies. Up to the present, only the conidia from the free conidiophores of *X. polymorpha* have germinated[1]. They give rise to a thin mycelium which slowly spreads over the substratum and which is very unlike the dense flocculent masses so rapidly formed from ascospores, although the young mycelia as seen under the microscope are very similar in both cases.

The structure of these conidiophores as seen in microtome sections is remarkably like that of certain Basidiomycetes. The centre of the organ consists of a dense mass of hyphae, somewhat interlacing but running approximately parallel to the long axis. Towards the periphery the tissue becomes looser and bends outwards, forming parallel branches which stand close together like a hymenium. The branches are septate and each forms a single spore on a sterigma-like process which, after the abstriction of the spore, remains as a pointed prolongation of the spore-bearing branch.

The action of the fungus in destroying the wood which forms its substratum shews some points of interest. Pieces of wood were hardened in Flemming's solution, and transverse, radial, and tangential sections were cut. The hyphae had penetrated into all the tissues and could be seen running along the medullary rays and down the vessels, wood-fibres, and wood parenchyma. The hyphae varied considerably in thickness, the larger ones being chiefly in the wide vessels. In all cases the mycelium passed from cell to cell by means of the pits. One of the most striking points is the way in which the wood-fibres are affected. In radial sections some of the hyphae in the fibres are seen to run in a straight line, while others take a spiral course and are united to the straight hyphae by connecting branches. The explanation of this seems to be that the hyphae make their way first into the often narrow lumen of a fibre, and that from these hyphae there are formed branches which penetrate the pits and grow spirally round in the wall, decomposing it as they go, evidently by excreting some wood-destroying enzyme. Often two or three hyphae may be seen forming as many spirals round a fibre and lying in channels which they have made in the cell-wall.

[1] Since the above notes were written conidia from old conidiophores of *Xylaria polymorpha* have germinated.

The Habits and Development of some West African Fishes. By J. S. BUDGETT, M.A.

[*Read* 4 February 1901.]

Our knowledge of the fish fauna of the fresh waters of Africa has of late been greatly extended by the study of collections from the great African lakes, the Nile, and the rivers of the West Coast. Up to the present time, however, nothing has been known about the breeding habits and development of any of the most interesting forms, including Polypterus, Protopterus, Gymnarchus, Mormyrus and Heterotis. It was with a view to investigating the development of these fish and especially of Polypterus that I spent the summer of 1900 on McCarthy Island on the river Gambia. The flooded lands of this island I searched persistently from June to September, but failed to obtain the eggs of Polypterus; I did, however, obtain a very young larva of Polypterus measuring 1¼ inch in length.

In this larva the dermal bones are not yet developed over the general body surface, though some of the dermal bones of the head have already begun to ossify. The dorsal finlets at this stage are merely a continuation forwards of the finfold of the tail. The heterocercy of the caudal fin is scarcely more apparent (even in section) in this larva than in older specimens. The external gill is of very great size. The base of the shaft is situated immediately behind the spiracle, and is supported by a short segmented rod of cartilage borne upon the hyomandibular bar. Each pinna of the external gill bears a double row of pinnules. Alternate pinnae on each side are smaller and directed at a different angle of the intermediate pinnae, giving the appearance of two rows of pinnae on either side. The internal gills are very small and can as yet be of little functional importance, as the combined section of the arteries to the internal gills is certainly not a tenth part of the section of the artery supplying the external gill. The arteries to the two halves of the swim-bladder are likewise very small indeed. In this young larva the roof of the mouth is perforated by a duct from the pituitary body, as has been shown to be the case in Calamoichthys[1]. The oviduct appears to develop in a similar manner to that of Lepidosteus[2],

[1] Bickford, Elizabeth E., "The Hypophysis of Calamoichthys," *Anat. Anz.* x., 1895.

[2] Balfour and Parker, "The Structure and Development of Lepidosteus," *Phil. Tr. Roy. Soc. London*, Part II., 1882.

being an included portion of the body cavity into which there still open a number of nephrostomes. These nephrostomes open upon a slight groove which will eventually become the dorsal wall of the oviduct.

While searching for the nests of Polypterus I discovered the underground nests of *Protopterus annecteus* and obtained a complete series of eggs and larvae. The entrance to the burrow is in but a few inches of water, and when the water around the mouth of the nest dries up, the parent (who lives in the nest with the eggs and larvae) is seen to lash the surface of the water with its tail. The larvae are provided with four pairs of plumose external gills and a ventral sucker as in Lepidosiren; soon after hatching they attach themselves to the sides of the nest by the sucker and hang in a vertical position. The larvae hatch in eight days, and leave the nest in twenty days. The external features differ from those of the Lepidosiren only in unimportant details; there is in the larva of Protopterus, however, an indication of a spiracular cleft.

I also found the nests of *Gymnarchus niloticus*. These are made in about three feet of water and float on the surface. The nests are two feet long and a foot wide, the wall of the nest standing several inches out of water, except at one end where it is two or three inches below the surface, and leaves an entrance to the nest. The eggs measure 10 mm. in diameter; the larvae hatch in five days, when they greatly resemble the embryos of Selachians. The gill arches are not covered by an operculum and bear rows of gill filaments which later become of great length and very numerous. The yolk-sac becomes drawn out into a long cylindrical bag which is completely absorbed by the time the larva leaves the nest. Each nest contains about 1000 eggs.

The nests of *Heterotis niloticus* were very abundant. They are built on the swamp bottom in two feet of water. They measure four feet across, the walls reaching the surface of the water. When completed this nest is perfectly round and the bottom is quite smooth. The eggs measure 2½ mm. in diameter, are quite round and bright orange in colour. The larvae soon after hatching form a swarm in the centre of the nest and are provided with long protruding gill filaments.

In the same swamps *Sarcodaces odoë*, Bl., lays its eggs in masses of froth on the surface of the water. The eggs measure 3 mm. and are transparent. The hatched larvae are provided with conspicuous adhesive organs on the front of the head with which they hang to the under side of the surface.

I also found nests containing eggs which apparently belong to *Hyperopisus bebe*, Lacep., one of the Mormyridae. These nests are scooped out from the swamp bottom; the eggs are attached

to the rootlets thus laid bare. The hatched larvae are provided with six cement organs on the surface of the head. From them a delicate rope of mucus is spun often nearly the length of the body of the larva; by this fine rope the larvae hang suspended from the rootlets until the yolk-sac is absorbed.

It is remarkable that the larvae of Gymnarchus and Heterotis are both provided with long protruding gill filaments which have hitherto, I believe, been only once recorded in the Teleostomi; and that Sarcodaces and Hyperopisus are provided with conspicuous cement organs on the head; these cement organs on the head of the larva have usually been regarded as characteristic of the Ganoidi.

It is thus seen that the conditions by which fishes, which breed in tropical fresh waters, are surrounded is conducive to the development of very various accessory organs in the larva, both for the purpose of respiration and also of preserving them from harmful contact with their surroundings.

On the Interference bands produced by a thin wedge. By H. C. POCKLINGTON, M.A., St John's College.

[*Received* 8 January 1901.]

Let P be a source of monochromatic light, let OA, OA' be two surfaces which partially reflect and partially transmit the incident light. It will be assumed that the coefficient of reflection is so small that the intensity of the beam reflected from each surface is the same, and that the intensity of a beam that has undergone three reflections is practically zero. Let P', P'' be the images of P in OA, OA', and let PRQ, $PR'Q$ be the paths of the two rays that interfere at Q. Let $AOA' = \alpha$, a small angle, and let the polar coordinates of Q and P'' be r, θ and ρ, $-\phi$ respectively. Then those of P' are ρ, $-(\phi + 2\alpha)$.

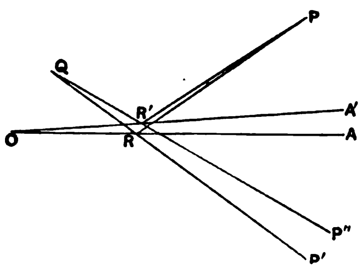
The distance QP'' is

$$\sqrt{r^2 + \rho^2 - 2r\rho \cos(\theta + \phi)},$$

and hence by differentiation with respect to ϕ, the difference of the distances QP'' and QP' is

$$\delta = \frac{2\alpha r\rho \sin(\theta + \phi)}{\sqrt{r^2 + \rho^2 - 2r\rho \cos(\theta + \phi)}} \quad \text{.................}(1),$$

and this is also the difference of the distances $PR'Q$ and PRQ.



If δ is equal to an odd number of half wave-lengths of the light emitted by P, there will be a dark band passing through Q; if equal to an even number of half wave-lengths, a bright band passes through Q. On examining any part of the reflected beam by a lens, light and dark bands will be seen.

In the actual case, however, the source will not be a point, but will cover a certain space, and it will in general happen that the dark bands due to one position of P do not fall on those due to a neighbouring position of P. In this case the bands will appear

confused or be invisible when received on a screen at Q or when examined by a lens which is focussed on Q. The bands will in short appear out of focus. In order that they may be in focus, it is necessary that δ shall not change when the position of P is slightly changed, *i.e.* that

$$d\delta/d\rho = 0 \text{ and } d\delta/d\phi = 0.$$

The first of these gives

$$r - \rho\cos(\theta + \phi) = 0 \qquad \text{.....................(2)},$$

and the second gives

$$(r\cos\overline{\theta + \phi} - \rho)(r - \rho\cos\overline{\theta + \phi}) = 0,$$

so that the condition that the bands are in focus reduces to (2), and the bands lie on a surface, the principal section of which is a semicircle described on OP' as diameter.

If the incident light is a parallel beam, P is at infinity,

$$\rho = \infty,$$

and the phenomena are given by

$$\delta = 2\alpha r \sin(\theta + \phi);$$

equation of principal section of surface on which the bands lie is

$$\theta + \phi = \pi/2.$$

Hence the bands lie in a plane passing through the edge of the wedge and lying perpendicular to the reflected beam. The bright band of order n is given by $\delta = n\lambda$, and hence by $r = n\lambda/2\alpha$. The bands are equidistant, and their distance apart is independent of the angle of incidence of the light from the source.

The only wedge that is practically available is the air wedge produced *e.g.* by two pieces of plane glass which make contact along a line and are kept apart at their further ends by a piece of paper. The refraction through the glass can cause a displacement of the bands and of the plane in which they lie, but leaves the distance apart of the bands unaltered if the incident light is parallel.

With a view to testing the accuracy of the formulae, I have determined the wave-length of sodium light by means of these bands. The angle of the wedge was determined by causing the reflections of two slits to coincide, the distance between the bands (which was about ·01 cm.) was determined by direct comparison with a rule graduated in millimetres placed in the plane of the bands, a hand-lens being used to see the coincidences with. The result was correct to about 1 per cent. Greater accuracy could of course be obtained by using a vernier microscope, but in this case it seems to be as easy to illuminate by vertical light, when the bands will lie in the plane of the air film.

On the most volatile gases of the atmosphere. By Professors G. D. LIVEING and J. DEWAR.

[*Read* 18 February 1901.]

Atmospheric air, at ordinary pressure, was liquefied directly by contact with the walls of a vessel cooled below $-200°$ C.; when about 200 cc. of liquid had condensed, communication with the atmosphere was closed, and a fraction of the liquid, still kept at $-210°$ C., allowed to distil into a second, still colder, vessel immersed in liquid hydrogen. When about 10 cc. had collected in the second vessel, in the solid state, communication between the two vessels was closed, and the gas above the solid in the second vessel was found to have a pressure of 10 to 15 mm. of mercury. Some of this gas was pumped out, and 43 per cent. of it was found to be hydrogen. Also tubes previously exhausted and sparked to remove hydrogen from the electrodes, and then filled with gases from the liquid air, at atmospheric pressure, gave the spectrum of hydrogen very strongly. Calculated as a percentage by volume of the atmosphere, hydrogen is present in very small quantity, nevertheless it forms a sensible percentage, and accords with the supposition that there is an interchange of gases by diffusion between our atmosphere and interplanetary space.

In other experiments the gas above the solid in the second vessel was allowed to pass through a U-tube cooled in liquid hydrogen into tubes previously exhausted; and the spectra given by electric discharges through them examined. These spectra are brilliant with the red, orange and yellow rays of helium and neon, but shew besides a vast number of other rays, belonging to substances hitherto unknown, which are most brilliant at the violet end, so that they can be photographed, notwithstanding the opacity of the glass tubes, up to a wave-length 3142. Their brilliance depends on the character of the discharge, and is greatest about the negative pole, and with no leyden-jar in circuit.

These spectra were searched for the characteristic nebular, coronal, and auroral rays. Tubes filled as above described shew no ray at about the wave-length 5007, the brighter green ray of nebulae, though they give a weak ray near, but not in the exact position of the other green nebular ray, λ 4959, and a strong ray close to the strongest ultra-violet nebular ray λ 3727. A tube which had been filled with gas from the second vessel, without first passing through the U-tube cooled in liquid

hydrogen, shewed a ray which seemed to be identical in position with the brighter green nebular ray; so that it is probable that the nebular stuff may yet be found in the earth's atmosphere.

The evidence of the existence in the earth's atmosphere of the gas, or gases, which produce the spectrum of the solar corona is stronger, for not only do the spectra of the most volatile atmospheric gases include a ray which fits, within the limits of probable error of observation, the place of the chief green coronal ray, but several other rays which correspond with strong rays which have been observed in the corona.

No ray, however, has been traced in these spectra, corresponding to the green auroral ray, which is perhaps due to the less volatile gas krypton. On the other hand one of the strongest rays, emitted mainly at the negative pole, at λ 3587, is at about the place of a strong auroral ray, and the authors suggest that the electric discharges to which the aurora is due may be of a nature similar to that which makes the glow about the negative pole in an exhausted tube.

The spectra suggest the presence of more than one unknown substance in the most volatile portion of the atmosphere, but further observations are needed to unravel their complexity.

On a method of comparing affinity-values of acids. By H. J. H. Fenton, M.A., and H. O. Jones, B.A., Clare College.

[*Read* 18 February 1901.]

When the hydrazone of oxalacetic acid is heated with pure water it yields the hydrazone of pyruvic acid with evolution of carbon dioxide, but in presence of dilute acids of sufficient concentration a totally different change occurs; in this case no gas is evolved and pyrazolon-carboxylic acid results.

$$\text{I.}\qquad \begin{array}{l} CH_2COOH \\ | \\ C : N_2HPh \\ | \\ COOH \end{array} = \begin{array}{l} CH_3 \\ | \\ CN_2HPh + CO_2 \\ | \\ COOH \end{array}$$

$$\text{II.}\qquad \begin{array}{l} CH_2COOH \\ | \\ C : N . N\begin{matrix} \diagup H \\ \diagdown Ph \end{matrix} \\ | \\ COOH \end{array} = \begin{array}{l} CH_2 \;.\; CO \\ | \qquad\quad | \\ C : N . NPh + H_2O \\ | \\ COOH \end{array}$$

Based upon these changes a very simple method has been devised for comparing the affinity-values of acids, and the results agree remarkably well with those obtained by the well-known methods.

In order to explain the nature of the changes involved, the authors attribute the evolution of carbon dioxide to the instability of the negative ion, and they have now made further experiments in order to test this hypothesis, the results being in all cases favourable. It is further shewn that it is possible by this method to compare the ionizing capabilities of various solvents, and experiments have now been made with pyridine in order to throw light upon the disputed question of its behaviour in this respect.

Von Laszczynski and von Gorski have found that inorganic salts have marked electric conductivity in pyridine solution, whereas Werner on the other hand found normal molecular weights from such solutions.

Examined by the present method the following result was arrived at:

0·1006 gram hydrazone dissolved in 7·5 c.c. of dry pyridine gave 6·12 c.c. of carbon dioxide (corr.).

Under exactly similar conditions the following results were obtained with other solvents:

	Wt. of hydrazone	Vol. of CO_2
Water	0·0975	7·96
Amyl alcohol	0·0989	3·56
Toluene	0·1078	2·29
Nitrobenzene	0·1026	2·29

It will be seen therefore that, on the hypothesis mentioned above, pyridine exerts a very considerable ionizing function.

On isomeric esters of dioxymaleic acid. By H. J. H. FENTON, M.A., and J. H. RYFFEL, B.A., Peterhouse.

[*Read* 18 February 1901.]

It was shewn by one of the authors on a previous occasion that the ethyl ester of dioxymaleic acid exhibits the remarkable property of becoming liquid when kept in a desiccator in presence of air, although it is relatively stable in presence of moisture or in absence of oxygen. This property has now been further investigated, and it is shewn that oxidation and loss of water take place with the formation of the liquid ester of dioxytartaric acid which

has the anhydrous form, *i.e.* dioxosuccinic ester. Another modification of the dioxymaleic ester has also been prepared which appears to be quite stable under the circumstances above mentioned, and it is considered probable that they represent the maleic and fumaric forms respectively.

Note on the constitution of cellulose. By H. J. H. FENTON, M.A., and Miss MILDRED GOSTLING.

[*Read* 18 February 1901.]

Certain carbohydrates when acted upon by dry hydrogen bromide in etherial solution at the ordinary temperature yield an intense purple colour which was shewn to be due to brom-methylfurfural. This substance has been isolated in the crystalline state, and it was further demonstrated that its production is characteristic of *ketohexoses* or of substances which give rise to these on hydrolysis. Carbohydrates of the aldose type yield none of this product. Exactly similar results have been obtained by operating in other solvents at 100°, and under the latter condition it is found that all forms of cellulose give large yields of brom-methylfurfural and it is concluded that the results definitely indicate the existence of a ketonic nucleus in cellulose.

Some substituted ammonium compounds of the type $NR'R''R_2'''X$.
By H. O. JONES, B.A., Clare College (communicated by Mr Fenton).

[*Received* 4 March 1901.]

The isomerism of substituted ammonium compounds has long been the subject of numerous researches; but until quite recently no very definite positive results had been obtained. The course of such investigations is much hampered and complicated, first, by the peculiar mobility of groups attached to the nitrogen atom, necessitating the use of 'heavy' radicals to render the compounds stable enough to shew isomerism, and further, by the great difficulty or even impossibility of producing the desired compounds in a crystalline state or even at all.

Le Bel (*Compt. Rend.* CXII. 724) found that an ammonium compound containing four different alkyl radicals acquired a small but fugitive rotatory power under the action of moulds. No isomerism was observed. This remained the only isolated observation on the optical activity of the nitrogen atom until 1899.

Le Bel also observed certain differences of crystalline form in compounds having three radicals the same. These were attributed to dimorphism.

Schryver and Collie (*Chem. News* LXIII. 174) prepared diethylmethylisoamylammonium chlorplatinate in the three possible ways, and found that the crystalline form of one compound differed from that of the other two. This difference however readily disappeared on recrystallisation.

Wedekind (*Ber.* XXXII. 517, 3561) found two definite and stable isomers of phenylmethylallylbenzylammonium iodide. All attempts to resolve these compounds into optically active portions were unsuccessful.

Pope and Peachey (*Jour. Chem. Soc.* 1899, LXXV. 1127) using Reychler's dextro-camphorsulphonic acid, succeeded in resolving one of Wedekind's compounds into two portions of equal and opposite rotatory power. The optical activity is here due to the asymmetric nitrogen atom.

Of the many configurations which have been proposed for the groups attached to the nitrogen atom, two seem to explain the phenomena observed satisfactorily, namely Bischoff's 'pyramidal' configuration, in which the five groups are supposed to be distributed around the nitrogen atom somewhat after the manner of

the angular points of a tetragonal pyramid about a point inside it (Fig. 1), and Willgerodt's 'double tetrahedral' configuration,

Fig. 1.

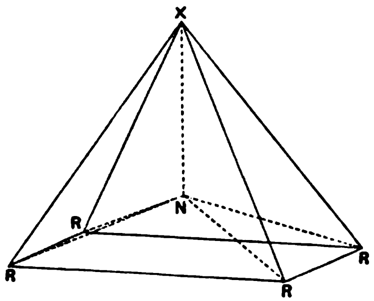
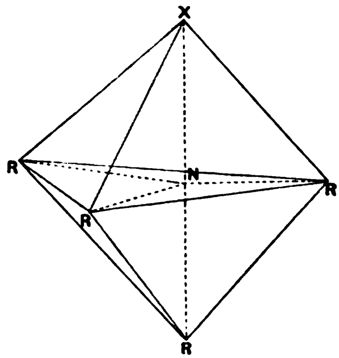


Fig. 2.



which represents three of the groups as lying in the same plane with the nitrogen atom while the other two are placed along a line at right angles to it on opposite sides (Fig. 2).

On either of these configurations a substance of the type $NR'R''R_2'''X$ should exist in isomeric forms. Taking the first configuration and using a plane projection for simplicity, the possible isomers would be represented as follows:

Fig. 3.

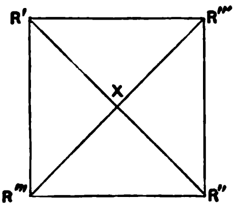
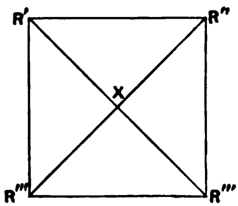


Fig. 4.



The first compound is planisymmetric and should therefore exist in one form only, the other compound having no plane of symmetry would consequently be expected to exist in two forms, non-superposable mirror images of one another and optically active (Figs. 5 and 6).

The present investigation is an attempt to detect this characteristic isomerism, which may be compared to that observed in compounds with two asymmetric carbon atoms, such as the tartaric acids. This note describes some of the compounds which have been prepared in the course of the work.

Phenyldimethylbenzylammonium Iodide. This compound was prepared both from dimethylaniline and benzyl iodide and from methylbenzylaniline and methyl iodide. The latter method gave

FIG. 5.

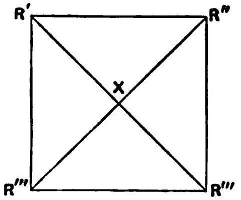
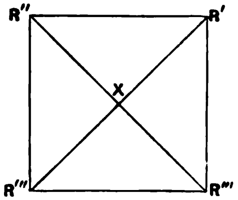


FIG. 6.



the better yield and purer product. The compounds obtained were found to be identical, even when the reaction was carried out entirely at 0° C. An internal transformation, in which a methyl and benzyl group are interchanged, must therefore have taken place. The salt is readily soluble in water and hot alcohol, somewhat less soluble in cold alcohol. It crystallises from alcohol either in long or flat prisms of the prismatic system which melt to a brown liquid at 163–4°.

The corresponding dextro-camphorsulphonate was prepared by heating the iodide, or the corresponding chloride prepared by Michler and Gradmann (*Ber.* x. 2079), with the calculated quantity of the silver salt of Reychler's acid and moist ethyl acetate (Pope and Peachey's method, *loc. cit.*).

The white salt thus obtained is very soluble in water, alcohol and chloroform, but sparingly soluble in benzene, ligroin, dry ethyl acetate and acetone. It crystallises from a mixture of chloroform and benzene and ligroin in plates of the oblique system which melt at 189° with decomposition.

Phenylmethyldibenzylammonium Iodide. A mixture of methylbenzylaniline and benzyl iodide sets in a few hours to a solid mass of the above salt: the yield is nearly quantitative. After washing with ether and crystallising from warm alcohol the salt is obtained in colourless prisms of the prismatic system, which melt and become brown at 134–5°. It is very sparingly soluble in water, a characteristic property of dibenzyl compounds, but dissolves readily in chloroform.

The corresponding chloride was not formed from a mixture of the tertiary amine and benzyl chloride after standing for months or heating to 100° for some hours. It was therefore obtained by digesting the iodide dissolved in warm alcohol with freshly prepared silver chloride. The solution after filtering was evaporated in a vacuum desiccator: a colourless crystalline mass was thus

obtained. This salt, which is very soluble in cold alcohol, crystallises from a mixture of alcohol and ether in colourless prisms very similar to those of the iodide, which melt at 160–1°. Although the chloride is not so readily formed as the iodide it appears to be more stable; the iodide when boiled with alcohol smells strongly of benzyl iodide, whilst the chloride does not seem to be affected by similar treatment; no odour of benzyl chloride was noticed.

All attempts to prepare the compound of dibenzylaniline and methyl iodide have hitherto failed. There is no visible action between the compounds at the ordinary temperature. On heating the mixture to 100° in a sealed tube a reaction takes place with the unexpected and somewhat remarkable result, that, instead of the desired compound, phenyltrimethylammonium iodide was obtained. The two benzyl groups have been removed as benzyl iodide and three methyl groups and an iodine atom added on.

Phenylmethyldibenzylammonium Dextro-camphorsulphonate was prepared from the iodide by the method already described. It is a colourless salt sparingly soluble in all solvents except chloroform, alcohol and methylene-diethylether. When crystallised from a mixture of chloroform and benzene it forms colourless needles which shew a great tendency to form aggregates, melt, not very sharply, between 135° and 138° and contain a molecule of chloroform of crystallisation.

The corresponding dextro-bromcamphorsulphonate was obtained in the same way, using the silver salt of Kipping and Pope's dextro-bromcamphorsulphonic acid. It was at first obtained as a gum which after rubbing several times with ether and standing over sulphuric acid solidified. It however separates from solvents in a gummy form and has not yet been obtained crystalline.

Molecular Weight of Glycogen. By HENRY JACKSON, M.A., Downing College.

[*Received* 14 March 1901.]

In an investigation on the chemistry of glycogen it became necessary to redetermine its molecular weight. The previous results are those of Külz and Bornträger (*Pf.* XXIV. 19), who, by noting the elevation of the boiling-point of water after introducing a weighed quantity of glycogen, concluded that the formula was $(C_6H_{10}O_5)_6$. Sabanéff (*J. Russ. Chem. Soc.* 21) applying Raoult's method of determining the molecular weight found a higher value, viz. $(C_6H_{10}O_5)_{10}$.

All recent work has pointed to a more complex molecule than either of these. The boiling-point method is not suited for very complicated carbohydrates, and the glycogen used by Sabanéff does not appear to have been completely free from mineral salts, and the presence of small quantities of inorganic substances, soluble in water, would increase the depression.

It therefore became necessary to obtain the glycogen in a state of purity, taking especial precautions to remove proteids and mineral salts: the following method of preparation and purification was adopted, which is a slight modification of that described by Lebben (*Zeit. Oeffenth. Chem.* 6). Horse liver was treated in a porcelain dish with water and a little 15 per cent. potassium hydroxide and boiled over wire gauze for 20 minutes. The extract was filtered through glass wool, and was then precipitated with 80 per cent. alcohol. On the following day the precipitated crude glycogen was collected on a filter and washed with alkaline alcohol.

For further purification the glycogen was dissolved in boiling water and, when cold, was just neutralized with 10 per cent. hydrochloric acid. To this solution dilute hydrochloric acid and Brücke's reagent were added very carefully until no further precipitation of proteids occurred. Absolute alcohol was now added until about 55 per cent. was present in the liquid. The precipitated glycogen was quickly filtered on the pump, washed with absolute alcohol and finally with absolute ether.

The glycogen, obtained as a white amorphous powder, was then dissolved in water, and dialysed for a week. It was again precipitated with 55 per cent. alcohol and again subjected to dialysis.

The glycogen, thus prepared, was tested for proteids and no trace of nitrogen could be found; a weighed quantity was ignited in a platinum crucible and no weighable ash was left. It may therefore be assumed that the sample used was free from inorganic salts.

In determining the molecular weight the cryoscopic method was followed, 30 cc. of water were taken and successive quantities of 2 grams of glycogen were added until 6 grams were present in solution. After each addition the depression of the freezing point was noted and it was thus possible to obtain three readings from each experiment.

The depressions were extremely small, and the values of the molecular weights deduced from them varied between 9,500 and 10,000.

These results point to glycogen having a highly complicated molecule, which links it closely to the dextrins derived from starch.

The application of Raoult's method to colloidal solution has been criticised. Recently Lobry de Bruyn (*Rec. Tr. Chem.* 1900), in considering the size of particles present in colloidal solutions, shews that their diameters deduced from purely physical considerations, such as the size capable of polarizing the light scattered by them, are in agreement with those calculated from chemical data, such as the molecular weight determination. There is no distinction between true solution and colloidal solution; there is no criterion of the homogeneity or heterogeneity of a liquid, and it is possible to pass continuously from undoubted solutions to liquids containing obvious particles in suspension.

The author would like to express his thanks to Mr W. A. Hoffmann, B.A., of Christ's College, for assistance in this investigation, and to the Grant Committee of the Royal Society for funds kindly placed at his disposal.

On the condensation of formaldehyde and the formation of β-acrose. By Henry Jackson, M.A., Downing College.

[*Received* 18 February 1901.]

An aqueous solution of formaldehyde, obtained by boiling paraformaldehyde, was treated with basic lead carbonate and heated on a water bath for one hour. After filtering it was evaporated in vacuo at 50° C. and the syrup treated with a mixture of methyl and ethyl alcohols. The insoluble lead salt was separated and the alcohol distilled off from the sugar. A two per cent. aqueous solution of the sugar was heated with phenyl hydrazine acetate on the water bath for four hours. The crude osazone was boiled with water; and acrosazone (Fischer and Passmore, *Ber.* 1899) remained undissolved. The filtrate from this on cooling deposited a mass of fine yellow crystals. These were re-crystallised twice again from hot water. This was found to be a mixture of osazones which have been separated by a long series of fractional precipitations into the following:

(i) Readily soluble in ether and benzene, ethyl acetate and ethyl alcohol. Melting point 131°—136°.

(ii) Sparingly soluble in ether and benzene, soluble in ethyl acetate. Melting point 158°. This appears identical with the β-acrosazone obtained by the condensation of glycollic aldehyde (Fenton and Jackson, *Rep. Brit. Ass.* 1900; Jackson, *J. C. S.* 1900) and of glycerose (Fischer and Tafel, *Ber.* 1887; Wohl, *Ber.* 1900).

(iii) Sparingly soluble in most solvents, but may be re-crystallised from methyl alcohol. Melting point 194°—196°.

On the effect of a magnetic field on the resistance of thin metallic films. By J. PATTERSON, B.A., Toronto, 1851 Exhibition Scholar. (Communicated by Professor Thomson.)

[*Read* 4 March 1901.]

A. C. Longden in the *Physical Review*, XI. 2. 40, described a method of making standard high resistances from thin films of metals deposited on glass by means of the kathode discharge. He has shewn that the resistance of these films is much greater than that calculated from the ordinary specific resistance of the metal. It would be of interest to try what effect a magnetic field would have on the resistance of a thin film deposited in this manner from a bismuth kathode.

The apparatus for making the films was the same as that used by Longden. To secure good contact between the film and the electrodes, the ends of the glass plates were silvered in a silvering solution and fine copper wire was wound round the silvered ends; then copper was deposited electrolytically on the wire and on part of the silver. The bismuth film was deposited on the glass plate thus prepared.

The films were examined under the microscope to see if there was any visible discontinuity, but none could be detected, and as there were no interference fringes produced by the reflected or transmitted light the films were continuous to the order of the wave-length of light. They were also tested by removing half the width of the film, the resistance was then doubled, shewing that the contacts with the electrodes were good and that, if there were any discontinuities in parts of the film, they were equally distributed.

The thickness of the films was measured by Wiener's method (*Wied. Ann.* 31. 629) and the measurements are correct to 15 per cent.

The resistance was measured by the ordinary Wheatstone bridge and a D'Arsonval galvanometer, which had a resistance of 100 ohms and gave a deflection of 1 mm. on the scale for 2×10^{-9} ampère.

The following table gives the results obtained:

Dimensions of film in cm. Length	Breadth	Thickness	Resistance in ohms	Resistance calculated from specific resistance	Magnetic field in lines per sq. cm.	Increase of resistance	% increase	% increase of bismuth wire
1·4	1·3	1×10^{-5}	58·04	1·38	26200	·16	·27	146
					27500	·18	·31	154
1	1·3	6×10^{-6}	103·05	1·44	26200	·055	·053	146
					27500	·061	·059	154
1·2	1·3	4×10^{-6}	968·7	3·46	26200	·1	·01	146

These results shew that the change of resistance in the magnetic field is not only very much smaller than that of bismuth in form of wire but that the change of resistance decreases with the decrease in the thickness of the film.

A film of cobalt 1·4 × 1·3 cm. with a resistance of 682·2 ohms was made, but in a field of 27500 lines no change could be detected.

In concluding this preliminary note, the author desires to thank Mr C. F. Mott for kindly placing his apparatus for making the films at the author's disposal, and to Prof. J. J. Thomson for his interest and suggestions.

On the theory of Electric Conduction through thin metallic films. By J. J. THOMSON, M.A., F.R.S., Cavendish Professor of Experimental Physics.

[*Read* 4 March 1901.]

The study of the electric resistance of thin metallic films affords a very direct method of testing a theory of electrical conduction through metals which was developed by the writer in a report presented to the International Congress of Physics at Paris, in 1900. According to this theory the current through a metal is carried by means of corpuscles, those small negatively electrified particles which constitute the cathode rays; which are given off by incandescent metals and also by metals when exposed to ultra-violet light. These corpuscles are assumed to be distributed throughout the volume of all metals, being produced by the corpuscular dissociation of the molecules. These particles, like the particles of a gas, are supposed to be moving rapidly in all directions, their kinetic energy, like that of the molecules of a gas, being proportional to the absolute temperature. Under the action of an electric field these charged corpuscles acquire a drift in a definite direction—the opposite direction to the electric force—since their charge is negative. This drift of the corpuscles under the electric field constitutes the current through the metal. If n is the number of corpuscles per unit volume of the metal, u the velocity of drift in the negative direction of x, e the charge on a corpuscle, then the intensity of the current parallel to the axes of x is equal to neu. If X is the electric force, m the mass of a corpuscle, t the average time between two collisions of a corpuscle, u is equal to $\frac{1}{2}\frac{Xe}{m}t$; if λ is the mean free path, c the velocity of mean square, $t=\lambda/c$; thus the current equals $X\frac{1}{2}\frac{ne^2}{m}\frac{\lambda}{c}$; and the conductivity of the metal is therefore $\frac{1}{2}\frac{ne^2}{m}\frac{\lambda}{c}$. The conductivity thus depends on the free path of the corpuscle. In the case of one metal, bismuth, we have data which enable us to find the value of this mean free path, which turns out in this case to be between

10^{-4} and 10^{-5} cm.; thus the free path is large compared with molecular dimensions, indeed it is so long that it is possible to make metallic films whose thickness is much less than the mean free path of the corpuscle in bismuth. Let us consider the case of a metallic film whose thickness is comparable with λ—the mean free path of the corpuscle in an unlimited mass of metal. The limitation imposed by the thickness of the film will diminish the free path. I find that if d is the thickness of the film and if we suppose the direction of motion of the particles uniformly distributed then when d is less than λ, the mean free path λ' is given by the equation

$$\lambda' = d\left\{\frac{3}{4} + \frac{1}{2}\log\frac{\lambda}{d}\right\}.$$

When $$d = \lambda \qquad \lambda' = \frac{3}{4}\lambda,$$

$$d = 2\lambda, \qquad \lambda' = \frac{7}{8}\lambda,$$

$$d = 3\lambda, \qquad \lambda' = \frac{15}{16}\lambda,$$

....

$$d = 2n\lambda \qquad \lambda' = \left(1 - \frac{1}{8n}\right)\lambda.$$

Thus when the thickness is greater than λ the mean free path changes but slowly with the thickness of the film, but when the thickness of the film becomes less than λ the free path diminishes rapidly as the thickness of the film diminishes. Now the conductivity of the metal contains the mean free path as a factor, hence we see that the conductivity of metallic films ought on this theory to diminish slowly as the thickness diminishes, until the thickness of the film is reduced to the free path, then any further diminution will be accompanied by a rapid diminution in the conductivity. From observations of the way the specific resistance changes with the thickness we may hope to approximate somewhat closely to λ.

There is another way in which the effect of the thinness of the thin film might be expected to make itself felt. In the Report already referred to it is shown that if $\delta\sigma$ is the increase in the specific resistance σ of a metal produced by a transverse magnetic field H, then

$$\frac{\delta\sigma}{\sigma} = \frac{1}{12} H^2 \frac{e^2}{m^2}\frac{\lambda^2}{c^2},$$

where λ is the mean free path. Thus when the film becomes so thin that the mean free path is diminished, not merely is the specific resistance increased, but the effect of a magnet on the resistance is very materially diminished. This is strikingly shown by Patterson's experiments[1], which prove that in thin films of bismuth the effect of the magnetic field on the resistance is very small. The electrical properties of thin metallic films as investigated by Longden and Patterson are qualitatively at any rate in accordance with the results of the theory discussed in this paper; further experiments are however needed on films of the various metals and with as wide as possible range in thickness to see if the numerical results obtained by experiment are in agreement with those predicted by theory.

[1] *Proc. Camb. Phil. Soc.* Vol. XI. p. 118.

The Prevention of Malaria. By J. W. W. STEPHENS, M.D., Gonville and Caius College[1].

[*Received* 25 February 1901.]

The various hypotheses put forward as to the nature of the crescentic and flagellating forms of the malaria parasite, and the suggestions made, originally by Laveran and subsequently by others, as to the possible *rôle* of mosquitoes in the transmission of malaria need not be considered here. We may pass directly to the actual work which has resulted in our present knowledge of the relation of mosquitoes to malaria. Ross, in India, at Manson's instigation, worked at the question whether parasites further developed in the gut of mosquitoes. He worked principally with *Proteosoma* of birds' blood, and for these parasites worked out very fully their complete cycle in the mosquito from the first stages in the stomach-wall to the final stage of sporozoites in the salivary glands. He further was able to shew that the parasites could be transmitted to healthy birds by means of mosquito punctures (in fact by inoculation of the sporozoites). Here then was at last the complete proof for birds of what had been for long a matter of speculation with regard to human malaria.

Ross had seen earlier in the gut of certain mosquitoes that had bitten malarial patients pigmented bodies which he considered to be further stages in the development of the human parasite, but this could not then be proved. In the meantime the Italians had been working at the question from different points of view. It was shewn by Grassi that the distribution of *Anopheles claviger* bore a close relation to the distribution of malaria, that in fact it was always present most plentifully in those regions where malaria was most severe; whereas *Culex pipiens* and other varieties of *Culex* were more frequent in regions not infested with malaria and were often absent in malarial regions. Bignami, who, since 1894, had been trying to infect patients with malaria by mosquito-bites had always failed, but now, using mosquitoes from malarial regions, he eventually succeeded; the result was certainly a fortunate one, for among the mosquitoes used, *Anopheles* were very few.

But the fact was established; and later Grassi, Bastianelli and Bignami worked out the cycle of development of the human

[1] Most of the data on which this paper is based will be found in

1. *Reports to the Malaria Committee of the Royal Society*, 1899–1900, by J. W. W. Stephens, M.D. Cantab., and S. R. Christophers, M.B. Vict. (Harrison and Sons, London).
2. *Further Reports to the Malaria Committee*, 1900 (Harrison and Sons).
3. *Reports.* Third Series, 1900 (Harrison and Sons).
4. *Reports.* Fourth Series, 1901 (Harrison and Sons).

parasite in *Anopheles* as Ross had previously done for birds. It need only be added that up to the present time it has not been shewn that any *Culex* is capable of transmitting malaria.

And further, only those mosquitoes (*Anopheles*) which contain sporozoites (*i.e.* the final stage in the development of the parasite) derived from a previous case of malaria are capable of transmitting malaria. Seeing then that *Anopheles* solely are concerned in the transmission of malaria, it becomes very important to know where they are found, and where they breed, and generally to study the habits of this genus. Much work has been done by Ross, by the Italians, by Koch, and by others on this point. We may give shortly here our results in tropical Africa. We investigated the distribution in Freetown itself and in the surrounding bush country.

Freetown.

In many well-lighted houses it was impossible to detect *Anopheles* when we visited them early in the morning, but it was quite possible that they had been present and had flown away to seek some dark retreat, so that we devised the method of making artificial pools of cement in the streets which were kept filled with water. In these *Anopheles* laid their eggs and the larvae hatched. By this means we detected the presence of this mosquito in many districts where search in the houses had been negative.

The distribution of *Anopheles* in Freetown is in the main this; it is dependent on the distribution of the water there, streams, drains and natural pools. In the huts and houses bordering on these the mosquitoes occur in large quantities, especially in the dark huts, but our pools shewed that *Anopheles* also lurked in the houses, gardens, and vegetation of districts remote from these, but in regions where, during the rains, pools had existed.

Anopheles larvae constantly occurred in the rock pools of the hill-streams of Sierra Leone, though it was impossible to detect the larvae elsewhere in the stream.

Adult insects were also caught by us occasionally when camping out in situations remote from human dwellings. They were very few, however, and often in the heart of the bush no mosquito of any kind was caught.

In mangrove swamps *Anopheles* larvae are extremely difficult to find, but we have found them in a few instances at the edge of shallows.

In the bush wherever it is unusual to meet with shallow pools, *Anopheles* are certainly uncommon, but there exists a set of conditions under which they may be found in quantity. Wherever in a clearing in the bush we find a collection of native huts it is safe to predict that here *Anopheles* will be detected by careful

search, or by making of pools if necessary. The same is true of all native villages, at least all those we have ever visited, for it is generally impossible to exclude local breeding grounds in the rains, and possibly in the dry season. It is a peculiar condition that in such native quarters mosquitoes may still be found even after a drought of some months.

We may sum up by saying that *Anopheles* occur very widely distributed. Often it is difficult to detect them, but if a native village or small collection of huts occurs here they congregate even if breeding-places are absent or only exist during the rainy season. It is worth noting that to the breeding-places of the genus, which have been adequately described by many authors, we may add as not infrequent spots, native canoes and deep wells 30—40 ft.

Accra.

In the country around Accra we were unable to find a single natural breeding-place, but artificial ones—excavations made by the natives around their huts for various purposes—exist in many hundreds. With the advent of the dry season most of these pits become dry and breeding-places are now only found where the ground water is sufficiently near the surface to be reached by the deeper excavations (6—10 ft.). On the higher parts of the district, 40—60 feet above sea level, in the dry season the ground water is not reached in the numerous pits existing everywhere and in consequence large areas are now free from breeding-places.

Around the borders of the lagoons which are a feature of the coast the conditions are different. Here the ground water is reached within a few feet and in the driest season numerous pits dug by the natives along the margin of the lagoon contain *Anopheles* larvae.

This condition shews that the effect of salt lagoons and low-lying salt marshes is an indirect one, as in the lagoons themselves larvae were not found.

Lagos.

Lagos is situated upon low-lying alluvium and is surrounded by extensive lagoons. The highest point of the island is not above 20 feet above sea level. A considerable tract in the centre of the town lies about 10 feet above sea level. This central area is approximately level but its margins sink rapidly towards the lagoons. Between this sloping ground and the lagoons there is a strip of land of varying width which lies almost at lagoon level. This strip is bounded by the 5-foot contour line. The heaviest rains are rapidly absorbed in the central elevated portion of the island so that in a few days after continuous rain all surface pools have disappeared. The subsiding water however emerges again around the borders of this tract forming a line of oozing water extending around nearly the whole island. It is here that

breeding grounds occur in the greatest profusion and in the huts adult *Anopheles* are to be caught in myriads.

But a more important side of this question still remains for consideration, viz. where do we find infected *Anopheles* and by what means do they become infected, for it is not so much the existence or absence of *Anopheles* that is the vital question as whether or no the mosquitoes contain sporozoites in their salivary glands?

We found that an examination of the salivary glands of *Anopheles* caught in native huts always gave a certain proportion of infected ones. This proportion was generally about 10—20 °/₀; in some cases as high as 50 °/₀ and in others again as low as 2 °/₀.

The highest percentages we unexpectedly found in those places where *Anopheles* were at first sight scanty and where there might be a complete absence of breeding grounds for at least some months, whereas our lowest value 2 °/₀ was found in Lagos island where there were innumerable breeding grounds; and it may be that the difference was dependent solely on the fact that in the latter case the large number of insects always present was sufficient to reduce the percentage of infected ones.

All native huts examined by us contained infected *Anopheles* in variable proportion.

Source of infection of Anopheles. The source of this universal condition of infection was moreover made clear when systematic blood examinations of the inmates of the huts were made. For it appeared that we had in the huts a universal condition of infection with malarial parasites and that this infection was confined almost entirely to the youngest children, especially those of 1 to 5 years old. It is not that these children were suffering from attacks of "fever" in the ordinary acceptance of the term, but that they, although to all intents and purposes quite healthy, almost all contained parasites in their blood. Here then was revealed a complete explanation of the mode of infection of the *Anopheles*. Prof. Koch working at the same time in the East Indies found an exactly parallel condition.

Since these results were published some authors have attempted to claim that this had been previously recognized. It is not denied that the existence of malaria in children has long been known, but it would appear that these authors have not realized the actual condition that exists among native children which is, that in children apparently perfectly healthy there exists an almost universal condition of infection with malaria parasites; this, it will be recognized, is quite a different state of things to the existence of cases of malaria among children: just as in small birds in Africa there exists a very large infection (about 50 °/₀) of *Halteridium*, so in native children below a certain age, who are apparently quite well, as many as 100 °/₀ may contain parasites in their blood.

An example of this is the following Table:

TABLE I.

Accra	Babies	Children up to 8	Children up to 12	Children over 12
	per cent.	per cent.	per cent.	
Village A	90	57	28	very rarely infected
Village B	75	50	...	
Cantonment ...	71	75	30	
Accra	23	20	...	

Or again an idea of the malarial infection of two small villages may be got from Tables II. and III.

TABLE II.

House		Ring forms	Pigmented leucocytes	Crescents	Total infections	Total number examined
1	...	...	2	1	2	2
2	...	1	1	...	2	4
3	Not examined	...	...	...	...	...
4	...	1	...	...	1	4
5	...	...	...	1	1	2
6	Not examined	...	...	...	...	...
7	...	1	3	...	4	7
8	Not examined	...	...	...	...	...
9	...	...	1	1	2	4
10	...	...	1	...	...	1
11	Not examined	...	...	...	...	...
12	...	...	...	3	3	4
13	...	2	1	1	3	3
14	No children	...	...	...	. .	...
15	No children	...	...	...	...	...
16	...	2	...	1	3	4
17	...	...	1	...	1	1
1	Isolated house ...	1	...	...	1	2
2	No children	...	...	...	...	...
3	Isolated house ...	...	...	2	2	3
	Total...	18		10	25	41

TABLE III.

House		Ring forms	Pigmented leucocytes	Crescents	Total infected children	Total children examined
1	...	...	1	2	2	4
2	No children	...	...	...	...	...
3	...	...	...	1	1	1
4	...	1	2	3	4	5
5	...	...	1	...	1	1
6	...	...	...	2	2	3
7	One child only ...	...	...	...	...	1
8	...	...	1	...	1	1
9	No children	...	...	...	...	...
10	No children	...	...	...	...	...
	Total...	6		8	11	16

We have briefly described the distribution of *Anopheles* in some typical malarial parts of W. Africa and also the conditions which determine the existence of breeding grounds. A conception of these conditions is necessary before undertaking any measures for the destruction of *Anopheles* or their breeding grounds; but an actual knowledge of the places can only afford a really adequate idea of the conditions, and it is a want of complete knowledge on these points that has led to many immature schemes for wholesale destruction of mosquitoes and to an unrealised predicted freedom from malaria. But as we have already pointed out it is not so much the question of distribution of *Anopheles* that is important as the absolutely fundamental question, what is the source of the infection of *Anopheles* and how can Europeans protect themselves from infection?

Before this work on native children was done, it had been thought that the source of infection was the European convalescent from an attack of fever; for it is in this state that gametes usually occur (we shall point out later however that as a matter of fact in W. Africa they rarely do occur). But it was always difficult to understand how the same mosquitoes which had got infected from such (supposed infective) cases should again find out

other Europeans living some distance away and so infect them. But the actual condition is a very different one. Europeans are in towns surrounded not by isolated cases of European fever but by thousands of cases of native malaria (parasites and gametes in the blood) and in native villages where Europeans live or sleep the night, every hut may contain children with parasites and in these huts we find a percentage of infected *Anopheles* reaching occasionally as high as 50 %. So that the European everywhere is living in the midst of infection. How then can this be avoided?

We may consider some of the schemes that have been advocated.

(α) Destruction of *Anopheles* larvae by the use of kerosene or tar applied to the pools. Such a procedure can only be of the most limited application and experiments made in Freetown for a period of some months shewed that when the application ceased *Anopheles* larvae reappeared everywhere.

(β) Drainage: filling up of pools and hollows.

This method has the great advantage that it is permanent in its results. It is naturally expensive, but it should be used as a subsidiary means whenever possible. In Lagos for instance much might be done by filling up with sand the tracts of oozing water that we have described.

(γ) Construction of mosquito-proof houses.

These have been made use of in the Roman Campagna with good results, but those who advocate them have little knowledge of the climatic conditions of tropical Africa. Life in such a house would be a terrible experience and sooner or later *Anopheles* are certain to be found inside.

(δ) Destruction of parasites in their host by the use of quinine (Koch). Even if this method were a certain one, and the experience of the Italians is that it is not, its application to a native population as existing in tropical Africa would be a Herculean task. The eradication of native malaria in a town of 50,000 inhabitants or even in a small native village is not a practical measure under existing conditions.

(ϵ) Segregation of Europeans.

There fortunately remains a simple and practical means of avoiding this great source of infection. That is the method of segregation.

We may summarize the conclusions at which we arrived after an examination of several hundred native children. The parasites found were solely of the malignant tertian type. The frequency with which gametes occurred was very striking, and the constant presence of infected *Anopheles* in native quarters was thus readily explained.

But when we consider Europeans, the conditions are quite different. Parasites are rarely found except in definite attacks of

fever, and even when present they bear no comparison to the large infections with the ring forms or with gametes of native children. Thus of twenty-one Europeans who habitually slept without mosquito nets, and were so exposed to the constant risk of infection, only two shewed parasites and these were extremely rare. Of another set of Europeans who used nets and adopted ordinary precautions none contained parasites. Again it is a peculiar fact that in Europeans who are convalescing from an attack of fever in West Africa, gametes are rarely found. This also is the experience of Ziemann in Cameroons, so that they are actually not in the necessary condition for the infection of mosquitoes.

These facts then shew that the generally received idea that Europeans derive malaria from pre-existing cases in Europeans requires considerable modification. This factor it seems to us sinks into complete insignificance beside that of infection derived from native sources. The normal condition of native children is one of almost continuous infection, and there are therefore many thousands of cases of malaria in large towns. This enormous source of infection has we believe so far entirely escaped recognition. Whilst Europeans live in the midst of native quarters exposed to infection on all sides, the isolation of such Europeans only as have fever is manifestly a futile procedure.

Malaria then is a contagious disease, the contagion being conveyed by the mosquito, and we deduce as the direct result of these observations the conclusion that malaria can be avoided most readily by avoiding this source of contagion and by living as far removed as possible from native huts.

Now it is notorious that among men employed on railway works in W. Africa malaria is always rife and the explanation is clear when we know the conditions under which they live. We found without exception in all the railway camps examined by us the following conditions. A single European house or a small group surrounded by native huts, or with huts in the immediate vicinity, and in these huts we always found, as already stated, infected *Anopheles*.

This dangerous source of infection can be however readily avoided by locating the European dwellings at a distance from native quarters, and on railways this can be effected with great ease. A site then should be selected as far as possible from the native villages. We should fix a mile as a perfectly safe limit and we shall give reasons for believing that half-a-mile or even less would be sufficient.

We must consider here, as having a direct bearing on this question of segregation, the distance of flight of *Anopheles*, for it may be objected that even if segregation be effected there is no certainty that infected *Anopheles* will not fly this distance.

But even did this occur it is quite clear that the risk is now very largely diminished, for granted that occasionally an *Anopheles* did fly this distance, the risk is very different from that encountered in the midst of native quarters in which *Anopheles* may be caught in hundreds or even thousands.

But our experience of two years' residence in Africa, under many different conditions, leads us most emphatically to deny that the objection of the flight of *Anopheles* is at all a practical one. Our experience for instance in Freetown itself absolutely contradicts this. Now in the outskirts of Freetown, *Anopheles* exist in myriads, and myriads of larvae are to be found in the pools. But there is a central area of Freetown which has no pools and in which houses (not huts) occur, and these are of stone and fairly well lighted. We have at different times slept during some months in five different houses in this area, and never have we found an *Anopheles* on our mosquito net at night or at daybreak, and we invariably searched at these times, and this is convincing evidence that they were not present. Now if it be true that *Anopheles* fly extraordinary distances as occasionally stated, we should have found them in the centre of Freetown, but we never did. Our experience at Accra was exactly similar. We there occupied a bungalow, a quarter to half a mile from a native village, but again we never observed a single *Anopheles*.

This too brings us to the question of isolation of Europeans in large towns which at first sight might seem to present considerable difficulties, but we see from our experience in Freetown that such isolation, to be effective, need not necessarily imply a separation by a large distance. If under the worst conditions, even in towns, the European quarter is well constructed and cleared of native huts and provided with well-drained streets, then we shall find that it will enjoy a considerable protection as compared with the houses surrounded with the really dangerous native quarters.

Segregation then we believe affords a very simple and practical means of avoiding the danger of contagion. In villages in the bush the process can be readily effected, and even in W. African towns the difficulties are not really great.

We may add solely that this mode of protection applies to those conditions we have knowledge of in West Africa. We cannot discuss its application to other conditions unknown to us.

A Theorem on Curves in a Linear Complex. By J. H. Grace, M.A., Peterhouse.

[*Received* 4 March 1901.]

When all the tangents of a twisted curve belong to the same linear complex, the curve cannot have a proper point of superosculation; for at such a point the osculating plane contains three consecutive tangents not in general meeting in a point, which is contrary to the supposition that they belong to a linear complex. In fact two of the tangents must coincide in order that the three may be concurrent, i.e. the curve must have a stationary tangent. The point of contact of a stationary tangent may be regarded as the coalescence of two points of superosculation, and hence we may say that:—

The points of superosculation of such a curve coincide in pairs, and each point of coalescence is the point of contact of a stationary tangent.

A rigorous algebraic proof of this proposition for the case of rational curves has been given by Picard—the method incidentally shews that the equation giving the parameters of the points of superosculation is of the form

$$T = 0$$

where T is rational.

Suppose the curve is of order n, then it is easy to see that the equation for the parameters just mentioned is of order $4n - 12$, so that the curve has in general $2n - 6$ separate stationary tangents. I proceed to prove the converse of this, viz. *If there are $2n - 6$ stationary tangents to a rational curve of order n, then all the tangents to the curve belong to the same linear complex.*

The proof is very simple for $n = 3$, 4, or 5, and we therefore consider these cases separately.

(i) $n = 3$. Here there are no stationary tangents and the rank is 4, so that a linear complex drawn to contain five of the tangents contains them all.

(ii) $n = 4$. Here there are two stationary tangents and the rank is 6. Obviously therefore a linear complex containing more than six of the tangents contains them all. But the linear complex containing the two stationary tangents and any other three tangents has seven lines in common with the tangent developable because each stationary tangent counts for two. Hence such a complex contains all the tangents.

(iii) $n = 5$. Here the rank is 8, and a linear complex drawn to contain the four stationary tangents and one other tangent contains in reality nine tangents and therefore contains all the tangents.

For $n > 5$ a different method is necessary, and it applies equally well to $n = 3$, 4, or 5.

Suppose the curve is defined by

$$x = f_1(\lambda),\ y = f_2(\lambda),\ z = f_3(\lambda),\ w = f_4(\lambda),$$

where λ is a variable parameter and the f's are rational integral functions of order n.

Then the coordinates of the tangent at any point of the curve are the six Jacobians of the f's taken in pairs, i.e.

$$J_{23},\ J_{31},\ J_{12},\ J_{14},\ J_{24},\ J_{34},$$

and each of these is of order $2(n-1)$, so that the rank of the curve is $2(n-1)$.

Now consider the points of the curve at which a linear complex contains six consecutive tangents; it is easy to see that their parameters are given by

$$\begin{vmatrix} J_{23}, & J_{31}, & J_{12}, & J_{14}, & J_{24}, & J_{34} \\ \frac{dJ_{23}}{d\lambda} & \dots & \dots & \dots & \dots & \dots \\ \frac{d^2J_{23}}{d\lambda^2} & \dots & \dots & \dots & \dots & \dots \\ \frac{d^3J_{23}}{d\lambda^3} & \dots & \dots & \dots & \dots & \dots \\ \frac{d^4J_{23}}{d\lambda^4} & \dots & \dots & \dots & \dots & \dots \\ \frac{d^5J_{23}}{d\lambda^5} & \dots & \dots & \dots & \dots & \dots \end{vmatrix} = 0 \dots\dots\dots (A),$$

and that the order of this equation in λ is $6(m-5)$ where m is the order of each J. (Of course the easiest way to see the last fact is to make the J's homogeneous for a moment by the addition of a variable μ.) Since $m = 2n - 2$ we infer that there are in general $12n - 42$ such points on a rational curve of order n. If there are more than this number the equation (A) is an identity and then all the tangents belong to a linear complex.

Now in our case there are $2n - 6$ stationary tangents and each of these[1] counts for six of the points defined by (A); hence the equation (A) has $6(2n-6) = 12n - 36$ roots and therefore it is an identity. Consequently when there are $2n - 6$ stationary tangents all the tangents belong to the same linear complex.

[1] *Camb. Phil. Soc. Proc.* XI. 28.

Preliminary note on the function of the root-tip in relation to geotropism. By FRANCIS DARWIN.

[*Received* 7 March 1901.]

In my paper[1] "On Geotropism and the Localisation of the Sensitive Region" I described a new method of testing the point of view brought forward in the *Power of Movement in Plants* that in certain geotropic parts of plants the apex is a percipient organ while the more basal motor region is set in action by an influence transmitted from the sensitive region.

The present paper is an attempt to apply this method to the case of roots[2].

If a seedling bean is supported by its cotyledons, the root projecting freely in damp air in a horizontal position, a downward curvature soon begins which continues until the apex points vertically downwards, when growth continues in the vertical line. According to the theory set forth in the *Power of Movement in Plants*, the tip of the horizontal root is stimulated by the force of gravity and an influence is conveyed to the motor part of the root, in which a curvature consequently takes place: when however the tip is vertical it is no longer stimulated by gravitation and therefore curvature ceases. If instead of fixing the cotyledons and leaving the tip of the root free, the arrangement is reversed, so that the tip is fixed in a horizontal position and the cotyledons are free to move, the result should according to the theory be quite different. For the downward curvature of the root does not in this case alter the position of the tip, which remains horizontal and should therefore continue to transmit an influence to the motor region. In the case of the apogeotropic hypocotyls of *Setaria* and *Sorghum* and of the cotyledons of *Phalaris*, this has been proved to be the case, and the remarkable coils and spirals so produced are figured in the paper above quoted. To apply the method to roots is a matter of some technical difficulty; the tip of the root (e.g. in the bean) is a smooth and slimy cone and is not easily

[1] *Annals of Botany*, Dec. 1899.

[2] The Pfeffer-Czapek method is generally held to have proved the truth of the theory in question, but Wachtel's paper shows that the technical difficulties of repeating Czapek's experiment are not insignificant, so that a new method of attack is not superfluous. See Pfeffer, *Annals of Botany*, 1894: Czapek, *Pringsheim's Jahrb.*, 1895, 1900: Wachtel, *Bot. Zeitung*, 1899 (abstract of the original Russian paper).

fixed into a tube or other support, and since the cotyledons are of relatively great weight the difficulty appears insuperable without some special device. This has been supplied by Mr H. Darwin who devised a "Root-Lever" which has been made for me by the Scientific Instrument Company. It will be described in a fuller paper; it will be sufficient for the reader to know that while the weight of the cotyledons is supported, they are free to move in any direction in obedience to the curvature of the root. The cotyledons being supported by the root-lever, the tip of the root is inserted into a fixed horizontal tube and the root is kept from withering by water slowly and gently dripping on it. The great difficulty is to keep the tip of the root from slipping out of the fixed tube. A great majority of the experiments failed from this cause, but this does not invalidate the theory, although it points to the need of improving the method. Omitting from consideration these cases and a few in which the roots did not curve geotropically, my results point clearly to a strong tendency in the root to continue curving. In several cases the cotyledons travel through 180° and in one case through about 360°, so that the root looked as though a knot had been tied in it.

In the case of peas the difficulty of keeping the root in place is less, the failures are fewer, and the tendency to continued curvature in the root is strongly marked. These experiments are still in progress and will be described in detail in a fuller publication. There too will be fully discussed the interpretation of the results which presents some difficulties depending on the distribution of growth in roots (as compared with stems) and on the existence of other forms of irritability in the root-tip. One consideration must not however be omitted. The tip of the root is sensitive to unequal pressure or unilateral injury, and it may be supposed that the continued curvature of the root is due to pressure exercised on the tip of the root by the fixed tube. But the results of a number of experiments in which a root was placed tip downwards in a *vertical* tube do not support this view. There seems no reason why a vertical tube should not be equally effective with a horizontal one in producing contact stimulation. But there was no tendency to continuous curvature when the tube was vertical.

Note on some factors in the spore-formation of Acrospeira mirabilis (Berk. and Br.). By R. H. BIFFEN, B.A., Emmanuel College.

[*Read* 4 March 1901.]

Acrospeira mirabilis is a fungus found as a parasite on Spanish chestnuts[1] and so far nothing beyond the mode of development of its characteristic chlamydospores, traced by Berkeley in 1861, has been published.

This note on some of the factors concerned in its spore-formation is but a portion of the results obtained while making the attempt to trace its life history and so to find its true place in the present system of classification.

The spores found by Berkeley and Broome were developed from the subterminal cell of a row of three or four, coiled into a spiral of from one and a half to two turns. These cells all became fused together so that the spore was partially invested. When ripe it was brown, thick-walled, and covered with rounded warts. The investing cells were similar, but thinner-walled and poor in protoplasmic contents.

From its thick walls one might conclude that this spore-form was a resting-spore like the chlamydospores (or paulospores of Klebs) of *Hypomyces* spp. but it seems to be the spore-form which serves for the rapid reproduction of the fungus.

In spite of the fact that *Acrospeira* is a parasite it may be grown readily as a saprophyte on various media. On germination the large cell only puts out a hypha, the further development of which was determined chiefly by the food supply available.

Thus if sown on an extract of peas and ten per cent. gelatine the mycelium grew slowly and remained sterile altogether, though transferred repeatedly to fresh supplies of pea extract. If however it was transferred to pieces of sterilized chestnuts it gave rise, in the course of four or five days, to numbers of the invested chlamydospores, similar to those described by Berkeley.

On sowing on slices of sterilized melon chlamydospores were again formed but a large percentage of them were abnormal and resembled the teleutospores of *Puccinia* or of *Phragmidium* in shape, owing to the development of two or of three of the cells to form spores without the preliminary coiling.

[1] Berkeley and Broome, *Ann. and Mag. Nat. Hist.*, 1861, p. 449.

If the chlamydospores were sown on a watery extract of chestnuts made up with agar-agar a further crop of chlamydospores was produced and at the same time the submerged hyphae formed small bunches of flask-shaped structures from which endoconidia arose. The more dilute the chestnut extract the greater was the abundance of these endoconidia, so that on germinating the chlamydospores in water or in agar-agar practically a pure culture of them could be obtained.

When the chlamydospores were germinated in beer-wort gelatine however no sign of these endoconidia could be found on the submerged hyphae but they became shortly septated into barrel-like cells, some of which grew rapidly into large ellipsoidal structures in which the protoplasm became finely vacuolated while the walls thickened and became carbonized. On placing these structures in water they cracked in the course of a few days and let free swarms of minute, slightly motile, spherical bodies. Presumably these structures were sporangia but so far all attempts to germinate the "spores" have failed. At the same time the aerial portions of the mycelium again formed the characteristic chlamydospores. Sowings on gelatine or agar-agar made up with Klebs' solution and 5 % cane sugar or glucose gave rise to pure cultures of these "sporangia."

In cultures on chestnut-agar five weeks or more old or in the cotton-wool plugs, used to keep the chestnut cultures moist, a further spore-form has been produced which consists of a group of chlamydospores surrounded by a thick envelope of several layers of cells. They arise from the same coils as the chlamydospores already described when little nutriment is available.

These spore-balls have been germinated on the media previously used with the following results: on pea-extract the mycelium was sterile, on chestnut-agar endoconidia were produced but not so abundantly, and on beer-wort gelatine "sporangia." On the two first media numbers of cell-fusions occurred especially when the food supply was limited. At the same time on the chestnut-agar large crops of spore-balls were produced but they differed from those formed on the old chestnut cultures in having only a single layer of envelope cells—like a typical *Urocystis*. I have since found this same form in diseased chestnuts, so it must be assumed to be the normal one. The development of the abnormal form will be considered later.

The cultures on beer-wort gelatine besides forming "sporangia" gave rise to arches or spirals of cells which became strongly swollen and carbonized. They resembled the ascogonia described by Tulasne and others in *Ascobolus*[1] and were externally very

[1] Tulasne, *Anns. Sci. Nat.* 5me Sér. VI. p. 215. Janczewski, *Bot. Zeit.*, 1871, p. 257.

different from the *Eurotium*-like helices from which the spore-balls developed. In spite of their carbonized walls these structures are not chlamydospores, at all events at present, and some facts point to their further development along the lines of *Ascobolus* or some such form.

It is thus peculiarly interesting to note that one can not only determine which form shall be produced by varying the food supply but even intermediate forms can be raised by mixing the two media. Moreover by transferring a culture producing these "ascogonia" to a dung decoction, on which spore-balls are normally formed, the fungus makes the attempt, often it is true unsuccessfully (for the cells of the ascogonia are sometimes 70—80 μ in diameter), to enclose them and so produce the spore-ball type.

So far these results may, I think, be attributed merely to the effect of the varying food supply. Within the last year or so though Klebs has shown that altering the rate of transpiration is a potent factor in determining the form of spore-development. It may merely alter the external character of the spore or a totally different spore-form may be produced.

In *Acrospeira* I have only found cases in which the external characters of the spores have been changed. Thus the chlamydospores first described when grown under water instead of in the air had thin, smooth, and uncarbonized walls instead of the thick, brown, warted walls[1]. If however transpiration was abnormally rapid, as for instance when the cultures were placed in a dry incubator or removed from the tubes and left exposed to the air, the chlamydospores had exceedingly thick, smooth, carbonized walls, and were only about half the normal size. The case of the spore-balls was still more interesting for the difference between those produced on old chestnut cultures and on the chestnut-agar plates was great enough to found separate species on.

It was found that by keeping the cultures producing the helices from which the spore-balls developed very moist that instead of short balloon-like branches being put out from the helix which at once became adpressed to it to form the single envelope layer, long hyphae were formed which coiled together in a snake-like fashion and ultimately gave rise to the many-layered envelope.

These branches themselves appeared to be potential sporophores, for on liberally feeding them in the early stages of their formation they too grew out into the characteristic helices.

[1] Cf. Klebs, *Prings. Jahrb.*, 1900, p. 80, for similar experiments with *Hypomyces*.

On a reserve carbohydrate, which produces mannose, from the bulb of Lilium. By J. PARKIN, M.A., Trinity College.

[*Read* 4 March 1901.]

Besides starch the bulbs of members of the genus *Lilium* contain another reserve carbohydrate. It exists as a sort of mucilage in the cell-sap of all the parenchymatous cells of the bulb-scales. Alcohol precipitates and hardens it, so that sections of scales preserved in spirit show each cell filled with a solid block of mucilage, in which the starch grains are embedded. On treatment with water the mucilage swells and gradually dissolves, liberating the starch grains. Thus the alcoholic material presents somewhat the same microscopic appearance as that of other monocotyledonous reserve organs containing starch and inulin, for example the Hyacinth bulb described in a former paper[1]. *Lilium*, however, did not respond to the microchemical tests used there, and so these bulbs were merely referred to as possessing starch and not inulin.

In some recent papers by Leclerc du Sablon[2] on reserves of bulbs and tubers, he speaks of the carbohydrate soluble in water but insoluble in strong alcohol as dextrin. No conclusive evidence is given to prove that it is such. He seems merely to have found that it is readily hydrolysed by an acid to produce sugar reducing Fehling's solution. The monocotyledons he mentions more particularly as possessing dextrin in their reserve organs are *Lilium candidum, Hyacinthus orientalis, Tulipa Gesneriana* and *Ophrys aranifera.*

Proof has previously been given[3] that his supposed dextrin in the Hyacinth bulb is inulin, or at any rate a carbohydrate producing fructose (levulose) on hydrolysis.

The carbohydrate occurring along with starch in the bulb of *Lilium candidum* has now been chemically examined, and found to be one yielding mannose and not glucose on hydrolysis.

Chemical examination. The bulbs used were taken from the ground on September 20th, at a time when the foliage was withering. The scales were removed, cleaned, sliced in pieces and allowed to dry at the ordinary temperature in a current of air They were then ground up.

About 17 grams of this dried material were taken and

[1] Parkin, *Phil. Trans. Roy. Soc. Lond.*, ser. B, vol. CXCI. (1899), pp. 61—64.
[2] Leclerc du Sablon, *Rev. Gén. de Bot.*, 1898.
[3] Parkin, *Annals of Botany*, vol. XIV. 1900, pp. 155—157.

extracted several times with cold water, till the mucilaginous matter was dissolved out. The extract was filtered to remove particles in suspension, such as starch grains; then evaporated down to a small bulk, refiltered and finally poured into a large excess of alcohol, when a quantity of white flocculent matter separated out. This was collected, dissolved in water and re-precipitated by alcohol. Then treated similarly a third time, and finally dried on a porous plate. About 1·5 grams of a translucent almost colourless tough solid were thus obtained. This substance swells and dissolves slowly in water to form an opalescent solution, which does not give any marked colour with iodine. On account of the opalescence its behaviour to polarised light could not be studied with any degree of accuracy. Dilute solutions indicated a dextrorotary power.

A definite quantity, 0·959 gram, of this substance dried till constant in weight, was dissolved in water, and the volume made up to 100 c.c. Of this 25 c.c. were taken and boiled for one hour with 2 per cent. sulphuric acid. The hydrolysis is not so quickly effected as in the case of inulin, but one hour's boiling with this strength of acid sufficed to complete it; alcohol ceased then to cause any precipitation, and the opalescence had quite disappeared, leaving a perfectly clear solution. This hydrolysed portion first showed levorotation, gradually changing to the opposite sign, and after standing a few hours the rotation became constant.

Angle of rotation in 200 mm. tube when constant $= + 0^\circ{\cdot}23$.

The cupric reducing power of the hydrolysed liquid was estimated by ascertaining the weight of copper obtained from the cuprous oxide formed on boiling with Fehling's reagent.

Weight of copper calculated for the 100 c.c. = 1·7 grams.

These two results do not agree at all closely on the assumption that the sugar is glucose (dextrose), arising from the hydrolysis of dextrin. The 1·7 grams of copper are equivalent to 0·954 gram of glucose, which should give a rotation at least four times as great as that found. A second experiment with another portion of the solution revealed the same discrepancy.

The feeble dextrorotary power and the fact of the opticity being at first left-handed, suggest at once mannose which exhibits this form of multirotation. If it is this sugar, then, the characteristic hydrazone should be deposited in the cold on the addition of phenylhydrazin and acetic acid—a reaction which distinguishes mannose from all the other hexoses.

Some of the hydrolysed liquid, previously neutralised with calcium carbonate, was concentrated on the water-bath, filtered to free it from the calcium sulphate deposited, and then added to a mixture of phenylhydrazine and acetic acid. After standing a short time a bulky yellowish amorphous precipitate settled down.

This was removed, well washed and boiled with water. It dissolved and on cooling and standing, separated out in rhombic crystalline plates. As finally prepared by crystallization from dilute alcohol it was nearly colourless with a melting point of 195°—200°. Dissolved in hydrochloric acid it showed levorotation. Hence it agrees in all respects with mannose-hydrazone.

The numbers obtained for the opticity and cupric reducing power agree closely with the view that the carbohydrate is one which hydrolyses to mannose. According to Fischer and Hirschberger[1] mannose reduces Fehling's fluid rather more strongly than glucose. From their figures the 1·7 grams of copper obtained above are equivalent to 0·874 gram of mannose. This amount should give a rotation in the 200 mm. tube of $+0°{\cdot}25$[2], which approaches very near the observed angle $+0°{\cdot}23$. Thus there is little room for doubt that practically the whole of the carbohydrate extracted from the bulb of *Lilium candidum* by cold water and precipitated by strong alcohol, hydrolyses to mannose.

The same substance has also been prepared from the bulb of *Lilium auratum*, and mannose-hydrazone made from it after hydrolysis.

The bulbs of six other species examined, viz. *L. bulbiferum*, *L. croceum*, *L. dauricum*, *L. lancifolium*, *L. longiflorum* and *L. Martagon*, contain in their parenchymatous cells mucilage, which is in all probability the same carbohydrate, so that this additional reserve material may be considered a characteristic of the genus.

Distribution of mannose in plants. A mannose-producing carbohydrate occurs in certain seeds in the form of reserve-cellulose. Reiss[3] showed that this variety of cellulose is capable of being hydrolysed by strong acids to give a new sugar, which he termed seminose, now reckoned to be identical with mannose. He proved the presence of this mannocellulose in seeds of several palms, of *Allium cepa*, *Asparagus officinalis*, *Iris pseudacorus*, *Strychnos nux vomica* and *Coffea arabica*. His results have been confirmed and extended by other investigators.

Another mannose-producing carbohydrate differing from reserve-cellulose in being soluble in water and capable of being hydrolysed by dilute mineral acids has been called mannan. Its presence was first proved in the drug, salep, which is prepared from the tuberous roots of certain species of *Orchis*. The mucilage it contains is converted into mannose on boiling with dilute acids[4].

[1] Fischer and Hirschberger, *Ber. Deut. Chem. Ges.* 1889, XXII. p. 365.

[2] $[\alpha]D$ is taken to be equal to $+14°{\cdot}25$, the value obtained by Ekenstein for crystalline mannose.

[3] Reiss, *Ber. Deut. Chem. Ges.*, 1889, XXII. p. 609.

[4] Gans and Tollen, *Ibid.*, 1888, XXI. p. 2150.

Tsuji[1] has shown that the Japanese food, "namakonujaku," prepared from the root of an Aroid (*Conophallus Konjak*) consists largely of this mannan. Kinoshita[2] has further investigated the carbohydrate in the root of the plant itself. Still more recently another Japanese investigator[3] has found it in the leaves of the same Aroid, which he names *Amorphophallus Konjak*[4], and has also made the interesting discovery of free mannose in the leaf-stalks of this plant.

Bourquelot and Hérissey[5] have lately shown that the reserve carbohydrates of the seeds of certain leguminous plants, e.g. *Ceratonia siliqua*, *Trigonella Foenum-graecum*, *Medicago sativa*, consist of a mixture of mannan and galactan or a combination of the two, for the carbohydrate-reserve hydrolyses fairly readily to mannose and galactose.

The carbohydrate found in *Lilium* is perhaps identical with the mannan of these previous investigators. Consequently the reserve organs of the following monocotyledons may be considered to contain mannan—species of *Orchis* and *Lilium*, and the Aroid, *Amorphophallus Rivieri*. Further research will very likely prove that it is of much wider distribution in bulbs and tubers.

Mannose has also been shown to be present in or obtainable from other vegetable matters, where it can hardly be reckoned of nutritive value, such as the wood of conifers[6], orange peel[7] and probably the glucosides[8], scammonin and ipomoeïn.

Hence mannose seems destined to become quite an important sugar in plants.

[1] Tsuji, *Bull. Coll. Agric. Imp. Univ. Tokyo*, 1894, 2, p. 108.
[2] Kinoshita, *Ibid.*, 1895, 2, p. 205.
[3] Tsukamoto, *Ibid.*, 1897, 2, p. 406. I have not been able to see the original papers but only abstracts in the *J. Chem. Soc.*, 1894, 1896, 1897, and in *Land. Vers. Stat.*, Bd. 45, 1895, p. 433.
[4] This plant is named *Amorphophallus Rivieri* in the Index Kewensis.
[5] Bourquelot and Hérissey, *Comp. Rend.* cxxix. 1899, pp. 228, 391.
[6] Bertrand, *Ibid.*, p. 1025.
[7] Flatau and Labbé, *Bull. Soc. Chim.*, 1898, p. 408; Abstr. *J. Chem. Soc.*, 1899, p. 445.
[8] Kromer, Abstracts, *J. Chem. Soc.*, 1893, 1, pp. 428, 482.

The colour vision of the Eskimo. By W. H. R. RIVERS, M.A., St John's College.

[*Received* 9 March 1901.]

The observations described in this paper were made on the same party of Labrador Eskimo of whom Mr Duckworth and Mr Pain have already given an account in the *Proceedings* of this Society[1]. I am indebted to Mr Taber for his permission to examine these people.

A few previous observations have been recorded.

Bessels[2] obtained the names of coloured papers from sixteen individuals belonging to Smith Sound. All could distinguish red, blue, yellow, green, black and white, but had no names for gradations of intensity. The same name was given to both brown and blue, and Bessels believed that these people were unable to distinguish the two colours.

In 1880 Virchow[3] obtained the names of colours from five Eskimo from Labrador. He gives the names for the chief colours and notes that difficulties only occurred in the cases of orange and yellow on the one hand, and violet and brown on the other.

Holmgren[4] records that Almquist, a member of the Vega expedition, examined 125 Eskimo in Port Clarence in Behring Straits, and found only one to be colour-blind.

I have also consulted Erdmann's Eskimo dictionary[5], which is based on the work of missionaries in Labrador, and the comparative vocabulary drawn up by Rink[6].

I examined eighteen individuals (ten males and eight females) with Holmgren's wools. All understood the method quite readily and none were colour-blind. Nearly all put red and pink wools together but, as the names given later to the colours showed, they evidently distinguished the two colours well. Blue and green

[1] *Proc. Camb. Phil. Soc.*, vol. x., p. 286, 1900.
[2] *Arch. f. Anthropol.*, vol. VIII., 1875, p. 107.
[3] *Verhandl. d. Berlin. Gesellsch. f. Anthropol.*, 1880, p. 266.
[4] *Upsala Läkareförenings förhandlinger*, vol. xv., 1880, p. 222.
[5] *Eskimoisches Wörterbuch*, by F. Erdmann, Budissin, 1864.
[6] *The Eskimo Tribes*, by H. Rink, Copenhagen and London, 1887.

were also commonly put together, but as the examination had unfortunately to be carried on by the illumination of the electric light, not much importance can be attached to this.

The names of the colours were obtained by showing the series of coloured papers sold by Rothe of Leipzig, supplemented by grey and brown papers, and also by showing various wools from Holmgren's set. The names were obtained from different individuals independently of one another. The methods used were exactly the same as those employed in my work in Torres Straits[1].

Red was called aupaluktak[2] by all.

Orange was called quqsutak[3], or quqsuangaijuk by most individuals. It was also called aupalangaijuk (reddish), or aupalulutuk (light red) by some.

Yellow was called quqsutak by nearly all. Two men called it quqsutaklarik (real yellow) to distinguish this colour from the orange to which they had already given the same name. One woman called yellow quqsutakqaqoangaijuk (whitish yellow).

Green and yellowish green were both called iviujuk. One or other of these colours was also called by several natives iviujuk-iviangaijuk (greenish green) meaning probably that the paper in question was green, but not so green as the other. One man called the green paper iviangaijukqaqonetuk, a compound of the word for green and white with the affix "-netuk."

Blue green was called by most tungajuangaijuk (bluish); also tungajuksolutak-qaqoangaiuk, a compound of the words for blue and white. This colour was also called iviangaijuk (greenish), and iviujuk-kenangaijuk (blackish or dark green).

Blue was called tungujuktak by most; also tungajuangaijuk (bluish), and tungajuktak-tungalangaijuk (bluish blue), the same form as was also used for green.

Indigo was tungujuktak, also tungajuktamerik (probably pure blue) and tungajuksorituk (pure blue).

Violet was called tungajuangaijuk (bluish), by most. It was

[1] See *Rep. Brit. Ass.*, 1899, p. 586.

[2] I have adopted the spelling of Eskimo words which is employed by Boas (*Sixth Ann. Rep. Amer. Bureau of Ethnology*, p. 413). The letter "q" stands for a guttural sound which Rink describes as something between g, rk, and rkr, and the latter has also used the letter q to express it. The letter χ stands for the German "ch" sound. "Au" = the "ow" in how, "ai" the "i" in hide, and "j" corresponds to the English y.

[3] I was very doubtful about the correct spelling of this word. It was certainly pronounced differently by different individuals. Alternative spellings are qutχutak and quqjutak. The word is not given in Erdmann's dictionary and the nearest words to it in Rink's vocabulary are qorsuk, green or yellowish, and qussok or qudjok, white. Three of Virchow's natives used this word, which Virchow spells korsutak or kuksutak, as written by the natives themselves. I also made some of the Eskimo write down their words but found that they used a very simple alphabet and neglected all the phonetic difficulties.

also called aupalangaijuk (reddish) and aupalanganniusaijuk (said to mean light red, more probably dull red), while one woman called this colour tungajuktak-aupalangaijuk (reddish blue).

Purple (magenta) was called aupaluktak by most, while several called it aupaluktak-tungajuangaijuk (bluish red).

On giving various brown papers and wools to be named there was at once much greater hesitation than for any other colours. Some failed to think of any suitable names, but by others, browns of different shades were called by the following names: kaijuk, kaijuangaijuk, aχjangatuk, sinanuk, sinanangaijuk, aupalangaijuk, aupalanniusaijak, aupaluktakaupalangaijuk, quqsangaijuk, iviuangaijuk, and tungajuangaijuk.

White was called qaqotak by nearly all; it was also called qaqotamerik.

Black was kĕnētuk[1]; deep black was called kenelarik (real black) and kenetamerik-merik. A duller black was called kenangaijuk, aχjangatuk and aχjanganiusaijak.

A dark grey was called kenangaijuk, aχjangatuk, aχjangaijuk, sinanangaijuk, and also kenetangaijuluatuk (probably light black).

A light grey was called qaqoangaijuk by most; also sinanuk, and aχjangatuk.

These names agree in general with those given by Virchow except that the natives examined by him did not indulge in the refinements of nomenclature which I describe, but limited themselves to the chief forms together with those ending in "angaijuk." In addition to the six chief words, aupaluktak, kuksutak, iviujak, tungujuktak, kakortak and keiuitak, Virchow gives one other word "songapaluktak," used for orange and yellow, which I did not meet with. This word is not given in Erdmann's dictionary, but Rink gives sungarpoq as a word for yellow in Greenland and sungaktok as having the same meaning in the northern part of the American side of Behring Straits.

The first interesting feature of the Labrador vocabulary is the definiteness of nomenclature for green and blue. Every one of the Eskimo examined by me used iviujuk and tungajuktak constantly and definitely for green and blue, and named shades of those colours by suitable modifications of these terms. The definiteness of the word for blue was shown in a very striking way by the fact that several individuals called purple (magenta) aupaluktaktungalangaijuk (bluish red) and that one called violet tungajuktakaupalangaijuk (reddish blue). These individuals seemed to have recognized in giving these names that both colours contained red and blue, and that one contained more red and the other more blue.

[1] Rink spells the word qernerpoq (the Greenland form).

This definiteness of nomenclature for green and blue, and especially for blue, is very exceptional in the languages of people in stages of civilization similar to that of the Eskimo. Other subarctic races, as the Chukchis[1] and the Samoyeds[2], are said to show the same absence of definite terminology for green and blue which is characteristic of nearly all primitive languages.

The other divisions of the Eskimo race do not appear to be as advanced as those examined by me. Bessels found that the Eskimo of Smith Sound confused brown and blue and had no names for gradations of intensity, and Almquist found that the colour names of the Eskimo of Behring Straits were indefinite. It is possible that the Labrador Eskimo examined by me had become exceptionally definite in their nomenclature owing to European influence, for in Erdmann's dictionary tungajoktak is given as meaning green as well as blue, and two of the Labrador Eskimo examined by Virchow called violet "kirnitangajuk" (blackish).

The tendency to confuse blue and black in nomenclature which is present in nearly all more or less primitive languages seems to have been completely absent in the Eskimo examined by me, though possibly the tendency may have been shown in a slight degree by the woman who called blue-green "iviujukkenangaijuk" (blackish green). The Eskimo of Hudson Bay (Central Eskimo) called black "mugtuk" while those of the southern part of the American side of Behring Straits use this word for blue (Rink). The latter also use a word "tanaqtok" for black which Rink states is possibly related to or confused with the word used elsewhere for blue.

One may also see in the words used by the Labrador Eskimo some traces of the prominence of red in colour nomenclature which is the most characteristic feature of primitive colour languages. Several individuals called orange and violet by some modification of the word for red. They were more influenced by the red component in these colours than by the yellow and blue components respectively.

In another respect the Eskimo language resembles all the other more or less primitive languages with which I am acquainted, viz. in the absence of a word for brown. When asked the names for brown wools and papers, the indecision and delay in answering was in marked contrast with their behaviour with other colours. The browns shown were most commonly named by means of some modification of the word for red, while others were termed yellowish or greenish or even bluish. Two other words "sinanuk" and "aχjangatuk" were used which were also applied to grey. One

[1] See Almquist, *Die wissenschaft. Ergebnisse d. Vega-expedition*, 1883, vol. I. p. 42.

[2] See Kirchhoff, *Das Ausland*, 1883, p. 546.

word "kaijuk," which was said to be the name of the yellow fox, was used by three individuals for brown wools and seemed more like a word for brown. It was only used spontaneously for brown, but the natives agreed that it would be correct to call a bright yellow by this name and it was also said to be properly used for grey. It was evidently far from being a generally accepted term for the colours which we are in the habit of classing together as brown. Of the five individuals examined by Virchow, three called brown aupalangajuk, one tongulangajok and one kojoangajok, no doubt the same word which I have written kaijuangaijuk.

One of the men examined by me also called two different brown wools tungajuangaijuk, showing the same tendency to confuse blue and brown in nomenclature which Bessels found among the Eskimo of Smith Sound. This confusion of blue and brown is very common and is still met with among German peasants[1].

The most characteristic feature of the Eskimo language appears in the colour vocabulary in the very extensive use of qualifying affixes. If one excepts three words, sinanuk, aχjangatuk and kaijuk, which were comparatively rarely used, all hues, shades and tints of colour were named by various modifications of the six words for red, yellow, green, blue, white and black. I have given in brackets the meaning of the various affixes used so far as I was able to ascertain them. There seemed to be no doubt that the termination "-angaijuk" corresponded exactly to our "-ish"; the terminations -larik, -lulutuk, -sorituk, -niusaijak, were said to mean real, light, pure, and light or dull, respectively, and the way in which they were used corresponded with these meanings.

Another termination "-tamerik," once used in the form "tamerikmerik," was said to mean "dark," but it was used both for deep black and for white and was probably used in the sense of "intense" or "pure." I was unable to ascertain the meaning of the termination "-netuk," but it may be negative; one object was called "kenelariknetuk" which was said to mean "not real black." I have been unable to find any of the above examples in the list of affixes given by Rink.

The Eskimo colour vocabulary presents a marked development of what I believe to be an uncommon tendency of colour language. In every language there are a certain number of well-established definite terms for colour with which every individual of the race is familiar. When names are given to other colours it is most common to use words derived directly from comparison with some natural object. This usage is generally found in Australian and Melanesian languages and, among the languages of which I have

[1] See Kirchhoff, *Das Ausland*, 1883, p. 546.

had the opportunity of investigating, it reaches its most marked development in the island of Mabuiag in Torres Straits where a man might give more than thirty names to different colours all derived from familiar natural objects.

It seems to be much less common to denote differences of colour by using modifications of a few colour names. Among the languages of which I have had previous experience I have only met with a slight tendency in this direction in one or two cases.

Psychologically the latter usage would seem to stand much higher, for it implies the presence of definite abstract ideas of colour while the multiplication of names is only one instance of the tendency to specialisation which is one of the chief features of the stages of mental development found among savage races[1]. The use of affixes is the characteristic feature of the Eskimo language generally but it is perhaps suggestive that this higher psychological development of nomenclature, as shown by the presence of distinct abstract terms for colour, should exist together with exceptional definiteness in the nomenclature for green and blue.

I endeavoured to ascertain from the natives the derivation of the terms they used. They told me that the word for green was derived from ivik, grass; that kaijuk[2] was the name of the yellow fox and that aχjangatuk was derived from aχjak[3], powder, but could not tell me the derivation of the other terms used.

Rink states that the word for red is derived from auk, blood[4], the affix -paluqpoq (Greenland) meaning "has the appearance of." The word songapaluktuk obtained by Virchow is obviously formed in the same way. This word is derived from songaq (sungaq), bile. The words for yellow and green in other subarctic races are derived in the same way. The Chukchis use a word "dlilil" for yellow and green which means bile (Almquist)[5] and the word used by the Samoyeds for both green and blue, "padiraha," is closely related to the word "padea," bile (Kirchhoff)[6]. The Voguls are said to call green and yellow "vosrem ospe," meaning "like bile" (Budenz, quoted by Kirchhoff).

Erdmann and Rink give a word "tungo" blue or black berry

[1] Both usages are found in the languages of civilized races. The tendency towards specialisation in colour names is found in the colour vocabulary in popular use, while in scientific methods of nomenclature all colours are described as modifications of a few standard sensations.

[2] The word kaijuk is also said to mean 'blood.' See Rink, pp. 107, 117, and also Herzog, *Zeitsch. f. Ethnol.* vol. x., 1878, p. 449.

[3] Rink gives what is probably the same word as argsak, ashes.

[4] Hall (*Life with the Esquimaux*, 1864, vol. II., p. 207) found this word ("oug") was used for anything red when the Eskimo were talking with a stranger not well versed in their language.

[5] *loc. cit.*

[6] *loc. cit.*

juice, which is obviously related to, and may be the source of, the word for blue.

It is worth noticing that the women of the party were quite as good as, if not superior to the men, both in matching and naming colours. The individual who seemed to me to have the most highly developed colour vocabulary was a middle-aged woman, Akopejok, and my opinion seemed to have been shared by the natives themselves, for on several occasions when men were unable to find a suitable name, they went to consult this woman.

The superiority of women in colour nomenclature is familiar among ourselves, but I believe it to be unusual in low stages of culture.

Erdmann mentions that women often have special names, different from those used by the men, and I looked out carefully for any indication of this in the names used for colour but failed to detect any difference. In some languages such differences are only present as slight modifications of termination, etc., and if this were the case in Eskimo, they may have escaped my notice.

My previous experience of very defective colour nomenclature has been derived from races inhabiting the tropics and it seemed somewhat unnatural to find a far more highly developed language for colours in the inhabitants of a subarctic country such as Labrador.

The Eskimo, however, told me that in the autumn they could see all the colours I had shown them in the hills of their country and it is possible that when colour is only a transient occurrence in the year's experiences, it may excite more attention and therefore receive more definite nomenclature than in those parts of the world where luxuriance of colour is so familiar that it receives little notice.

So far as I can gather from reading accounts of Eskimo life, colour does not appear to be largely used in the dress or decorations of these people. Coloured beads are generally mentioned as part of the dress of the women and I noticed that those whom I examined were working with beads of various colours, including blue.

Variations in the ossification of the Occipital bone. By Prof. A. MACALISTER, President.

[*Read* 4 March 1901.]

In one occipital boue out of every four some recognisable divergence from the described normal type is to be found specially in the region of the planum interparietale. These are either varieties in degrees of curvature, ridging or other qualities of surface, or varieties in the union of the component elements. At present I propose to deal only with the latter series.

The portion of the occipital behind the foramen magnum originates partly in cartilage, partly in membrane. The former portion, the *supraoccipital,* includes all the planum nuchale as far as the linea nuchæ suprema, the latter includes all that part of the planum interparietale which extends from the linea nuchæ suprema to the lambdoid suture. The supraoccipital region ossifies from a pair of centres which arises about the ninth week close to the region of the protuberance. These unite medially about the twelfth week. Apparently there is not much variation in these except in the extent to which they spread. I have already described the occurrence of intercalary centres between the exoccipital and the supraoccipital, and the os Kerckringii has been long known. When it does not exist a median spur derived from the united supraoccipital centres can be usually found in the fœtal skull which enters and, by expanding and uniting with the adjoining exoccipital borders, fills up the infra-occipital fontanel.

It is the region of the planum interparietale which is most prolific of varieties. My observations lead me to believe that normally there are at least four spots at which ossification begins in this membranous area. These begin about the twelfth week most commonly as a little cluster above the region of the protuberance which generally unite on the median part of the space about the fourteenth week. As the position of these clusters of bony nodular reticulations is inconstant, there is sometimes a difficulty in their analysis, and often they appear to be grouped from the first as a pair of almost confluent nuclei.

The four normal elements, when distinct, are arranged two laterally and two medially and superior. The former are named interparietals, and the latter preinterparietals. The union of interparietal and preinterparietal on each side takes place in general with the fourth month, and the combined lateral plates begin to unite medially and below by the end of that month. At birth, as is well known, the entire hinder part of the occipital is usually a single piece shewing marginal traces of the suture below supraoccipital and interparietals as well as medially and above, between the contiguous preinterparietals. Almost all possible deviations from normal union of these elements may occur, and these may be tabulated by a simple series of formulæ.

Taking the type form in which all the elements coalesce, supraoccipital, interparietals and preinterparietals, and distinguishing the right from the left of these paired elements by affixing a dash to the initial of the latter, it may be represented by the formula $sii'pp'$. On the other hand, the cases in which each element remains discrete so that the single occipital bone is represented by an archipelago, the formula would be

$$s + i + i' + p + p'.$$

Between these extremes there are about 28 possible intermediate combinations, of which the most interesting are the forms known as the interparietal bone which is $s + pp'ii'$, and that called epactal bone which is $sii' + pp'$. The other possible combinations are

5. $s + pp' + i + i'^*$;	6. $si + pp' + i^*$;	7. $si' + pp' + i^*$,
8. $si' + pp'i^*$;	9. $s + pp'i + i'^*$;	10. $si + pp'i'$;
11. $s + i + pp'i^*$;	12. $sp'ii' + p$;	13. $sii' + p + p'$;
14. $si' + p + p' + i^*$;	15. $si + p + p' + i'$;	16. $spii' + p'^*$;
17. $sp + p' + i + i'^*$;	18. $sp'i' + pi$;	19. $s + pi + p'i'$;
20. $si' + p' + pi$;	21. $s + pi + p' + i'$;	22. $spi + p'i'^*$;
23. $si + p + p'i'$;	24. $s + p'i' + p + i$;	25. $spp'i + i'^*$;
26. $spp'i' + i^*$;	27. $spp' + i + i'$.	

Those with a star appended are represented in our Museum.

Another source of variation is the union of one element, the preinterparietal, with the parietal bone. Of this I have only one specimen ($spii' + p'$ *par*).

Over and above the varieties of the combination of the normal centres of ossification there is a second and more prolific series of varieties due to the occurrence of new centres along the line of the lambdoidal suture. These are numerous and perplexing

but may be reduced to three groups, lambdoidal, intermediate and asterial.

The angle of the lambda may be formed by a single bone or of a pair of lambdoid bones which differ from the preinterparietal in the depth to which they penetrate into the squama. By overgrowth they sometimes seem to push the preinterparietal element very hardly, but there is generally little difficulty in the way of distinguishing them. Outside these there are often paralambdoid bones, single, paired or multiple; occasionally a pair of paralambdoids may exist while the angle is preinterparietal.

About the middle of the lambdoidal suture on each side there is a more or less conspicuous reentrant angle on the hinder edge of the parietal into which a projecting angle of the occipital projects. This is the angulus intermedius. On the whole skull this lies along the line of that indefinite rounded ridge that forms the boundary between the medial strip of skull-roof and the lateral skull-wall, which continues from the frontal tuber through the parietal tuber, and here crosses the lambdoid suture to end in the linea nuchæ suprema. The parietal notch probably owes its existence to the unequal growth of the twin centres of that bone the limits of whose posterior territories it marks. On the occipital it corresponds nearly to the limit between the preinterparietal and the interparietal. At this spot an angular bone is very commonly developed, often flanked medially by one or more metangular and distally by one or more parangular.

The third region at which wormian bones may be found is the asterion or point of confluence of the parietal, mastoid, interparietal and supraoccipital bones. Here there is often a galaxy of ossicles, sometimes only an asterial or a metasterial bone, flanked perhaps by others not belonging primarily to the lambdoid suture, but to the occipitomastoid or parietomastoid.

To one or other of these types the larger sutural bones may be referred. There are also minuter ossicles of another category representing detached portions of sutural teeth separately ossifying and often embedded, not penetrating the whole thickness of the bone.

Geometrical Notes on Theorems of Halley and Frégier. By C. TAYLOR, D.D., Master of St John's College.

The two subjects of these notes are Halley's construction for the normals or normal from a given point to a parabola by means of a circle, and Frégier's theorem that a chord of a conic which subtends a right angle at a fixed point of the curve passes through a fixed point on the normal thereat.

A.

HALLEY.

1. Apollonius shewed how to draw the normals to a conic from a given point by means of a rectangular hyperbola.

This cuts the conic in four or fewer real points, each lying on a normal to the conic from the given point.

In the course of his investigation Apollonius found the coordinates of what we call the centre of curvature, and thus virtually the equation of the evolute of a conic.

From the given point H in certain positions normals can be drawn to a central conic at four points P, p, Q, q.

When the conic is a parabola one of the four points, as q, is at infinity, and the remaining three, P, p, Q lie on a circle through the vertex A. Halley shewed how to draw the normals HP, Hp, HQ by means of this circle.

2. Draw the ordinates PN, pn, QM, supposing Q to be either of the two points on the parabola having the same abscissa AM and such that

$$QM = PN \pm pn.$$

The chords AQ, pP make equal angles with the axis, and in this section they may be parallel.

Taking the upper sign, draw PpI to the axis, and let $4a$ be the latus rectum.

Then $\quad AM = IN + In = AN + An + 2AI.$

Writing $(PN + pn)^2/4a$ for AM and subtracting $AN + An$ from both sides, we get

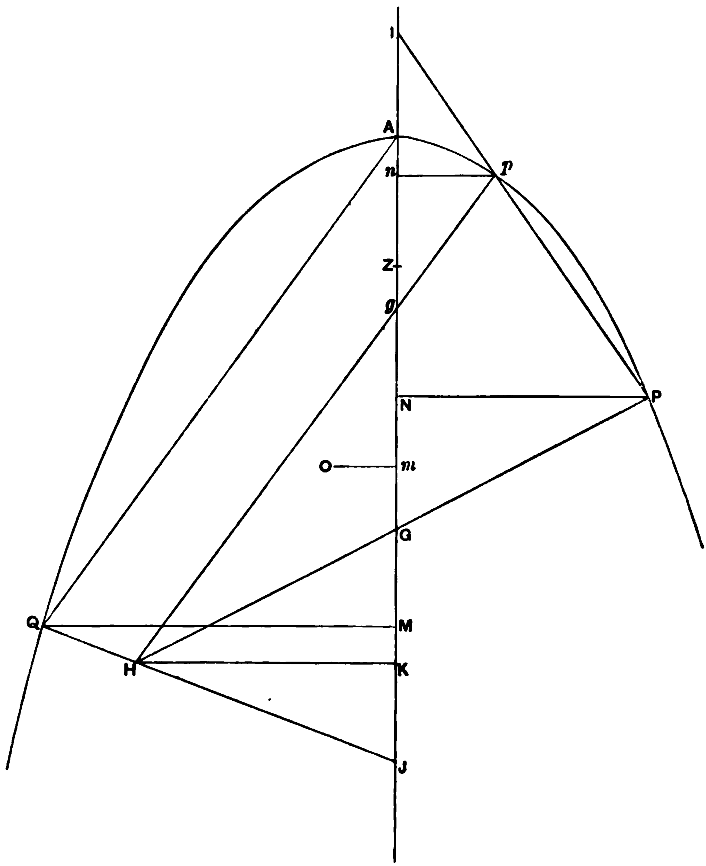
$$PN.pn = 4a.AI.$$

But $$In/pn = AM/QM = QM/4a.$$

Therefore $$PN.In = AI.QM.$$

In what follows Q is to be taken as in the figure and so that the algebraic sum of PN, pn, QM is zero.

3. Let the normals PG, pg produced meet in a point H whose ordinate is HK, and on AK produced take KJ equal to AI. Then JH *will be the normal at* Q.



For $$In/IN = pn/PN = KG/Kg,$$

the subnormals being each equal to $2a$.

Hence $$KG = In,$$

and $$AJ = IK = IN + In + NG = AM + NG.$$

Therefore JQ is the normal at Q.

And JHQ is a straight line, because

$$HK.NG = PN.In = AI.QM = KJ.QM.$$

4. Let O (on the same side of the axis with H) be the centre of the circle $APpQ$ and Om its ordinate. The point m is found from AK by taking an abscissa AZ equal to $2a$ and bisecting KZ.

For the diameters bisecting Pp and AQ meet the axis in points U, V such that

$$IU = \tfrac{1}{2}AM + 2a = AV,$$

and Om bisects UV.

Therefore $$2Am = AM + 4a - AI$$
$$= AK + 2a.$$

To determine Om from HK we have

$$Om/\tfrac{1}{2}AI = \tfrac{1}{2}QM/2a,$$

the ordinate of the middle point of AQ being equal to $\tfrac{1}{2}QM$.

Therefore $$Om = \tfrac{1}{4}HK.$$

The circle described with centre O and radius OA determines the points P, p, Q, and thus the normals HP, Hp, HQ.

5. A short proof, by means of another circle and a second parabola, of Halley's theorem that A, P, p, Q lie on a circle is given by Messrs Milne and Davis in their *Geometrical Conics*.

The focal perpendiculars SP', Sp', SQ' to the normals PG, pg, QJ bisect them, and S, P', p', Q' lie on the parabola whose equation is

$$y^2 = a(x - a).$$

This, by its intersection with the circle on SH as diameter, determines the points P', p', Q', and thereby the points P, p, Q.

The algebraic sum of the ordinates of P', p', Q' being zero, the same follows for P, p, Q, which therefore lie on a circle through A.

But as a practical construction it is of course simpler to draw this circle and find P, p, Q in Halley's way.

6. After finding O he gives a construction for the limiting point h on the ordinate of H such that, according as HK is less or greater than hK, two normals or none can be drawn from H across the axis. From h one normal only can be so drawn.

The point h is an intersection of consecutive normals, the position of H when P, p coincide.

Making AN, An, AI equal, we get

$$AK = 3AN + 2a; \qquad hK = AN.PN/a;$$

and for the equation to the evolute, the locus of h,

$$27ay^2 = 4(x-2a)^3.$$

To find h produce the axis outwards to B so that

$$BZ = \frac{27}{16} \text{ lat. rect.} = \frac{27}{4} a.$$

Then draw ZL at right angles to the axis to meet the circle on BK as diameter, and inflect Zh to HK parallel to BL.

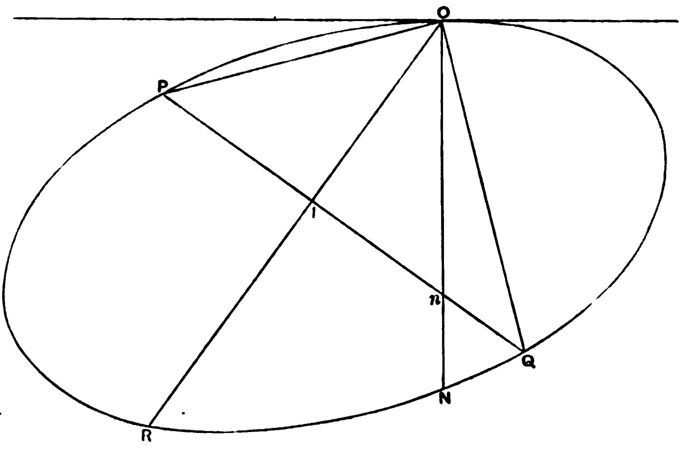
Having given his constructions without proof at the end of prop. 39 in his Serenus *De Sect. Coni*, Halley concludes,

"Horum omnium demonſtrationem, cum in nimiam excreſceret molem, totamque fere ſolidam Geometriam poſtularet, in præſentia omittendam cenſeo. Ex iis tamen quæ in quinto Conicorum [Apollonii] habentur, et quæ in *Philoſoph. Tranſact.* Num. 188 & 190 tradidimus, non multo opere comprobari poterunt."

B.

Frégier.

1. A chord PQ of a conic which subtends a right angle at



a fixed point O on the curve passes through a fixed point on the normal at O.

Draw the chord OIR across and at right angles to PQ, and let f, g be the focal chords parallel to OR, PQ.

Then $$OI^2/OI.IR = PI.IQ/OI.IR = g/f.$$

Therefore $$OI.f = IR.g = OR/(1/f \pm 1/g),$$

or $OI.f$ varies as OR, the chords f and g being at right angles.

But OR *varies as the projection of* f *upon the normal at* O[1].

Therefore, if PQ meets the normal in n, then OI varies as OI/On, and On is constant and n a fixed point.

2. Another proof is given as a problem in *The Ancient and Modern Geometry of Conics*, page 122 (1881), thus,

"279. If PQ be a chord of a conic which subtends a right angle at a given point O on the curve, and MN be the projection of the chord upon the tangent at O, shew that

$$\frac{PM}{OP^2} \pm \frac{QN}{OQ^2} = \text{a constant},$$

and that PQ passes through a fixed point on the normal at O."

The result follows from the lemma that, if $OPO'Q$ be a rectangle (or parallelogram), and if a line $OABC$ be drawn across PQ, $O'P$, $O'Q$, then

$$\frac{1}{OA} = \frac{1}{OB} \pm \frac{1}{OC}.$$

In the problem, this sum or difference being constant when $OABC$ is the normal at O, it follows that PQ passes through a fixed point A on the normal.

3. *Proofs from the Circle.*

a. If O be a fixed point on a conic and PQ any chord through a fixed point I, then OP, OQ are conjugate lines in an involution, and conversely.

This includes Frégier's theorem as a special case.

It may be proved by projecting the conic into a circle with I as centre.

[1] Take V on the normal chord ON, draw RVO' to the curve, and let VR, VN cut any parallel to OO' in r, n. Lastly let O' coincide with O. Then

$$VR.Vr/f = VN.Vn/h,$$

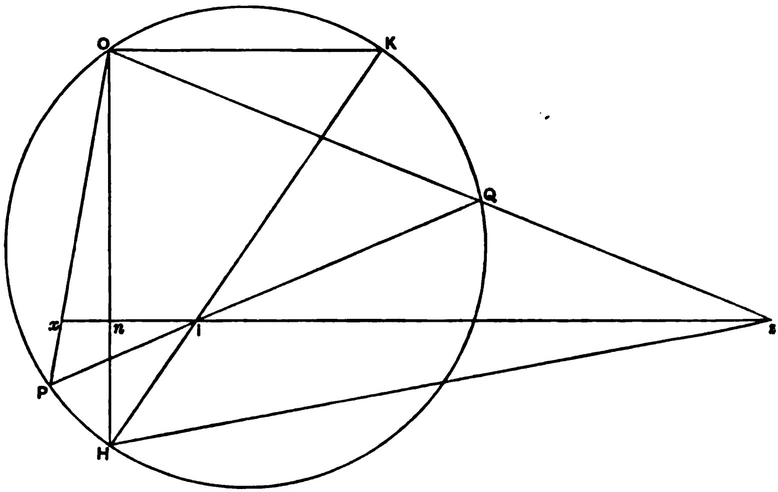
if f, h be the focal chords parallel to VR, VN, and in the limit

$$OR/f \cos NOR = ON/h.$$

b. Or in the circle, supposing that I is not the centre, draw a fixed chord HIK, and let the parallel through I to OK meet OP, OH, OQ in x, n, z.

Then by the circles $IHPx$, $IHzQ$,

$$\angle xHP = \angle xIP = \angle QIz = \angle QHz.$$



Hence $$nx.nz = nO.nH,$$
the points O, x, H, z lying on a circle because $\angle xHz = \pi - xOz$.

Or if these points lie on a circle, PIQ will be a straight line.

c. Supposing HIK to be a diameter of the circle, make the same construction in an ellipse with OK parallel to an axis.

Then, by orthogonal projection, nx.nz = *a constant.*

This constant may be determined by making OP, OQ coincide with OI and the tangent at O.

When OI is the normal at O the constant is On^2, and OP, OQ are always at right angles.

Hence Frégier's theorem and its converse for the ellipse.

Notes on new and interesting Plants from the Malay Peninsula. By R. H. Yapp, B.A., Gonville and Caius College, Cambridge.

[*Received* 26 March 1901.]

With one or two exceptions, the plants which form the subject of these notes were collected on Gunong Inas, a mountain nearly 6000 feet high, situated on the northern confines of Perak, one of the western states of the Malay Peninsula.

The mountain ranges of this region, which are only partially explored and somewhat inaccessible, possess a wonderfully rich flora, which is quite unaffected by the admixture of species introduced from other tropical countries, which form such a marked feature of the vegetation of the plains, especially in the neighbourhood of the rivers and sea-coast. A number of the species to be mentioned are probably new, and these will be described elsewhere in due course. Others are introduced here on account of some special interest, biological or otherwise, which attaches to them.

Dicotyledons.

In the Natural Order *Polygalaceae* there is a species of *Polygala*, probably new, at least to the Peninsula, for, although very closely allied to *Polygala arillata*, Hamilt., the latter species also is unknown from the Malay Peninsula, though it is found both in India and in the Malay Archipelago. The specimen in question was found at a height of about 5000 feet, growing on the ground in the jungle which, though dwarfed, still covers the mountains even at this elevation.

Among the *Celastraceae*, *Euonymus Wrayi*, King, a slender shrub, 6 to 9 feet high, is interesting on account of its rarity. It has been found only by two previous collectors, once in Perak and once in Pahang.

Of the *Rubiaceae* two plants merit attention. A species of *Lucinaea*, which climbs by adventitious roots, and was found in the low-lying lands at the foot of the mountain. It is identical with an undescribed specimen in the Kew Herbarium, also from Perak, collected by Ridley.

Cephaëlis stipulacea, Blume, of which I found two or three specimens in thick jungle, at an altitude of about 3 to 4 thousand feet, has hitherto been recorded only from Java.

Among the *Asclepiadaceae* there is an epiphyte, which is probably a new species of *Pentanura*, though the flowers are much larger than is usual in that genus.

This specimen was procured by the expedient of shooting several charges of heavy shot into a mass of foliage, many feet above the ground, where the red glint of flowers could be dimly discerned.

The *Gesneriaceae* are represented by a number of new species, belonging to the genera *Æschynanthus*, *Didymocarpus*, etc.

Many of the species in this Order, especially in the section *Cyrtandraceae*, to which all the Old World species belong, have an extraordinarily limited distribution[1], most (if not all) of the high mountain ranges of this region which have been yet explored botanically, having furnished their quota of new forms[2].

In the Order *Acanthaceae* there is a new species of *Asystasia*, and also a plant which will probably have to be put into a new genus, as it does not seem to fall at all naturally into any of those already recognised.

MONOCOTYLEDONS.

The *Orchidaceae* furnish a new species of *Dendrobium*. This is a rather pretty epiphyte, with bright rose-purple flowers, which was found at a height of 5200 feet. It is closely allied to *Dendrobium cornutum*, Hook. f., which is also a species from the Malay Peninsula.

In the *Gramineae* is a curious bamboo, *Bambusa Wrayi*, Stapf. This bamboo, which grows in large clumps at a height of 4000 feet to nearly 6000 feet above sea level, is of interest partly on account of its rarity, being known only from two localities, both in Perak, i.e. Gunong Inas, and the mountains at the source of the Plus river[3]: but especially from the fact that it is the plant from which the Sakeis and Semangs, two wild non-malayan races of aborigines inhabiting the Malay Peninsula, make their 'sumpitans' or blow-guns. The stems of this extremely graceful bamboo are from 40 to 60 feet high, and are remarkable for the extraordinary length of their internodes, which are often from 7 to 8 feet long, though they seldom reach a diameter of more than one inch.

These long thin internodes the wild tribes use for their blow-

[1] C. B. Clarke in De Candolle's *Monographiae phanerogamarum—Cyrtandraceae*, p. 5.

[2] H. N. Ridley, *Jour. Linn. Soc.*, vol. XXXII. p. 497.

[3] Letter from L. Wray, jr. in the *Kew Bulletin*, 1898, p. 17.

guns, the smaller ones forming the tube itself, the stouter the protecting sheath. The tubes are straightened while still green over fires, and are then hung up to dry in the smoky palm-leaf huts in which these people live.

The hollow internodes of this and several other species of bamboo I found were usually from $\frac{1}{3}$ to $\frac{1}{2}$ filled with apparently pure water.

This storing of water by bamboos, although mentioned by several authors[1], is a fact that appears to be far from widely known, though it certainly deserves to be so; for this supply of naturally-filtered water is, at least in the case of the larger species, at times invaluable, especially to those travelling through the hill-jungles of the tropics. For instance, on one occasion during the ascent of this mountain, we were quite dependent for a day or two on bamboo-water, for drinking, cooking and washing; an accident having occurred to the meagre supply of water we could carry with us, and there being no streams in our immediate line of march, as we were following the watershed line of the range.

That this water is actually pumped up by the roots of the plant, and is not merely collected rain-water is, I think, sufficiently proved by the fact that only the perfectly sound internodes contained clear water, while in those which had been damaged sufficiently to produce even a very small aperture leading to the exterior, the water was usually of a deep brown colour.

The extent to which this phenomenon is to be met with among bamboos, and its explanation, are, I believe, quite unknown. Possibly it occurs only in countries which, like the Malay Peninsula, possess a very damp climate, or, if in other tropical countries, only during the wet season. Probably the activity which finds expression in the enormously rapid growth of the young stems, is connected with the storing up of water, which can be used as the needs of the growing parts require. In any case, whatever be the causes of the phenomenon, the problem is an interesting one.

Vascular Cryptogams.

Amongst the *Filices*, *Hymenophyllum obtusum* Hk. et Arn. has never been recorded from the mainland of Asia before, though it has previously been found as near as Borneo.

Two species, a *Polypodium* and an *Acrostichum*, are probably new, though more or less closely allied to known forms.

Perhaps the most interesting of all, speaking biologically, are

[1] S. Kurz in the *Indian Forester*, vol. I. p. 239; also Hackel, *The True Grasses*, p. 199.

two curious epiphytic Ferns, *Lecanopteris carnosa*, Blume, and *Polypodium sinuosum*, Wall.

The stems of *Lecanopteris* form a thick crust, often 3 or 4 feet long and a foot or more thick, on the branches of trees. This Fern grows only on mountains of a considerable height; while the smaller Polypodium is found creeping on tree-trunks, often very little above sea-level.

Both of these remarkable Ferns may be classed amongst the so-called 'myrmecophilous' plants, as they agree in possessing thick, fleshy rhizomes, which, except in the very youngest parts, contain numerous hollows in the form of continuous galleries, which are invariably inhabited by colonies of ants.

These galleries are formed in both cases by the breaking down of a large-celled, thin-walled tissue, and are not excavated by the ants themselves, though the latter do occasionally tunnel out short passages leading from the galleries to the exterior.

PROCEEDINGS

OF THE

Cambridge Philosophical Society.

"*Ignoration of coordinates*" *as a problem in linear substitutions.*
By T. J. I'A. Bromwich, M.A., St John's College.

[*Received* 24 April 1901.]

The method known in dynamics by the name of "ignoration of coordinates" has received a good deal of attention at the hands of writers on dynamics; but it does not seem to have been examined at all from the point of view of linear substitutions. The object of the following is to explain how we are led to the usual results by simple examination of the linear equations from which we start.

Suppose that we have $(m+n)$ quantities $\xi_1, \xi_2, \ldots, \xi_m, \eta_1, \eta_2, \ldots, \eta_n$ given as linear functions of $(m+n)$ others $x_1, x_2, \ldots, x_m, y_1, y_2, \ldots, y_n$ by the equations

$$\xi_r = \frac{1}{2}\frac{\partial V}{\partial x_r}, \quad \eta_s = \frac{1}{2}\frac{\partial V}{\partial y_s}, \quad \begin{pmatrix} r = 1, 2, \ldots, m \\ s = 1, 2, \ldots, n \end{pmatrix},$$

where V is a quadratic function of the x's and y's. We are now going to express $\xi_1, \ldots, \xi_m, y_1, \ldots, y_n$ in terms of $x_1, \ldots, x_m, \eta_1, \ldots, \eta_n$; and to do so let the equations for the ξ's and η's be written in the symbolical form

$$\xi = Px + Qy,$$
$$\eta = Rx + Sy,$$

where P, S are square matrices of m and n rows respectively, while Q is a rectangular matrix of m rows and n columns, and R is one of n rows and m columns.

Solve for the y's in terms of the x's and η's, then we get

$$y = -S^{-1}Rx + S^{-1}\eta;$$

of course the determinant $|S|$ is supposed not to be zero, for if it were zero the quantities x, η would not form an independent set of variables, so that the method of transformation would fail entirely. Substituting in the ξ's, we have now

$$\begin{aligned} \xi &= (P - QS^{-1}R)\,x + QS^{-1}\eta, \\ y &= \quad -S^{-1}Rx \quad + S^{-1}\eta, \end{aligned}$$

which solves our problem. But, for the dynamical investigations, it is important to reduce the final substitution to a *symmetrical* one, so as to connect it with a quadratic form; now since V was a quadratic form, the original substitution was symmetrical, and so

$$P' = P, \quad Q' = R, \quad R' = Q, \quad S' = S,$$

where accented letters denote the matrices *conjugate* (or *transposed*) to the unaccented matrices. Using these facts we get

$$\begin{aligned} (QS^{-1})' &= S^{-1}Q' = S^{-1}R, \\ (P - QS^{-1}R)' &= P' - R'S^{-1}Q' = P - QS^{-1}R, \\ (S^{-1})' &= S^{-1}, \end{aligned}$$

and we see that the substitution giving ξ, y in terms of x, η is not symmetrical as it stands, but can be made so by writing it in the form

$$\begin{aligned} \xi &= (P - QS^{-1}R)\,x + QS^{-1}\eta, \\ -y &= \quad S^{-1}Rx \quad - S^{-1}\eta. \end{aligned}$$

The matrix of this substitution (A, say) is

$$\left(\begin{array}{c|c} P - QS^{-1}R & QS^{-1} \\ \hline S^{-1}R & -S^{-1} \end{array}\right) = \left(\begin{array}{c|c} 1 & -QS^{-1} \\ \hline 0 & S^{-1} \end{array}\right) \times \left(\begin{array}{c|c} P & 0 \\ \hline 0 & -S \end{array}\right) \times \left(\begin{array}{c|c} 1 & 0 \\ \hline -S^{-1}R & S^{-1} \end{array}\right),$$

where the zeros denote matrices all of whose elements are zero, and the unities are square matrices with units along the principal diagonal and zeros in all other places. This equation is transformable to the shape

$$A = C'BC,$$

for clearly the first matrix is the conjugate of the third; and it is to be observed that C is the matrix of the substitution expressing x, y in terms of x, η.

Now consider the quadratic form derived from V,

$$U = V - \Sigma y_s \frac{\partial V}{\partial y_s},$$

it is then easy to see that the matrix of the coefficients of U is the matrix denoted above by B. Suppose that when U is expressed in terms of x, η it becomes U_1, then its matrix is known to be

$$C'BC = A,$$

and we have thus the rule given by Routh (for dynamical cases)

$$\xi_r = \frac{1}{2}\frac{\partial U_1}{\partial x_r}, \quad -y_s = \frac{1}{2}\frac{\partial U_1}{\partial \eta_s}, \quad \begin{pmatrix} r = 1, 2, \ldots, m \\ s = 1, 2, \ldots, n \end{pmatrix}.$$

We may remark that the determinant of U_1 is

$$|A| = |B| \times |C|^2,$$

now $|B| = |P| \times |-S| = (-1)^n |P| \times |S|$, $|C| = |S^{-1}| = |S|^{-1}$,

so that $$|A| = (-1)^n |P| / |S|.$$

A result equivalent to the last was given in Part I. of the Mathematical Tripos, 1898.

Suppose that instead of applying the substitution C to U, we applied it to V; the resulting quadratic form in x, η would be, say, V_1, and its matrix would be

$$\left(\begin{array}{c|c} 1 & -QS^{-1} \\ 0 & S^{-1} \end{array}\right) \times \left(\begin{array}{c|c} P & Q \\ \hline R & S \end{array}\right) \times \left(\begin{array}{c|c} 1 & 0 \\ \hline -S^{-1}R & S^{-1} \end{array}\right) = \left(\begin{array}{c|c} P - QS^{-1}R & 0 \\ \hline 0 & S^{-1} \end{array}\right).$$

It follows that in V_1 the two sets of variables x, η are separated; a result which has important consequences in the theory of reducing quadratic forms[1] and in dynamics[2].

A somewhat different transformation occurs in Optics, when deducing the equations of a ray of a thin pencil (after passage through any optical instrument) from the properties of the characteristic function. In this case $m = n$, and it is necessary to express the y's and η's in terms of the x's and ξ's. Following a process similar to that adopted above, we find (if $|Q| \neq 0$)

$$y = -Q^{-1}Px + Q^{-1}\xi,$$
$$\eta = (R - SQ^{-1}P)x + SQ^{-1}\xi,$$

but I do not know of any means to bring the matrix of this substitution into a symmetrical form.

[1] Cf. a paper to appear in the July number of the *American Journal of Mathematics.*

[2] For instance, Thomson's (Kelvin's) and Bertrand's theorems on systems started from rest by impulses can be readily deduced from this.

There are, of course, certain relations amongst the coefficients of the matrix in consequence of the conditions

$$P' = P, \quad Q' = R, \quad R' = Q, \quad S' = S.$$

In the ordinary optical case ($n = 2$) these relations have been elaborated by Prof. R. A. Sampson[1]; and in this case I have used the substitution analogous to "ignoration of coordinates" with the object of shortening the discussion of the optical invariants[2]. More extensive substitutions of a similar character had been previously employed[3] by Prof. Heinrich Bruns of Leipzig, for the purpose of discussing aberration in an optical instrument and other similar problems.

It is perhaps worthy of remark that the matrix of the last substitution is equal to the product

$$\left(\begin{array}{c|c} 0 & 1 \\ \hline R & S \end{array}\right) \times \left(\begin{array}{c|c} 1 & 0 \\ \hline -Q^{-1}P & Q^{-1} \end{array}\right),$$

so that its determinant is

$$(-1)^n \,|\, R \,|\, \times \,|\, Q^{-1} \,|\, = (-1)^n \,|\, R \,| \,/\, | \, Q \,| = (-1)^n.$$

To illustrate the conclusions arrived at, take the simple case

$$V = ax^2 + 2hxy + by^2.$$

Then the original relations are

$$\xi = ax + hy,$$
$$\eta = hx + by.$$

The derived quadratic form is

$$U = V - 2y\,(hx + by) = ax^2 - by^2;$$

expressing this in terms of x, η, we find

$$U_1 = (a - h^2/b)\, x^2 + 2\,(h/b)\, x\eta - (1/b)\, \eta^2,$$

so that the derived substitution is

$$\begin{aligned} \xi &= (a - h^2/b)\, x + (h/b)\, \eta, \\ -y &= \qquad\quad (h/b)\, x - (1/b)\, \eta, \end{aligned}$$

which has a determinant $-(a/b)$, in agreement with what was proved before.

[1] *Proc. Lond. Math. Soc.* vol. 29, 1898, p. 33; it may be remarked that the relations occur (in a more general form) in connection with some types of contact-transformation.

[2] *Ibid.* vol. 31, 1899, p. 4.

[3] "Das Eikonal" (*Leipziger Abhandlungen*, Bd. 21, 1895, p. 325).

Also, expressing V in terms of x, η, we have

$$V_1 = (a - h^2/b)\,x^2 + (1/b)\,\eta^2,$$

in which the two sets of variables are separated, according to the general result.

In the corresponding optical transformation (which occurs in the theory of a symmetrical instrument) we express y, η in terms of x, ξ, and find

$$y = -(a/h)\,x + (1/h)\,\xi,$$
$$\eta = (h - ab/h)\,x + (b/h)\,\xi.$$

This substitution has the determinant -1 (as found in general), but here there is no other relation amongst the coefficients of the substitution.

On an attempt to detect radiation from the surface of wires carrying alternating currents of high frequency. By O. W. RICHARDSON, B.A., Coutts Trotter Student, Trinity College.

[*Read* 6 May 1901.]

This work was directly suggested by the theory of the mechanism of the conduction of electricity in metals recently put forward by Professor J. J. Thomson[1]. According to that view electric currents in metals are carried by negatively charged ions moving with a definite velocity under a given electric field. In the case of alternating currents of high frequency, almost the whole of the current is carried by a very thin 'skin' at the surface of the metal; so that if the current is carried by ions they must be either much more concentrated or move with considerably greater velocities in the parts near the surface of the metal. In any case, since the current tries to get as far out of the metal as possible, we might expect the ions to be driven away from the surface, if sufficiently strong currents of very high frequency were employed. The experiments cannot be regarded as a crucial test of the theory since we do not know anything about the forces which tend to retain the corpuscles within the body of the metal. But it seemed that a positive result might reasonably be expected, and so the experiments described below were undertaken.

On *à priori* grounds it seemed probable that the radiation to be looked for would be similar in character to that given out by radio-active substances. It might be constituted by either (*a*) charged ions moving with considerable velocity, or (*b*) some disturbance produced by the sudden stoppage or starting of such ions. The effect would therefore probably be of the nature of primary or secondary Röntgen rays. It would probably affect a photographic plate, and it would certainly ionise the gas through which it passed. It was therefore decided to look first for a photographic effect, and failing that a discharging effect emanating from the surface of the wire.

The alternating currents were obtained by the discharge of a large induction coil, each terminal of which was connected to the inside of a small Leyden jar. The jars were insulated in paraffin and their outsides were connected through the wire or rod to be investigated. The primary current was furnished by six large

[1] *Rapports Congrès de Physique*, Vol. III., Paris, 1900.

storage cells and a spark-gap of three centimetres was generally used. In this way the spark could be kept going almost continually for hours. The apparatus was set up in a dark room and in addition to this the excited wire was enclosed in a blackened wood box, inside which the photographic plates were exposed.

In the first experiments a brass rod about 1 cm. thick was used, but this was soon replaced by a brass wire ·7 mm. in diameter. Aluminium and copper wires of about ·6 mm. and a fine steel wire of ·09 mm. diameter were also tested. Exposures of 5, 10, 20 and 60 minutes were given successively to different plates. At first the plates were about 2 cms. from the wire, and it was expected they would be more fogged along the line of projection of the wire on the plate than elsewhere. The fog observed on developing was however quite uniform and so further exposures were given with plates covered by thick copper with a square hole in the middle. Nothing was observed except when the plate was close up to the wire, when the outline of the square was distinctly seen on developing. This was ultimately shown to be due to a very faint glow between the wire and the edge of the copper. Since, if the rays were very soft ones, they might have been absorbed by 1—2 cms. of air at ordinary pressures, the copper plate was now removed and the photographic plate brought up close to the wire. When the wire was only ·3—·5 mm. from the plate, an appearance similar to that produced by drawing an interrupted straight line on blotting paper was obtained on developing.

The interruptedness is due to the fact that the wire could not be made quite straight on account of its stiffness; so that it was nearer the plate in some parts than others. The edges of the linear spots presented in places a peculiar jagged appearance when magnified. The nature of the phenomenon was investigated by inserting thin plates of various material between the wire and the photographic plate. The action was found to be transmitted through transparent substances (*e.g.* thin glass and mica) but was stopped by the thinnest aluminium foil. It was traced finally to a very faint glow which occasionally passed along the wire. The effect seemed identical in every respect with that recently investigated by M. T. Tomassina[1]. It was found impossible to make sure of this glow never occurring, so that the only thing left was to try for an effect at low pressures when any radiation which might be given off would not be absorbed by the air.

In the form of apparatus used at first, the excited wire simply passed in at one end of a wide tube which could be exhausted and out again through a short side tube fixed at right angles to

[1] *Comptes Rendus*, May 1900.

the first. The wire was supported by sealing-wax both where it entered and where it left the tube. The other end of the tube was closed by a tight india-rubber stopper which could be removed to insert the plates. The outside of the tube was painted with several layers of black paint so that no light could get through. After getting rid of troublesome glows no photographic action could be detected. It was therefore concluded that no photographically active radiation was shot off at right angles to the wire.

In this apparatus there was no very ready means of discovering whether the discharge really passed through the part of the wire which was tested or not; although it could be proved to be passing through the parts of the wire external to the glass apparatus by the insertion of a coil with an electrodeless discharge bulb or of a spark gap. This defect was remedied in the form of apparatus described below by the insertion of a mercury cup into which the excited wire dipped, within the exhausted vessel. The wire could be drawn at will out of the mercury and a spark gap thus produced. By observing the light from the spark the intensity of the discharge could therefore be determined.

It was possible that although the ions did not move off at right angles to the wire they might still be produced close to the surface and pursue straight paths backwards and forwards parallel to the wire under the influence of the electric field. In this case the application of a strong enough transverse magnetic field would cause them to move away from the wire, so that a photographic effect might still be got if the wire were placed in a magnetic field. To accomplish this the wire was fixed axially in a narrow glass tube, which was placed between the poles of an electromagnet.

The apparatus finally took the form shown in figure 1. The discharge wire passed through the top of a T-tube, the top being about 30 cms. long and ·6 cms. diameter. The limb of the T-tube was cut off about 1 cm. from the shoulder and inserted into the cork *e*, so that it opened into the inside of a wide test-tube *f* which was cut off about 12 cms. from the open end. Another tube *g* also passed through the cork to the water-pump and manometer used before. The limb of the T-piece only penetrated half-way through the cork which was cut out so that the end of the tube was flush with the surface. This enabled the photographic plates, which were fixed by soft wax on to the end of the support *d*, to be brought as near the discharge wire as possible. The tight india-rubber stopper *h* allowed of the removal of the plates, while the other end of the tube was made air-tight by covering the cork etc. with sealing-wax. The central wire was free except for being fixed air-tight in *a* with sealing-wax. *A* was

joined to *b* by thin pressure tubing, and they were clamped independently so that by keeping *b* fixed and raising the clamp of *A*,

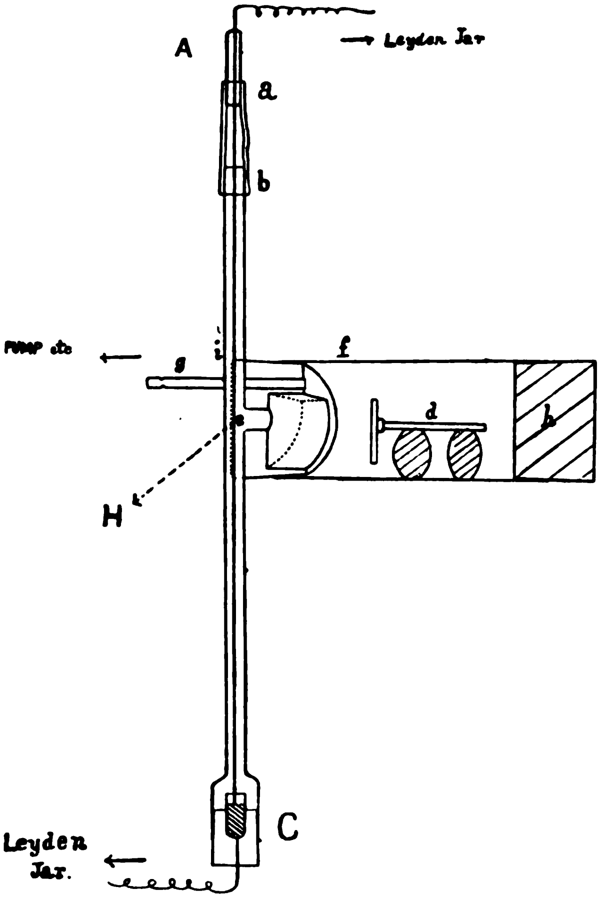


FIG. 1.

the wire was lifted out of the mercury cup and a spark gap produced. The apparatus was made light-tight with black paint, except for a small space near *C* which was generally covered with black paper held in place by a rubber-band. This enabled the spark gap to be observed when desired, and the quality of the discharge to be ascertained. The tube *i* was placed vertically in the gap between the pole pieces of an electromagnet, which were close up against it. The width of the air gap was thus equal to

the diameter of the tube *i* and the lines of force were perpendicular to the excited wire (*i.e.* they were along the dotted line *H* of the figure). It was found necessary, to prevent the wire from glowing, to put one end (a) of it to earth; this in no way interfered with the intensity of the discharge, as was shown by producing the spark gap at *C*.

With this apparatus experiments were made with copper, brass, soft iron and aluminium wires of about ·7 mm. diameter and also with the steel wire ·09 mm. diameter. The length of exposure varied up to three hours. The other conditions were changed as much as possible, *e.g.* the spark length was varied from 1 to 4 cms. and exposures were taken with and without the magnetic field on. To increase the frequency of the discharges, and therefore the intensity of the skin currents, the Leyden jars were replaced by two small paraffin condensers of less than one centimetre capacity.

The following measurements (for the case of the thin steel wire) indicate the maximum value of the current density attained at the surface. The capacity of the whole system including the wires was taken to be 10 cms. The self-induction was approximately equal to 4×10^4 electromagnetic units. The period of the oscillations was therefore $= 14 \times 10^{-6}$ seconds approximately. For the exponential factor $\epsilon^{-\left(\frac{2\pi\mu p}{\sigma}\right)^{\frac{1}{2}} x}$ giving the diminution of the current density with the depth x from the surface, μ was taken to be $= 10^3$ and $\sigma = 10^4$. If A is the current density at the surface that at depth x is $\therefore = A\epsilon^{-6\times10^3 x}$. The voltage for a spark length of 2·5 cms. is 75,000 or 250 electrostatic units of potential. The quantity of electricity set in motion at each discharge is therefore 2500 electrostatic units giving an average total current through the wire of 2×10^{10} units on the same system. Hence the current density at the surface of the wire during a discharge is given approximately by the relation

$$2\pi A \times \cdot 0045 \int_{\cdot 0045}^{0} \epsilon^{-6\times10^3 x}\, dx = 2 \times 10^{10},$$

i.e. $A = 4 \times 10^{15}$ electrostatic units or $1 \cdot 3 \times 10^6$ amperes per sq. cm.

Since the plates were unaffected in the above experiments it was concluded that no photoelectric radiation was given off from the wires. It was still possible that radiation might be given off which had no action in a photographic plate, or whose action was feeble compared with its power of ionising the air through which it passed. Further experiments were therefore made to test whether the air in the neighbourhood of the wire possessed conducting properties.

In the form of apparatus first used the air was drawn away from the wire and caused to pass between two parallel charged plates. One plate was put to earth and the other was connected to a sensitive gold-leaf electroscope whose rate of leak was measured. The air currents were generally produced by a dropping water bottle, but for the faster ones a water pump was used. The air in entering and leaving passed through sulphuric acid wash bottles, and the whole apparatus was further dried by a bulb of phosphorus pentoxide inserted in the middle. The supports of the plates were insulated with sulphur, and electricity was prevented from leaking from the discharge wire to the plates along the glass by putting an earthed ring of tin foil round both inside and outside. The electroscope was enclosed in a rectangular glass box and had a sulphur insulation. The movable leaf was of gold, and the readings were taken by a distant telescope. Both leaves were generally read in order to correct for any displacement of the instrument. It was found necessary to enclose the greater part of the apparatus in a wooden box to prevent leaking caused by ultra-violet light and gases from the spark gap. The insulation of the electroscope itself was very good, the normal leak being not more than ·4 scale divisions per hour, but that of the whole apparatus could not be reduced below two divisions per hour. In every case a blank experiment was made immediately after each real experiment. For this the outsides of the jars were disconnected from the apparatus and joined by a short wire. In this way the effect of the induction coil was made the same as in the actual experiments. Thus any appreciable difference in the leak in the two cases must be entirely due to radiation coming from the wire.

The following conditions were altered during the experiments.

(1) The velocity of the air varied from three small drops per second out of the water bottle to a rapid continuous stream drawn through by the pump.

(2) In some of the experiments, by tightening the pinch-cock of the inlet tube, the pressure of the air was reduced to about 5 cms. of mercury.

(3) The material and diameter of the wires and the capacity of the condensers was varied as before.

The following are typical of the observations taken:

Conditions of Experiment	Initial Reading of Electroscope	Final Reading	Leak in Scale Divisions	Time of Experiment	
Stream of air very quick	58·0	57·0	1·0	30 mins.	Current not passing through wire (Test Experiment)
Stream of air very quick	57·0	56·2	0·8	30 „	Current passing through wire
Using small Leyden jars	56·0	55·0	1·0	30 „	„
Using large Leyden jars	58·0	56·8	1·2	30 „	„
Pressure inside apparatus = 4 cm's. of mercury	54·7	54·0	0·7	30 „	„

It is thus seen that within the limit of the experimental error there is no extra leak produced by the current through the wire.

But it seemed possible that, even if ionisation took place near the wire, all the ions might have time to recombine before they reached the leaking plates. A new apparatus was therefore constructed in which the conductivity of the air in the immediate neighbourhood of the wire could be examined. The leaking part now took the form of a wire spiral forming a cylinder round the wire to be tested (Fig. 2 *b*). This was supported on a continuation (*a*) of the same stiff wire round which a sulphur cylinder was cast.

The wire was held in the end of the tube by sealing-wax *i*, which was carefully treated on the outside to form a good insulating surface. The exterior insulation was therefore of sealing-wax, the interior of sulphur. The end of the wire *g* could be put in contact with the electroscope used before. The wire through which the discharge passed was fastened (soldered when possible) to the stout copper wires ff_1. These were fixed air-tight into small tubes at opposite sides of the bulb *l* by means of sealing wax. To prevent a leak from the comparatively high potentials

of the wire ff_1 to the testing system *bg* an earthed ring of tin foil was put round the outside of the glass tube at *d*; and inside a fine steel spiral *c* which pressed tight against the glass was

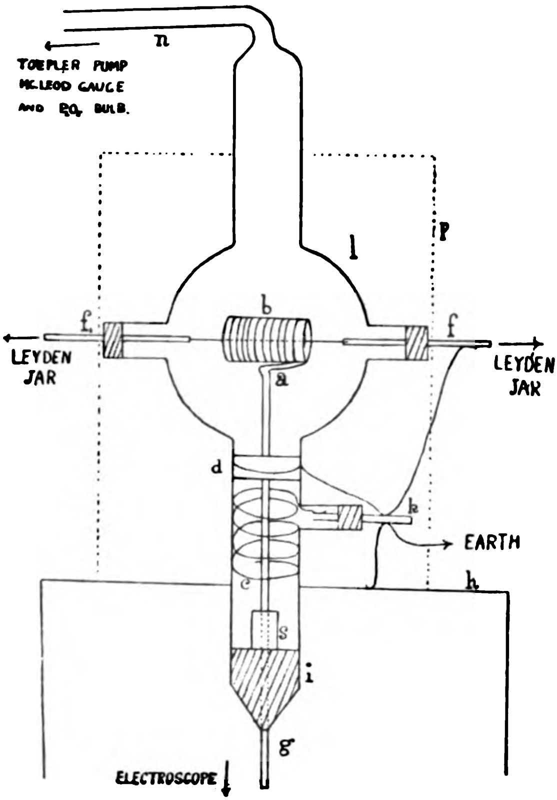


Fig. 2.

supported by the stout copper wire *k*. The wire *as* passed of course down the axis of this spiral. The lower part of the apparatus was protected from spark gases and ultra-violet light by being enclosed in the cigar-box *h*. It was found necessary to protect also the upper part of the apparatus from ultra-violet light. This was done by a cardboard arrangement *p* built to fit it. The air was exhausted through the tube *n* which led by way of a phosphorus pentoxide bulb to a Töpler pump and McLeod gauge.

The apparatus had now assumed its most sensitive form, since the capacity of the charged cylinder and parts connected with it was only three times that of the electroscope. In a slightly different form of apparatus, used for the experiments at ordinary pressures, and insulated entirely with sulphur, the normal leak was reduced to about 0·1 scale division per hour; and in the last described apparatus, used for low pressures the leak was never greater than 1 scale division per hour. The apparatus was therefore much better insulated than that with parallel plates when there was a normal leak of 2 divisions per hour. Experiments were made at ordinary pressures with aluminium and steel wires. The following numbers, for an aluminium wire, indicate the sort of readings obtained.

Pressure	Initial Reading of movable leaf	Final Reading	Leak in Scale Divisions	Time of Experiment	Leak per hour	
Atmospheric	46·0	44·2	1·8	18 hrs.	0·1	No current through wire
,,	44·2	44·0	0·2	3 ,,	0·07	Current passing through wire

From a large number of similar observations it was evident that no appreciable amount of ionisation was produced in air at ordinary pressures. It was still possible however that an appreciable amount might be produced at low pressures. For if a few ions were produced under ordinary circumstances, these, if the pressures were lower, would produce more by their motion in the electric field, *i.e.* when they collided with the molecules of the gas[1]. Thus the method might be expected to become more sensitive when employed at low pressures. A large number of observations were therefore taken (using an aluminium wire) at pressures varying from 3 to 0·1 mm. Two of the observations taken are subjoined :—

[1] J. S. Townsend, *Phil. Mag.*, Feb. 1901.

Pressure	Initial Readings of Electroscope	Final Readings	Leak in Scale Divisions	Duration of Experiment	Leak per hour	
mm. 1·81	Left Right 30·2—37·2	Left Right 30·2—36·4	0·8	44 mins.	1·0 Div.	Wire not excited
1·81	30·2—36·4	30·2—36·0	0·4	36 „	0·7 „	Current passing through wire
·36—·28	30·0—38·0	29·8—36·0	1·8	3 hrs.	0·6 „	No current through wire
·28—·37	31·0—39·2	31·0—37·2	2·0	3 „	0·7 „	Current through wire

The above experiments had been made with the cylinder charged positively; they were now repeated with a negative charge but no leak was obtained. The small Leyden jars (of about 400 cms. capacity) were next replaced by a paraffin condenser whose capacity was about half a centimetre, and the pressure was varied down to 0·01 mm. With these alterations several observations were taken, of which the following are examples.

1. *Aluminium wire,* ·6 *mm. thick.*

Time of Experiment	Pressure	Divergence of leaves Initial	Divergence of leaves Final	Leak	Rate of Leak	
mins. 50	mm. ·5—·08	13·4	12·4	Divns. 1·4	per hr. 1·5	No discharge through wire
50	·08—·3	12·4	11·4	1·0	1·1	Current through wire

2. *Steel wire,* ·09 *mm. thick at very low pressures.*

Time of Experiment	Pressure	Initial	Final	Leak	Rate of Leak	
100	·01—·02	41·5	40·2	1·3	0·8	No current through wire
106	·03—·07	43·2	42·2	1·0	0·65	Current through wire

The experiments therefore show that there is no leak produced by alternating currents in the wires. It remains only to consider the sensitiveness of the apparatus and the order of smallness within which the effect, if there be any, must lie.

Approximations to this were obtained in three ways:—

1. From the weight and dimensions of the gold leaf the quantity of electricity corresponding to a given reading was calculated. Now a leak from the 8th to the 7th millimetre per hour could easily have been detected, and the corresponding rate of diminution of surface density is $1{\cdot}11 \times 10^{-8}$ electrostatic units per second. This gives the least detectable loss of charge as $1{\cdot}42 \times 10^{-4}$ electrostatic units per second, taking into account the area of the leaves and the capacity of the whole apparatus. Since the charge on an ion is 6×10^{-10} electrostatic units, we get that an appreciable leak would have been obtained if 2×10^{5} ions had discharged against the conducting part of the apparatus per second. This is about 1000 ions per discharge since there were several hundred discharges per second.

2. A further test of the sensitiveness of the apparatus was afforded by the discharging action of ultra-violet light. A spark gap of 0·5 cm. was set up 15 cms. from the bulb *l*, and a disc was placed in front of the bulb so that all the light had to pass through a hole 1 cm. in diameter. Although most of the ultra-violet light must have been absorbed by the glass of the bulb *l*, nevertheless enough penetrated the interior to produce a leak of 9 divisions of the electroscope in 15 minutes.

3. A third test was afforded by the effect of radio-active substances. A little of the powder, which was of medium strength, was covered by thin aluminium, and after the discharge wire had been removed was inserted through one of the holes *f* by which it entered. It did not project so far as the charged cylinder, yet a leak of 9·5 divisions in 24 minutes was produced. On withdrawing the aluminium foil, three small grains of the substance dropped out and these produced a leak of 9 divisions per hour.

The experiments prove, if not that there is no radiation from the surface of metals carrying very rapidly alternating currents of great intensity, at any rate that it is smaller than the very delicate methods employed are capable of detecting. If the rays had been sufficiently active to produce 40 ions per c.c. per discharge throughout the volume of the gas in the small bulb (*l*) a measurable leak would have been obtained. This furnishes therefore an upper limit to the intensity of the radiation.

In conclusion I wish to thank Professor Thomson for suggesting this work to me, and for his advice and sympathy throughout the investigation.

Note on the Magnetic Deflection of Cathode Rays. By HAROLD A. WILSON, D.Sc., M.Sc., B.A., Trinity College, Clerk Maxwell Student.

[*Read* 6 May 1901.]

The experiments described in this note were undertaken with the object of proving, more completely than has hitherto been done, that the ratio of the charge carried by a cathode ray particle or corpuscle to its mass is independent of the nature of the metal of which the cathode is composed.

It was shown by Prof. J. J. Thomson[1] that this ratio is the same for cathodes of aluminium and iron and by W. Kaufmann[2] that it is the same for cathodes of aluminium and copper so that it is the same with aluminium, iron and copper cathodes.

In view of the great interest attaching to Prof. Thomson's corpuscular theory it seemed to be worth while to prove experimentally that the ratio in question is the same for a larger number of metals having a greater range of properties than the above three.

The method I have used is similar to that employed by Kaufmann (*loc. cit.*). A narrow beam of the rays is deflected by the magnetic field produced by a coil carrying a constant current and the deflection and the potential difference used to produce the rays are measured.

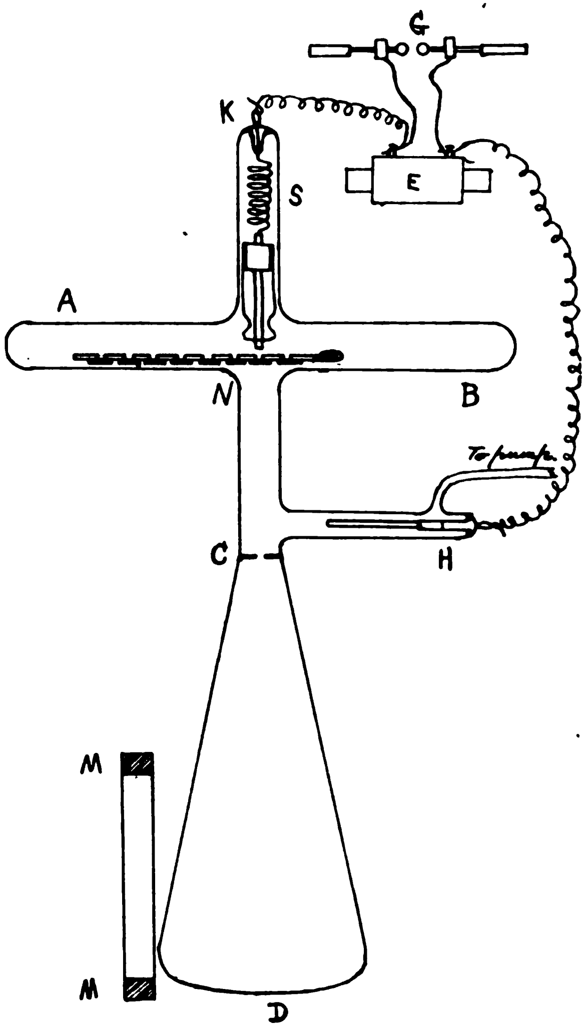
If C is the current in the coil, V the potential difference employed and d the deflection of the end of the beam of rays, then

$$d = \frac{AC}{\sqrt{V}},$$

where A is a constant which for a particular apparatus is proportional to $\sqrt{\frac{e}{m}}$, e and m being the charge and mass respectively of a cathode ray particle. This formula was found by Kaufmann to

[1] *Phil. Mag.* [5] XLIV., Oct. 1897.
[2] *Wied. Ann.* 61, pp. 544—552, 1897.

be very exactly true. In order to prove that $\frac{e}{m}$ for the cathode rays is independent of the nature of the metal composing the cathode it suffices to show that $d\sqrt{V}$ is the same whatever the



metal of which the cathode is composed, the magnetising current being kept constant.

The apparatus used is shown in the accompanying figure. KNC is a glass tube 2 cms. in diameter and about 20 cms. long. At C a conical bulb CD is blown 20 cms. long and 10 cms. in diameter. A tube AB was sealed on across the tube KNC and served to contain the cathodes of different metals.

The cathodes were each 1 cm. square and were fastened to the under side of a strip of mica 1 cm. wide and 10 long. They were fixed about 2 mms. apart. At one end of the strip a piece of thin sheet iron was fastened on by means of which and a small electromagnet the mica and cathodes could be moved along the tube AB so that any desired cathode could be brought over the tube NC.

Connection with the particular cathode to be used could be made by means of an aluminium wire fixed in a glass tube and suspended from the electrode K by a spiral spring S. A small cylinder of soft iron was fixed round this wire near its upper end which enabled it to be raised off the cathodes by means of an electromagnet.

A small hole was made in the mica strip, above each cathode, in which the aluminium wire rested and made contact with the upper side of the cathode.

Thus to change from one cathode to another it was merely necessary to first raise the aluminium wire and then slide the mica along until the desired cathode was in position when the wire could be lowered again.

The cathode rays passed down the tube NC and through a small hole in an aluminium diaphragm at C which allowed a narrow beam to enter the bulb CD. This beam produced a fluorescent spot on the glass at D by means of which its position was known. The anode was an aluminium wire in the side tube H.

The discharge was produced by an induction coil E and the potential difference was estimated by means of a spark gap G which was adjusted until a discharge passed both at G and in the tube.

The coil used to produce the magnetic field is shown at M, M; the current in it was always of the same strength in all the experiments.

The deflection of the fluorescent spot at D when the current in the coil was reversed was measured with a millimetre scale at D.

Cathodes of aluminium, iron, copper, zinc, tin, silver, lead, and platinum were fixed on the mica strip.

The following table contains all the observations made with this tube.

Metal forming Cathode	Atomic weight	Deflection (d)	Spark length	Potential Difference (V)	$d\sqrt{V}$
		mms.	mms.		
Aluminium	27	23	2·0	28·0	122
Iron	56	20	2·5	33·4	116
Copper............	63·6	18	3·0	39·0	113
Zinc	65	16·6	4·5	54·2	122
"	—	10	11·0	110	105
Silver	108	18	3·0	39·0	113
Tin	118	17	3·6	45·3	114
Platinum	195	16	4·0	49·3	112
"	—	16	4·0	49·3	112
"	—	11	10·0	105·0	113
Lead...............	207	18	3·0	39·0	113
"	—	15	5·0	59·0	115
				Mean...	114·2

The P. D.'s corresponding to the spark lengths are taken from Paschen's tables (*Wied. Ann.* 37, p. 79, 1889) for sparks between spheres of 1 cm. radius.

These results show clearly that $d\sqrt{V}$ is nearly a constant quantity whatever the metal of which the cathode is composed and potential difference used to produce the discharge. The variations from the mean value are not greater than can be readily accounted for by experimental errors. It thus appears that $\frac{e}{m}$ is the same for the cathode rays proceeding from each of the eight metals tried.

These experiments were carried out at the Cavendish Laboratory and my best thanks are due to Prof. Thomson for advice during the course of the work.

On a Diminution of the Potential Difference between the Electrodes of a Vacuum Tube, produced by a Magnetic Field at the Cathode. By JOHN E. ALMY, University of Nebraska.

[*Read* 6 May 1901.]

Characteristic effects of a magnetic field upon the discharge in vacuum tubes, especially with large field strength at the cathode have been noted and studied by Birkeland, Melani[1], and others. Birkeland found that with the lines of force parallel to the direction of the discharge, at gas pressures below 0·012 mm. of mercury, the P.-D. between the electrodes was made to diminish by the magnetic field, at first rather slowly as the magnetic intensity increases; when however a certain "critical intensity" is reached a large, abrupt, increase of this diminution is obtained with a very small increase of magnetic intensity. Further study of this effect was my purpose.

I found that not only fields parallel to the discharge, but also a field normal to the direction of discharge, gave this large and rather anomalous diminution of potential difference, and in fact the so-called "critical" intensity was much smaller in the latter case.

Using an induction coil giving a 6-inch spark to furnish the discharge, a spark gap, with two brass spheres in shunt to the tube to measure the potential difference between electrodes, a solenoidal electro-magnet with soft iron core 4 cm. in diameter, about 40 cm. long, to give the magnetic fields, the effects produced in a tube 3·5 cm. in diameter, 20 cm. in length, having plane aluminium electrodes (discs), were observed. The following series, with gas pressure of 0·09 mm. of mercury, are typical:

[1] Cf. Birkeland, *Comptes Rendus*, Vol. 126, p. 586; Melani, *Nuovo Cimento* (Series 4), Vol. 5, p. 329.

TABLE I. *Magnet perpendicular to tube.*

Spark gap (without magnet)...	11·5	11·4	11·5	11	11·2	11·3	11·5	11·5	11·5
Spark gap (with magnet)...	11·5	9	8	7	4	3	2	2	1·9
Potential-diff.......	104	84	76	68	42	33	25	25	24·5
Diminution in P.-D....	0	16	24	34	62	69	80	80	80·5
Magnetic field intensity ...	0	40	48	54	62	69	75	120	420

TABLE II. *Magnet parallel to tube.*

Spark gap (without magnet)...				12·5			
Spark gap (with magnet)......	12·5	10	7	5	3	2	1·7
Potential-diff....	110	90	68	51	33	25	21
Diminution of P.-D.	0	20	42	59	77	85	88
Intensity of magnetic field...	0	180	216	290	300	312	420

The magnetic intensities given are those at the centre of the cathode; spark gaps in millimetres, potential differences in static (c.g.s.) units, according to Heydweiller (cf. *Wied. Ann.* Vol. 48, p. 214).

That the same sort of effect is produced with continuous discharge was shown by using a large Holtz machine to give the discharge. Potential differences were measured with a Kelvin absolute electrometer; gas pressure being about 0·11 mm.

That the relatively large change in potential difference is only produced at fairly small gas pressures became very soon evident. A series of observations of the diminution produced by a certain magnetic field, normal to the tube, of intensity large enough to be safely above the "critical" intensity, with various

pressures were taken; this magnetic diminution is at first relatively large when fluorescence of the glass begins to be produced. Afterwards the magnitude of the effect increases very rapidly. (Table III.)

That the "critical" magnetic field, and also the magnitude of the diminution of potential difference in the tube, were both not independent of the size of the electrodes seemed so probable that the study of the effect of variation of electrode area upon these was made.

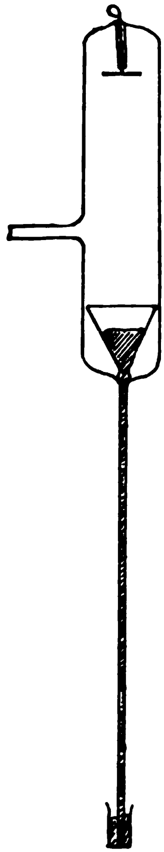


FIG. 1.

To enable variation of the area of an electrode to be made without variation of the other conditions of discharge the tube Fig. 1 was devised.

The fixed electrode being a plane aluminium disc, 15 mm. in diameter, the lower electrode was the top of a barometer column which ended in a funnel-shaped expansion of the tube. The area of this mercury surface could be varied at will by changing the position of the cistern of the barometer. The length of the discharge tube was made large so that the variations of the distance between the electrodes are relatively small.

The variation of the P.-D. between the electrodes as the area of the anode varies was observed. It was found that, with a given constant current passing through the tube, except for very small anode areas, the variation of P.-D. is directly proportional to the change of anode area. And with given constant potential difference applied, the current obtained varied in like simple proportion. The discharge from the Holtz machine, the Kelvin voltmeter, and a D'Arsonval galvanometer, as ammeter, were used here.

Using the induction coil and spark gap, without any measure, or regulation of the current in the tube, except that the coil was driven as uniformly as could be, the effect of variation of cathode area was noted. Also the effect of a magnetic field of 400 lines normal to the tube was observed.

We see here (Table IV) that the P.-D. between the electrodes increases, and somewhat abruptly as the cathode area increases above 1·8 sq. cm. but the P.-D. with the magnetic field on, while it increases with increasing cathode-area, does so at a more nearly uniform rate.

Experiments (Table V) were tried, using a large Wimshurst

Table III. *Potential difference in tube.*

Gas pressure	Without magnet	Magnet normal to tube	Magnet parallel to tube
0·28	3200	2600	2750
0·15	3600	3000	3100
0·10	4000	3250	3350
0·06	4800	4000	4100
0·04	5400	4500	4700
0·03	5800	4800	5000
0·02	6800	5500	5800
0·016	8400	6100	......
0·010	28000	7500	......
0·007	34000	9600	......

Table IV.

Cathode Area....	0·12	0·28	0·78	1·13	1·77	2·54	3·14	3·80	5·30
Without magnet, Spark Gap	1·93	2·01	2·27	2·36	2·6	5·55	9·00	12·4	14·1
Poten.-Diff.	27	28	28	31	33	63	96	110	132
With magnet, Spark Gap	1·90	1·93	2·01	2·06	2·14	2·27	2·40	3·00	4·10
Poten.-Diff.	27	27	27·5	28·5	29·2	30	30·3	31·5	48

electric machine to furnish the discharge, so that the current could be measured. A D'Arsonval galvanometer served as current-meter. Potential measurements were made by the absolute (Kelvin) electrometer. For the measurement of potential differences several times as large as the range of the instrument, a very high resistance, a long tube of xylol with several platinum electrodes at points along the tube, was connected in shunt to the discharge tube, and the potential drop over a part of this was read off with the voltmeter. The relation of a part to the whole was readily determined by observing the potential differences between the electrodes of the different sections of the resistance, with a constant total and summing the parts.

The following series of observations, taken each with constant current, show the character of the variation of the P.-D. between the electrodes of a tube, with variation of cathode-area:

TABLE V.

		I.	II.	III.	IV.	V.
Gas pressure		12 mm.	12	0·12	0·08	0·06 mm.
Current		1·0	50	60	42	50
Cathode areas 0·12	Potential-difference	3000	3600	......	......	36000
0·28		1500	2300	11200	29400	......
0·78		1300	1800	7000	25200	32300
1·77		1000	1300	5600	23800	26400
2·54		800	1150	6600	22000	30000
3·80		700	1000	8700	28600	37000
5·30		600	800	13000	32200	41000

In the discharge of the Wimshurst, at low pressures, there appears a noteworthy phenomenon. The character of the discharge is of two distinctly different types. With cathode area small the discharge takes place in streams, the discharge from the cathode seeming to come off mainly from certain points or small spots, at which a luminescence occurs, the discharge having a considerable similarity to brush discharge from a point at atmospheric pressure. Then, as the cathode area reaches a somewhat larger value the discharge passes to the steady glow or "dark" discharge common in the Crookes' tube at low pressure.

So long as the first type of discharge takes place the potential difference between the electrodes decreases as the cathode area increases, but from the point at which the character of the dis-

charge changes, the potential difference increases with increasing cathode area. And it is only with this second sort of discharge that the effect of the magnetic field is marked, or uniformly consistent.

The transition from the one type of discharge to the other is very largely dependent upon external conditions—in fact it may be entirely prevented. If, for instance, the discharge tube outside the cathode and clear up to the anode be coated with tinfoil, connected to earth (the cathode being also earthed) the character of the discharge remains the same independent of the cathode area. In this case the discharge takes place entirely by stream lines,—usually by a single stream—which pass along the side of the tube, accompanied by luminous flashes, often with crackling noise as a spark discharge.

The action of the magnetic field producing a diminution of potential difference between the electrodes is inseparably connected with the transition to the stream-like discharge, the action of the magnet is to always produce that type of discharge. Furthermore, *the tinfoil coating of the tube entirely eliminates the effect of the magnetic field*, and there is a diminution of the P.-D. between the electrodes due to the tinfoil, of the same magnitude as that produced by the magnet.

The suggestion offers itself that the effect of enlarging the cathode is to increase the charge which collects on the walls of the tube, and this charge acts as a counter E.M.F. to increase the P.-D. necessary for discharge in the tube.

To test this idea two different experiments were tried.

A tube, as shown in Fig. 2, was constructed. The diameter of the smaller tube was about 15 mm., that of the large part being about 60 mm. Aluminium cathodes of 1 cm. area were at each end, and a ring of aluminium wire at the centre was the anode.

The effect of a magnetic field, transverse and parallel, to the direction of the discharge was observed with each end of the tube; then the effect of surrounding the tube with tinfoil connected to earth, was tried.

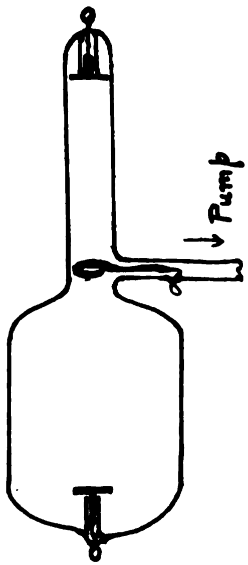


FIG. 2.

These results seem to indicate that the effect produced by the magnetic field is not due to the removal of a static field, for at the different ends of the tube

TABLE VI. *Potential difference between the electrodes.*

	Without magnet	Magnet transverse	Magnet parallel
With cathode *a*	2200	1550	1500
With cathode *b*	5000	3900	3600
a surrounded with tinfoil	1600	1550	1550
b surrounded with tinfoil	4000	4000	4000

the effects produced have magnitudes in the ratio 6 : 10 while the distance between the walls of the tube varies as 1 : 4.

As this experiment is hardly to be regarded as conclusive a tube of the form shown in Fig. 3 was tried. One electrode, a thick aluminium wire, partly enclosed by a thick-walled glass tube was inside the other electrode, a cylinder, with end closed by a disc of metal next the glass tube; by this form of electrodes the inner was shielded from the action of a static charge on the glass.

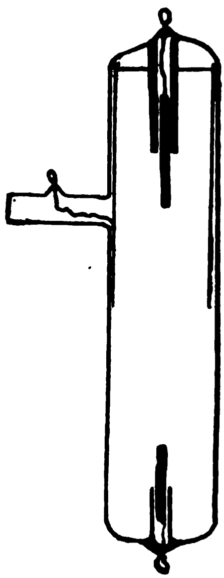


FIG. 3.

With this tube the magnetic diminution was produced, and, with the wire as cathode the effect produced is of remarkable magnitude, in some cases; namely at times the discharge reduces to a small luminous brush or bundle of lines, passing from the wire to the cylinder, resembling the arc discharge at atmospheric pressure. (Table VII.)

The potential differences in the two cases indicated as < 400 was much smaller than that necessary to give a single division deflection of the voltmeter; its actual magnitude was not obtained.

These results seem to show conclusively that the action of the magnetic field is independent of any static charge on the walls of the discharge-tube, since it is produced when the cathode is practically surrounded by the other electrode.

In short, the action of the magnetic field seems to be simply

TABLE VII. *Potential difference between the electrodes.*

	Wire as cathode	Wire as anode
Without magnet.........	7400	8400
Magnet parallel to tube	2800	<400
Magnet normal to tube	3000	<400

that of concentrating the discharge, so that the discharge through the gas takes place by a sort of brush—or arc—discharge, in fact with some similarity to disruptive discharge, instead of as usual in the Crookes' tube discharge. And the effect of this concentration is to very greatly increase the conductivity, or diminish the resistance to discharge, of the tube.

In conclusion, I wish to express my thanks to Professor J. J. Thomson for the many privileges afforded at the Cavendish Laboratory, and for his advice and help given in the course of this work.

Some Experiments upon Beams under endlong compression.
By H. E. WIMPERIS, B.A., Gonville and Caius College.

[*Received* 2 April 1901.]

1. To know at what particular load a beam will break requires a knowledge of several important and many obscure points. Among the more important points are the elasticity of the metal (supposing the beam to be of metal), the method of applying the load and the actual shape of the beam; other obscurer but not less important ones are the homogeneity and ductility of the material and the treatment it has undergone when being forged, rolled, or cast, etc. The ductility is roughly measured by the percentage elongation that occurs, say, on an 8-inch length when a tensile load great enough to break the rod is applied; this property though mentioned last is by no means the least important, as even though its effect is not marked during elastic extension or compression, yet when once the limit of elasticity is exceeded its effect is very strongly shown in distributing the stress more evenly over the surface strained. A bar with little ductility will break owing to an isolated fracture over one small area spreading over the whole surface[1], whereas a ductile bar will allow the stress at the elastic limit to be considerably exceeded before fracture.

The effect of ductility is very plainly shown in the experiments described in this paper, and its measure is indicated by the fact that the mild steel rods used, when stressed to the extent of 40 tons per square inch, showed an elongation of 12 per cent. on an 8-inch length.

The simple theory of the bending of beams assumes that a section of the beam originally plane remains plane during the elastic deformation that occurs on loading, this section being taken perpendicular to the plane of bending. In other words, the relation between the distance of any fibre from the neutral axis and the strain in that fibre is one of simple proportion. When the load is increased to such an extent that the maximum stress exceeds

[1] The state of affairs in a non-ductile rod has been well compared to a "Bridge structure in which several tension members have been replaced by Cast Steel Ties."

the elastic limit, then this linear law no longer holds, and the applied bending moment is equilibrated by the inner layers taking more than what may be called their "fair share" of the load; when this occurs the stress is no longer proportional to the strain. During the elastic deformation the section was everywhere plane and the stress diagram was of the form shown in Fig. 1 where the ordinates of the shaded triangles *AFC* and *CEH* represent the stress at different distances from the neutral axis; when the elastic limit is exceeded the stress diagram takes the form indicated at *ABCDE*. In the first case the equilibrating moment was just great enough to balance the applied without deforming the "plane

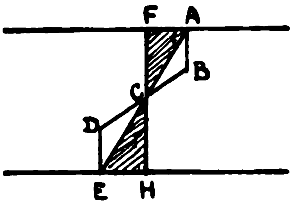


FIG. 1.

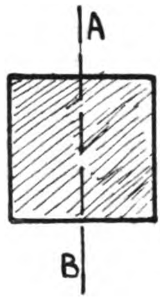


FIG. 2.

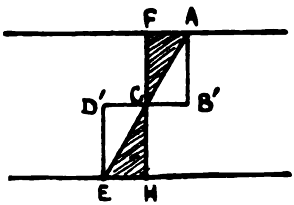


FIG. 1a.

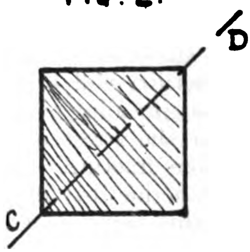


FIG. 2a.

section," at the larger moment however the beam takes up the load by increasing the shaded area *AFC* to the area *AFCB*, and the limiting case is reached when the triangle *AFC* becomes a rectangle standing on *AF* for base and of height equal to *FC* (this is the limiting case for an infinitely ductile substance) as shown in Fig. 1 *a*. In that case the resisting moment will be at its maximum value, i.e. about 50 per cent. greater than that at the elastic limit (this is nearly true for mild steel or iron) and any further increase of the load must produce fracture. The moment is said to be

about 50 per cent. greater since the moment of the "rectangular forces" is greater than that of the "triangular" in the ratio $\frac{3}{2}$.

As an example of this effect the case of a square sectioned strut may be cited; for any given elastic deformation the required bending moment will be the same whether bending occurs about an axis parallel to one side of the square or about a diagonal or about any intermediate axis. When the elastic limit is exceeded, this state of affairs no longer holds—thus if bending occurs about the axis AB in Fig. 2 the limiting bending moment, as previously explained, is about 50 per cent. greater than the maximum elastic one. In Fig. 2 a however, where bending occurs about an axis such as CD, the limiting bending moment is only about 40 per cent. greater than the maximum elastic one.

In other words, the strut is *weaker* when bent as in Fig. 2 a than as in Fig. 2, therefore when such a strut collapses it might be expected to do so by bending about a diagonal axis. The author tested this theory by taking a $\frac{3}{8}$-inch square-sectioned strut and loading it till instability set in and collapse occurred—it was found that the bending *was* about a diagonal axis, and seven repetitions of this experiment gave the same result in every case.

2. This helps to explain the experimental results obtained by the author in the following experiments. The material used was mild steel and was in the form of circular rods of diameter = 0·256 inch, the ends of the rods being coned to receive steel balls, and these balls being used to transmit the endlong forces and at the same time to allow bending to occur with as little friction at the ends as possible. Preliminary experiments showed that the elastic limit of this material was about 33 tons per square inch and that the breaking load in direct tension was about 41 tons per square inch. The rods, each 24 inches long, were placed upright in the apparatus shown in Fig. 3 and loaded with a force P at the top whilst being pulled out sideways by a weight W. The lower block LB and the screw above were both recessed conically to receive steel balls (0·312 inch in diameter) and between these balls was placed the rod to be tested. The block LB rested upon the cast-iron platform of a steelyard so that any downward force exerted upon LB was measured by reading the steelyard lever—this steelyard read up to about 1 ton. The downward force P was exerted by screwing down the wheel SW until the strut-beam being tested was subjected to the required degree of endlong compression, this compression being easily measured as stated above. In addition to this endlong loading a lateral one was introduced by suspending a weight W from the cord shown on the right of the diagram. Both P and W were next increased—one at a time—

until collapse occurred; at the point of collapse the values of P and W were carefully noted and readings were obtained over a

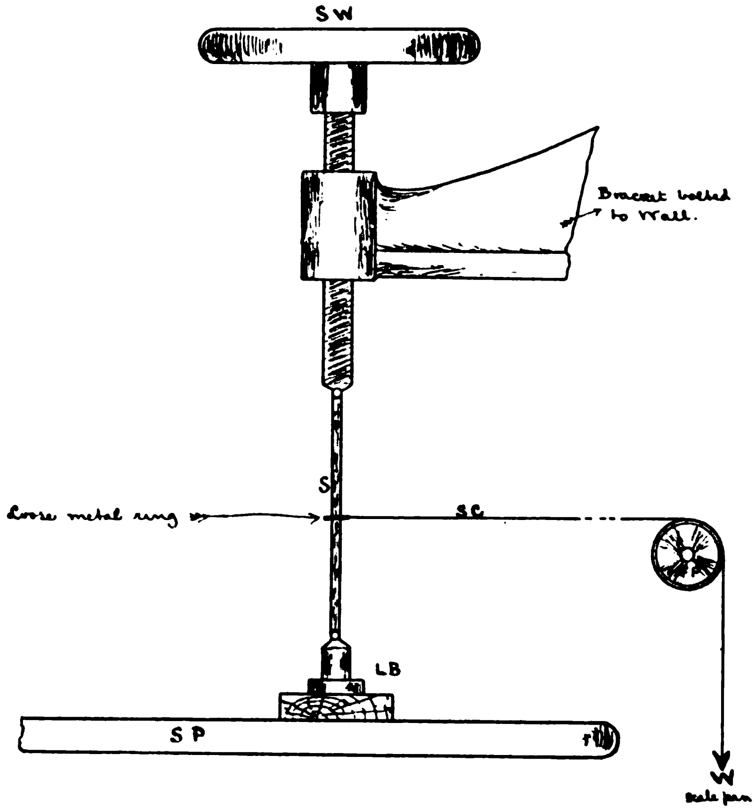


Fig. 3.

S, is the beam being tested.
SP, steelyard platform.
LB, lower block.
SC, is a cord for putting on side pull.
FP, is a fixed pulley, mounted on ball bearings.
SW, is a screw wheel and is used to compress the beam.

considerable range, in fact from the condition in which collapse was entirely due to the endlong load P to the condition in which the rod was very little different to a beam simply loaded at the centre—the endlong load in the extreme cases being incon-

siderable. A Table is given showing the values of P and W measured at the breakdown point:—

TABLE I.

W lbs.	P lbs.	Mean P lbs.	W lbs.	P lbs.	Mean P lbs.
0	108	104·5	12	70	70
0	101		12	70	
5	88	89	20	46	47
5	90		20	48	
10	73	74	30	27	26·5
10	75		30	26	

From this table it will be seen that when the value of W was 30 lbs., a vertical load equal to 26 lbs. was sufficient to break the strut-beam; with this lateral loading however an endlong load of 25 lbs. was withstood, and it was then noted that the central deflection was about 2 inches, though the calculated value on the usual elastic hypothesis was only 1·7 inch. The formula used to calculate the central deflection is[1]

$$y_1 = \frac{W}{2P}\left\{\frac{a}{\sqrt{P}}\tan\sqrt{P}\,\frac{l}{2a} - \frac{l}{2}\right\},$$

where y_1 = central deflection, l = length of strut, $a = \sqrt{EI}$, E = Young's Modulus and I = Moment of Inertia of section about the neutral axis.

[1] The differential equation to be solved is of the form

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y + \frac{W}{2EI}\,.\,x = 0,$$

where y is deflection at any point and x is distance measured from one end of the strut.

This, on solution, gives

$$\text{maximum deflection} = \frac{W}{2P}\,.\,\left\{\sqrt{\frac{EI}{P}}\tan\frac{l}{2}\sqrt{\frac{P}{EI}} - \frac{l}{2}\right\},$$

$$\text{and maximum stress} = \frac{Wz}{2}\sqrt{\frac{P}{IP}}\,.\,\tan\frac{l}{2}\sqrt{\frac{P}{EI}},$$

where z is half the thickness of the rod.

The observed value of the deflection is greater than that calculated, because the elastic limit had been exceeded and the material had "flowed" under the stress. Taking this observed value of the deflection, i.e. 2 inches, the tensile stress that would have had to be withstood on the elastic hypothesis comes out as 62 tons per square inch, which is considerably greater than the stress at which fracture in *direct tension* was found to occur in a similar rod. With a deflection as large as 2 inches, however, the plane section hypothesis breaks down and the *real* stress would be, as previously suggested, more nearly $\frac{2}{3}$ of this amount. Now $\frac{2}{3}$ of 62 tons is 41·3 tons, and this is in very close agreement with the 41 tons per square inch actually obtained as the breaking stress in direct tension. It therefore seems that for a ductile material the above hypothesis is very fairly approximate, and this is about as much as can be expected in such complex phenomena. These observations plotted with respect to P and W are shown in Fig. 4, the curve A is the curve outside (i.e. on the right of) which the stress in the strut-beam must have exceeded the elastic limit; the other two curves are obtained as follows. Curve B shows the result of assuming, on the elastic theory, that fracture occurs whenever the maximum deflection exceeds 1·75 inches, and curve

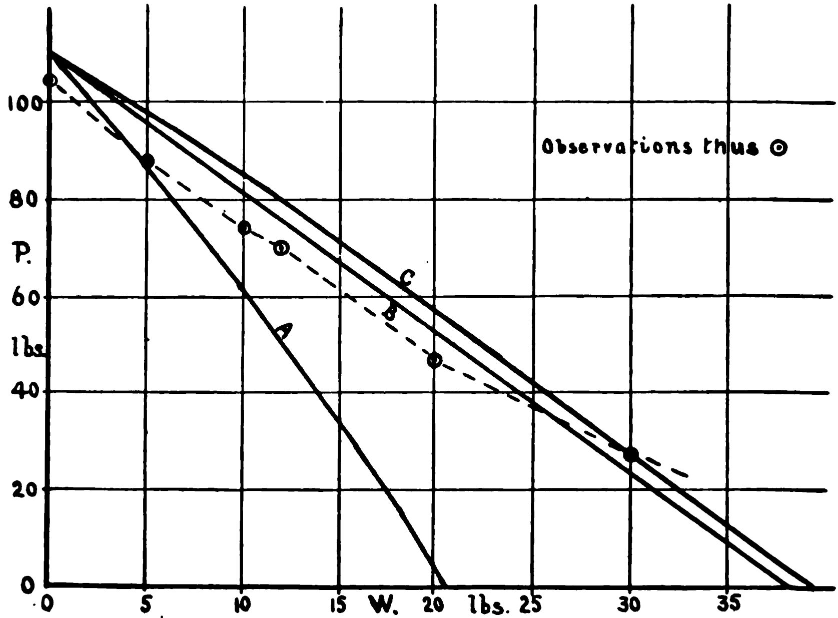


Fig. 4.

C that fracture occurs when the stress at the outermost layer exceeds 62 tons per square inch; both of these give a rough agreement to the experimental results, but it is worth while noting that the observed points are not far from a straight line of the type

$$P + \alpha W = \beta,$$

where α and β are constants.

This formula is very similar in appearance to

$$M = Py_1 + W\frac{l}{4},$$

where M = bending moment at the centre of the strut-beam, and the other symbols are as previously stated, the two formulae may be reduced to *identity* by assuming that collapse occurs when simultaneously the bending moment exceeds a certain value, and when the central deflection is greater than a certain amount, both amounts being independent of P and W. This also suggests that the simplest way to get an approximate relation between P and W is to find W when $P = 0$ and to join this point to the value of P when W is zero. This method should at any rate give a better idea of experimental conditions than that of assuming that the strut-beam collapses when the outermost layer reaches the Breaking Stress.

3. While upon this subject it is worth noticing that for somewhat similar reasons the ordinary Eulerian equation for struts is only an approximation. That this ideal formula fails to represent fact, is mainly due to the non-homogeneity of material, and is aided by want of straightness in the strut and by eccentric loading. Some experiments with struts were carried out on the same apparatus as that mentioned above (a larger machine being used for the higher loads) with a large number of struts of different lengths and with no side load applied.

According to Euler the relation between P and l for those struts should have been

$$Pl^2 = 63{,}400. \quad \text{(Formula, 1).}$$

For purposes of comparison a table of experimental results is given and Eulerian values are also shown tabulated. Professors Ayrton and Perry[1] have considered the effects of (1) non-axial loading, (2) non-homogeneity of material, and (3) want of straightness in the strut, and have justified the use of the formula (often known as the Gordon formula)

$$P = \frac{c}{1 + bl^2},$$

where b and c are constants.

[1] *Engineer*, 1886, pp. 464 and 513.

A formula of this type will be found to fit experimental results more nearly than the ordinary Eulerian equation, and as an example of its use the fourth and eighth columns of the table show values of P derived from the formula

$$P = \frac{60,000}{10 + l^2}. \quad \text{(Formula, 2.)}$$

TABLE II.

Length of Strut in inches	Maximum Load in lbs. (observed)	Maximum Load in lbs. from Formula 1	Maximum Load in lbs. from Formula 2	Length of Strut in inches	Maximum Load in lbs. (observed)	Maximum Load in lbs. from Formula 1	Maximum Load in lbs. from Formula 2
71·9	11·3	12·3	11·6	11·0	536	523	458
60·0	16·5	17·6	16·6	10·0	645	634	545
48·0	26	27·5	26·0	9·0	673	784	660
42·0	33	36·1	33·8	8·0	941	1,010	811
36·0	46	49·0	46·0	7·0	1,207	1,290	1,020
30·0	67	70·4	66·0	6·0	1,450	1,760	1,310
24·0	105	110	103	5·0	1,642	2,540	1,720
18·0	180	196	180	4·0	2,390	3,960	2,310
14·9	263	285	258	3·0	2,770	7,050	3,160
12·0	401	440	390	2·0	2,925	15,900	4,280

This fits as well perhaps as any expression of this type would do. Even a modified formula of this form only gives results over a limited range owing again to the complexity of the phenomena; one phenomenon for 3-inch and 2-inch struts, being the bursting of the conical ends under the great pressure applied.

The table here given ranges from a strut length of 6 feet to one of 2 inches, and naturally such a variation in length tests any formula very severely. If the series had been split up into several

short ranges an empirical formula of the Gordon type could be found for each range which would represent experimental results much more closely.

4. The subject of beams with endlong compression does not appear to have been tested by experiment to anything like the extent that the interest and importance of the subject would justify.

The strut-beams tested by the author had their lateral loads concentrated at the centre, but perhaps a more interesting case still would be that of a uniform lateral load such as is obtained in the coupling rods of a locomotive or in a horizontal strut loaded by its own weight, or in other ways.

The above experiments were carried out in the Engineering Laboratory, and the author's thanks are due to Professor Ewing for providing the facilities for the investigation.

The Oscillations of a Fluid in an Annular Trough. By B. COOKSON, B.A., Trinity College.

[*Read* 6 May 1901.]

The wave motion of a fluid in a circular trough can be discussed by the use of Bessel's Functions of the first kind. It may be found in Lamb's *Hydrodynamics*, Art. 187: but the case of an annular basin is passed over with the remark that it is "easily treated, theoretically, with the help of Bessel's Functions of the second kind." The analysis has therefore been carried out and applied to a particular case. The case is that of a trough containing mercury into which another annulus fits: this second annulus is floated by the mercury and on it is mounted an astronomical telescope, the combination forming a floating Zenith-Telescope. It is of interest to compare the period of a free wave of the fluid in the trough with the observed period of oscillation of the floating instrument and to know the character of the contour lines in the case of the free wave.

The fluid is assumed to start from rest and to be frictionless: the motion is irrotational and the velocity potential ϕ satisfies Laplace's equation

$$\nabla^2\phi = 0 \text{..............................(1).}$$

The motion is further imagined to be so small that the squares of the velocities may be neglected, so that the dynamical equation is

$$\frac{p}{\rho} + \frac{d\phi}{dt} = -gz \text{ ....(2).}$$

Here z is measured vertically upwards from the free surface when the liquid is in equilibrium. These two equations determine the small oscillations.

The second equation provides the condition for the free surface which is

$$\frac{\partial^2\phi}{\partial t^2}+g\,\frac{d\phi}{\partial z}=0 \qquad \text{.........................(3).}$$

when $z=0$. (See Lamb, *Hydrodynamics*, p. 371.)

There remain the boundary conditions, namely,

$$\frac{\partial\phi}{\partial z}=0, \text{ when } z=-h, \text{ the depth} \qquad \text{...........(4),}$$

$$\frac{\partial\phi}{\partial u}=0, \text{ all round the boundaries} \qquad \text{...........(5).}$$

As we are dealing with cylindrical boundaries, it will be convenient to express equation (1) in cylindrical coordinates: thus

$$\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2}+\frac{\partial^2\phi}{\partial z^2}=0 \qquad \text{..............(6).}$$

The equations (3) and (6) and the conditions (4) and (5) are all satisfied by

$$\phi=\{AJ_n(\kappa r)+BY_n(\kappa r)\}\sin n\theta\cosh\kappa(z+h)\cos mt \text{...(7),}$$

if
$$-m^2\cosh\kappa h+g\kappa\sinh\kappa h=0,$$

or
$$m^2=g\kappa\tanh\kappa h \qquad \text{.......................(8).}$$

If a is the inner radius of the trough and b the outer, then the second boundary condition may be expressed by the equations

$$AJ_n'(\kappa a)+BY_n'(\kappa a)=0,$$

$$AJ_n'(\kappa b)+BY_n'(\kappa b)=0,$$

or eliminating A and B, κ must be such as to satisfy the equation

$$\frac{Y_n'(\kappa a)}{J_n'(\kappa a)}-\frac{Y_n'(\kappa b)}{J_n'(\kappa b)}=0 \qquad \text{..............(8 }a\text{).}$$

If $x=\kappa a$, $\rho x=\kappa b$ and $\rho=b/a$, x must be a root of the equation

$$\frac{Y_n'(x)}{J_n'(x)}-\frac{Y_n'(\rho x)}{J_n'(\rho x)}=0 \qquad \text{....................(9).}$$

Substituting for B, we get as a solution

$$\phi=A_n\left\{J_n(\kappa r)-\frac{J_n'(\kappa b)}{Y_n'(\kappa b)}Y(\kappa r)\right\}\sin n\theta\,.\cosh\kappa(z+h)\cos mt \text{...(10),}$$

in which κ must satisfy (9) and m is given by (8). There are an infinite number of roots (9) for all values of n from $n=0$ to $n=\infty$: so that the complete expression for ϕ is a doubly infinite series of terms of the form (10).

For shortness put

$$B_n(\kappa r) = J_n(\kappa r) - \frac{J_n'(\kappa b)}{Y_n'(\kappa b)} Y_n(\kappa r),$$

thus $$\phi = \sum_{n=0}^{n=\infty} \Sigma_\kappa A_{n,\kappa} B_n(\kappa r) \,.\, \sin n\theta \,.\, \cosh \kappa(z+h) \cos mt.$$

Instead of the single trigonometrical factor $A \cos mt \sin n\theta$ in the typical term, we might put

$$(A \cos mt + B \sin mt) \sin n\theta + (C \cos mt + D \sin mt) \cos n\theta.$$

Let now η denote the elevation of the free surface at any moment above the mean level: then

$$\dot{\eta} = \left(\frac{\partial \phi}{\partial z}\right)_{z=\eta} = \left(\frac{\partial \phi}{\partial z}\right)_{z=0} \text{ to the order required,}$$

and since the liquid was originally at rest, $\dot{\eta}$ must be zero when $t=0$, so that we require $\sin mt$ and not $\cos mt$ in the expression for ϕ. The typical term is therefore

$$\phi = A_n B_n(\kappa r) \frac{\sin}{\cos} n\theta \,.\, \cosh \kappa(z+h) \,.\, \sin mt.$$

Hence $$\dot{\eta} = \kappa A_n B_n(\kappa r) \frac{\sin}{\cos} n\theta \,.\, \sinh \kappa h \,.\, \sin mt,$$

and $$\eta = -\frac{\kappa}{m} A_n B_n(\kappa r) \frac{\sin}{\cos} n\theta \,.\, \sinh \kappa h \,.\, \cos mt \quad\quad (10),$$

a possible form of the initial free surface is defined by putting $t=0$ in this expression for η.

By superposition of two fundamental modes of the same period but in different phases, we obtain a solution

$$\eta = A_n B_n(\kappa r) \,.\, \sinh \kappa h \,.\, \cos(n\theta \pm mt + \epsilon),$$

which represents a system of waves travelling unchanged round the origin with angular velocity m/n in the negative or positive direction of θ.

We may write (10) in the form

$$\eta = A_n B_n(\kappa r) \frac{\sin}{\cos} n\theta \,.\, \cos mt,$$

where A_n is a constant different from the A_n above, and with this form of η we can discuss the contour lines.

The two simplest cases are the cases of $n=0$ and $n=1$.

Case I. $n=0$.

$$\eta = A_0 B_0(\kappa r)\cos mt.$$

The motion is symmetrical about the origin, so that the waves have annular ridges and furrows. κ must satisfy $(8a)_{n=0}$ and when r is such that

$$B_0(\kappa r) = 0,$$

there is a nodal circle.

Case II. $n=1$.

$$\eta = A_1 B_1(\kappa r) \begin{matrix}\sin \\ \cos\end{matrix} \theta \,.\cos mt.$$

Besides the nodal circles given by

$$B_1(\kappa r) = 0,$$

there is a nodal diameter $\theta = 0$ or $\pi/2$, whose position, however, is indeterminate since the origin of θ is arbitrary. It does not follow that for every value of κ which satisfies $(8a)_{n=1}$, there will also be a nodal circle.

Returning to the general case, the period is seen to be

$$T = \frac{2\pi}{m} = 2\pi\sqrt{\frac{\coth \kappa h}{g\kappa}},$$

and it will be noticed that T and the whole motion in general are independent of the density of the liquid. The form of the free surface along a line through the origin is given by

$$\eta = B_n(\kappa r).$$

Theoretically the problem is now completely solved, but for its numerical application and the tracing of the contour lines we must know the solution of equation (9) and have the means of evaluating the Bessel's functions of the second kind: for I have not been able to find tables of those functions.

The solution of equation (9) will be found on page 242 of Gray and Mathews' *Bessel's Functions* and the expressions for Y_0 and Y_1 which were used to calculate their values, are given on page 22 of the same book.

$$Y_0 = J_0 \log x + 4\left\{\frac{1}{2}J_2 - \frac{1}{4}J_4 + \frac{1}{6}J_6 - \dots\right\},$$

$$Y_1 = J_1 \log x - \frac{J_0}{x} - J_1 + 4\left\{\frac{3}{2.4}J_3 - \frac{5}{4.6}J_5 + \frac{7}{6.8}J_7 \dots\right\}.$$

These series are absolutely convergent but are not very convenient for numerical work: it was necessary to include terms up to J_{17}.

We have also

$$Y_n' = Y_{n-1} - \frac{n}{x} Y_n,$$

an equation required in the calculation of $B_n(\kappa r)$.

The formula

$$J_{n+1} = \frac{2n}{x} J_n - J_{n-1}$$

was occasionally used as a check.

Large-scaled graphs were drawn of $J_1(x)$ and of $\frac{J_1'(\kappa b)}{Y_1'(\kappa b)} Y_1(x)$ and the values of these functions read off them. The contour lines were traced by assigning a value to the constant C in the equation

$$f(r) \,.\, \cos n\theta = C,$$

and by finding the values of θ corresponding to an assumed r.

Numerical Application.

Let a the inner radius of the trough be 12 inches,
„ b the outer „ „ „ 20 „
„ h the depth be $\frac{1}{4}$ inch.

Then $$\rho = \frac{b}{a} = \frac{5}{3}.$$

The first thing to do is to find κ from equation (9): the first two roots for the cases $n = 0$, $n = 1$ will suffice.

I. $n = 0$:

$$x_0^{(1)} = 4{\cdot}758: \quad x_0^{(2)} = 9{\cdot}449.$$

II. $n = 1$:

$$x_1^{(1)} = 4{\cdot}824: \quad x_1^{(2)} = 9{\cdot}481.$$

We have therefore

$$\kappa_0^{(1)} = \frac{x_0^{(1)}}{12} = 0{\cdot}396: \quad \kappa_0^{(2)} = 0{\cdot}787,$$

$$\kappa_1^{(1)} = \frac{x_1^{(1)}}{12} = 0{\cdot}402: \quad \kappa_1^{(2)} = 0{\cdot}790.$$

Hence in the symmetrical class the longest period is

$$2\pi\sqrt{\frac{\coth \cdot 396\,.\,\frac{1}{4}}{32\,.\,12\,.\,\cdot 396}} \text{ or } 1\cdot 62 \text{ seconds.}$$

The next longest is

$$2\pi\sqrt{\frac{\coth \cdot 787\,.\,\frac{1}{4}}{32\,.\,12\,.\,\cdot 787}} \text{ or } 0\cdot 82 \text{ seconds.}$$

In the unsymmetrical class where $n = 1$ the longest period is

$$\frac{2\pi}{K_1^{(1)}}\sqrt{\frac{1}{gh}} \text{ seconds or } 1^s\cdot 60.$$

and the next longest is

$$\frac{2\pi}{K_1^{(2)}}\sqrt{\frac{1}{gh}} \text{ seconds or } 0^s\cdot 81.$$

There is thus very little difference between the periods for the two cases of $n = 0$, $n = 1$: that is to say, the period is almost exactly the same when there is one nodal diameter as when there is not one. In either case each successive period is very nearly half the next longest.

Between the floating instrument mentioned in the first paragraph and the trough containing the mercury there is everywhere half an inch of free surface. Thus in the actual case, there are two narrow annular free surfaces each $\frac{1}{2}$ inch in width, the one having a mean radius of $19\frac{3}{4}$ inches, the other $12\frac{1}{4}$: the depth of fluid is about $\frac{1}{2}$ inch. The two longest periods of a free wave on these surfaces are the same for both the inner and the outer surface, viz. $0^s\cdot 13$ and $0^s\cdot 09$: these periods practically do not differ for the two cases of $n = 0$ or $n = 1$.

It may be here stated that the observed period of oscillation of the floating instrument is $9^s\cdot 40$.

It is interesting to compare the periods in the corresponding ones for a circular trough of the same outside diameter and with the same depth of liquid. The values of κ are given by Lamb, *Hydrodynamics*, Art. 187; they are

$$\kappa_0^{(1)} = 1\cdot 2197\,\frac{\pi}{20}: \quad \kappa_0^{(2)} = 2\cdot 233\,\frac{\pi}{20}.$$

$$\kappa_1^{(1)} = 0\cdot 586\,\frac{\pi}{20}: \quad \kappa_1^{(2)} = 1\cdot 697\,\frac{\pi}{20}.$$

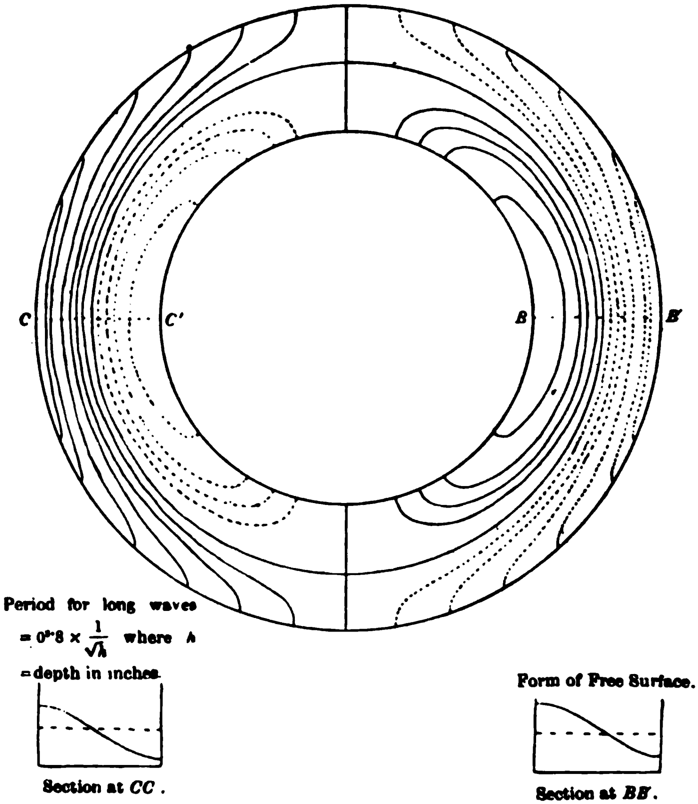


FIG. 1. ANNULAR TROUGH.

$n=1$: gravest mode.

The two longest periods for the symmetrical class are therefore

$$3^s{\cdot}35 \text{ and } 1^s{\cdot}83,$$

and for the unsymmetrical class

$$6^s{\cdot}97 \text{ and } 2^s{\cdot}40.$$

The oscillations in an annular trough are then much quicker than those in a circular trough. Returning to the case of the annular basin, it will be noticed that an additional nodal circle is introduced every time we proceed to a higher harmonic and that the nodal diameters are n in number. The contour lines between two nodal circles must be closed curves of the shape of those in Fig. 2, and the curves between a nodal circle and the boundary

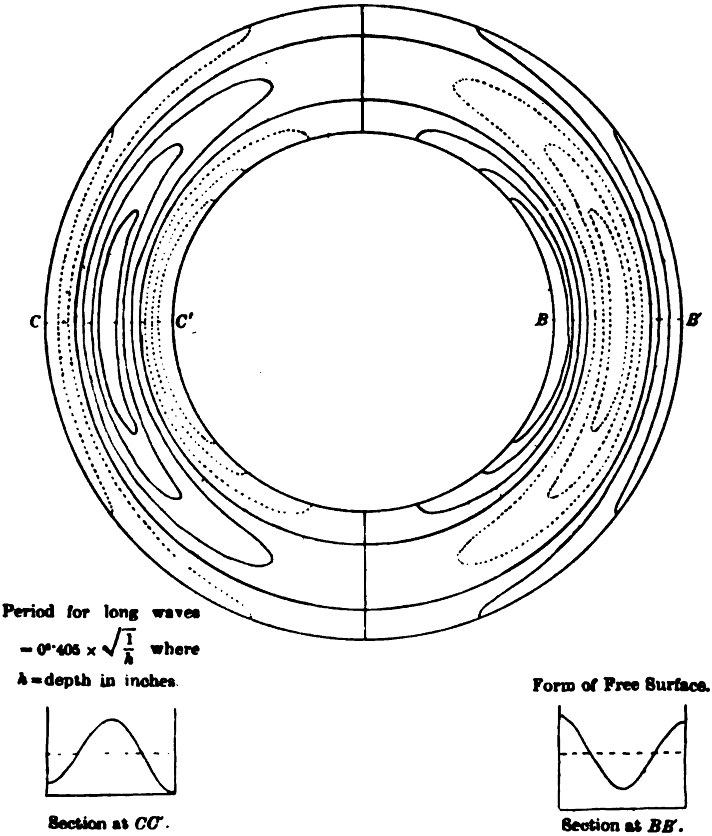


FIG. 2. ANNULAR TROUGH.

$n=2$: gravest mode.

will be of the same type as those in Fig. 2. It is therefore unnecessary to trace the contour lines for the higher harmonics: and it is obvious how the curves will be arranged when the number of nodal diameters is increased, that is when n increases. In the simpler case of a circular trough, we can see what the general character of the contour lines is by drawing a graph of $J_n(x)$: then the boundary of the trough will be at the 1st, 2nd, 3rd... maximum or minimum values of $J_n(x)$ for the 1st, 2nd, 3rd... gravest modes of the species considered. By simply examining the graphs we therefore see that for each species there is one nodal circle for the second gravest mode, two nodal circles for the next gravest and so on. A few cases are roughly illustrated below.

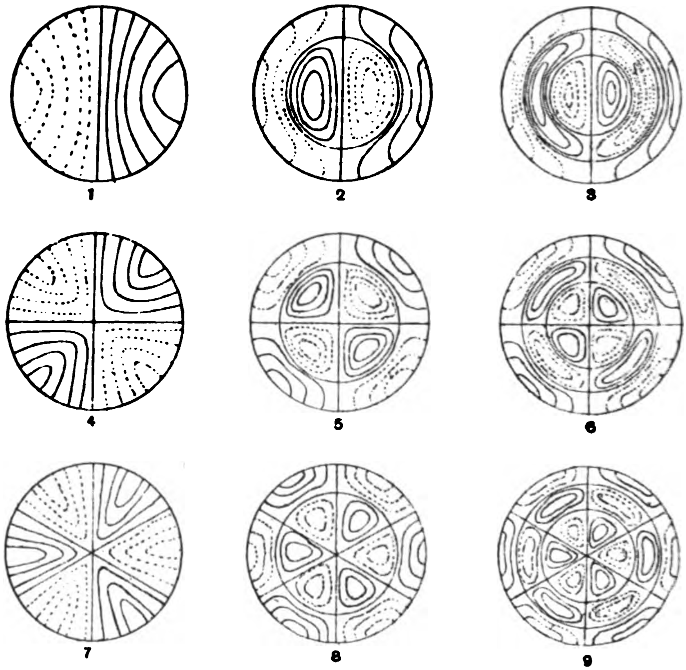


FIG. 3. CIRCULAR TROUGH.

1. 1st gravest mode, when $n=1$ These are
2. 2nd " " " " illustrated
3. 3rd " " " " by Lamb.

4. 1st gravest mode, when $n=2$
5. 2nd " " " "
6. 3rd " " " "
7. 1st gravest mode, when $n=3$
8. 2nd " " " "
9. 3rd " " " "

It will be noticed that the contour lines for an annular trough for the gravest mode when $n = 1$, can be obtained from those for a circular trough by cutting a concentric circular piece out of the circular basin, the radius being equal to the distance of the crest of the wave from the centre of the trough. The reason of this similarity is that for the annular trough of the dimensions here considered the part of η depending on r differs only very little from $J_1(r)$: this is seen at once on inspection of the tracing of the two curves

$$y = B_1(\kappa r): \quad y = J_1(\kappa r).$$

A little consideration will show why there is this similarity between the *gravest* mode for the annular trough and the *second gravest* mode for the circular trough. (See Lamb, p. 306.)

On a new species of Bothriocephalus. By A. E. SHIPLEY, M.A. Christ's College.

(With Plate III.)

[*Read* 20 May 1901.]

In a memoir entitled "A Description of the Entozoa collected by Dr Willey during his sojourn in the Western Pacific" and published in Dr Willey's "Zoological Results," Part v., Cambridge, 1900, I described a species of Tape-worm taken from the intestine of the sword-fish *Histiophorus* sp. of the Indian and Pacific Oceans. From an examination of the external features I came to the conclusion that the cestodes in question were examples of the species *Bothriocephalus plicatus* Rud. which lives in the better known sword-fish *Xiphias gladius* L. In acknowledging the receipt of a copy of the paper in question Professor Lönnberg of Upsala University most courteously pointed out that my description did not agree with what is known of *B. plicatus*. Dr Lühe in his paper on the Anatomy and Classification of the Bothriocephalidae[1] has in fact established a new genus *Fistulicola* under which he places the *B. plicatus* (Rud.). *Fistulicola* is grouped in the sub-family Triaenopharinae Lühe, of the family BOTHRIOCEPHALIDAE, whilst my species, as sections which I have recently prepared show, undoubtedly belongs to the sub-family Ptychobothriinae Lühe of the same family and to that genus which retains the generic name *Bothriocephalus*. To this genus are assigned (i) *B. bipunctatus* (Zed.) = *B. punctatus* Rud. 1808 = *Alyselminthus bipunctatus* Zed. 1800, described by P. J. van Beneden[2] from a Turbot and by Rudolphi from *Pleuronectes maximus* and *Cottus scorpius*; (ii) *B. claviceps* (Gze. 1782) Rud. 1810, from the intestine of *Muraena anguillae*, and of *Anguilla vulgaris* v. Matz[3]; (iii) *B. neglectus* Lönnb. from the alimentary canal of *Raniceps niger*[4]; both (iv) *B. laciniatus* (Lint.) from the Tarpum, *Tarpon atlanticus*, and (v) *B. occidentalis* (Lint.) from a Rock-cod, *Sebastodes* sp., probably belong to the same sub-family[5].

[1] *Verh. Deutsch. Zool. Ges.* 8th Jahresversammlung, 1898.
[2] *Mem. Ac. Belgique*, xxv. 1850.
[3] *Arch. Naturg.* 58 Jahrg., 1892.
[4] *Bih. Svenska Ak.* xviii. 1893.
[5] *P. U. S. Nat. Mus.* xx. 1898.

My specimens from the intestine of *Histiophorus* sp. undoubtedly belong to the sub-family Ptychobothriinae and to the genus *Bothriocephalus* as restricted by Lühe. The scolex is unarmed, and provided with longitudinal slit-like depressions which hardly attain the dignity of suckers situated in the dorsal and ventral plane. Laterally and more anteriorly is a still slighter depression. The anterior end of the scolex bears a flat cap something like a cook's cap but with four distinct lobes, symmetrically placed, two right and two left. Posteriorly the head is slightly constricted and then it expands again to terminate in a marked rim. (Fig. 1.)

There is no neck. The segments are anteriorly rather funnel-shaped with markedly salient angles. Towards the middle of the body the segments become much broader than long, but quite at the posterior end they lengthen again. The edges of the first six to ten segments which overlap for some distance the succeeding segment are divided up into four lobes but this lobation disappears behind. The posterior border of the last segment is rounded. The salient edges of the posterior border of the anterior segments overlap the succeeding proglottis for almost half its length.

The head is 1·5 mm. in length and 1·4 mm. to the constriction mentioned above, its breadth at the anterior end which bears the four-lobed cap is ·4 mm. The measurements of the proglottides vary very much in different regions of the body. The longest are those of the anterior third where a length of ·3 mm. is attained. Further back the segments shorten and broaden till they acquire the dimensions of about ·5 mm. broad by ·16 mm. long.

The ovary lies across the hinder part of the proglottis, and is produced into numerous rounded lobes. The ova are closely crammed together at the periphery but in the centre of the organ in the middle line the ova are more loosely packed and more spherical in outline and have passed into the chamber called the ootype. Their diameter here is some ·015 mm. Into this region opens the small shell-gland, and the ducts of the yolk glands. The shell-gland lies posteriorly to the ovary between the right and left halves of that organ and with the ducts of the yolk glands it opens into the ootype posteriorly. From the ootype the uterus arises and makes a few turns, coiling right and left and then opens into what the Germans call the "uterushöhle" or uterus-sac, a large spherical expansion of the uterus which opens by a very definite pore on the ventral surface of the proglottis. The difference between the spherical eggs in the ootype and those in the uterus is striking. The former are much smaller, well stained, with conspicuous nuclei and ·015 mm. in diameter, the latter are enclosed in a bright yellow egg-shell of an oval shape impenetrable to staining fluids, ·045 mm. long by ·035 mm.

broad and showing no structure but an unstained granular contents.

During the course of its twisting the uterus is so narrow that the eggs lie in a single row one behind the other so that in a transverse section but one egg is ever seen. In the uterus-sac however they are in considerable numbers, at least 100. The uterus-sac is not median as seems to be the case in other species of *Bothriocephalus*. It is pushed sometimes to the right and sometimes to the left by the presence of the cirrus-bulb, and whether it lies to the right or to the left seems to follow no certain law. In one series of sections through seven proglottides the position of the uterus-sac was R. R. L. L. R. L. R., in another series seven proglottides with the uterus-sac to the right were followed by six in which it was to the left. The opening of the uterus-sac to the exterior is circular, it does not seem to be provided with anything of the nature of a sphincter muscle, nevertheless it is a very definite and distinct structure. In a considerable number of the sections eggs were seen passing out of the pore. From what I have seen I think it probable that eggs pass out from the tape-worm into the alimentary canal of the host and that in *B. histiophorus* the eggs pass freely out from each ripe proglottis and do not wait until the posterior proglottides break off to make their escape from the parent.

Unlike the other species of the genus these specimens do not have their uterus-opening in the middle line or nearly in the middle line, but this aperture is quite distinctly pushed either to the left or to the right according to the side to which the uterus has been pushed by the conspicuous cirrus-bulb.

The yolk-glands are very numerous, in longitudinal sections they seem almost to run from segment to segment. In transverse section they run almost all round the segment but are broken by slight areas free from their presence at both sides and in the median line both dorsally and ventrally. They lie exclusively in the outer parenchyma outside the layer of longitudinal muscles which separates the central from the cortical parenchyma. Their four ducts unite and open by a common duct into the ootype close to the opening of the vagina. The brown or under a high power yellow yolk granules are unusually conspicuous in this form.

The vagina is a fine tube with thin muscular walls which passes almost straight from the ootype to the dorsal surface. It lies behind the cirrus-bulb and opens close behind the opening of the penis. Close to its opening is a well marked sphincter muscle clearly shown in Figure 3. This is the "napiform" muscular body of Linton, who regards the opening of the reproductive ducts as ventral whereas, as is indicated above, Lühe regards them as dorsal. There is no receptaculum seminis.

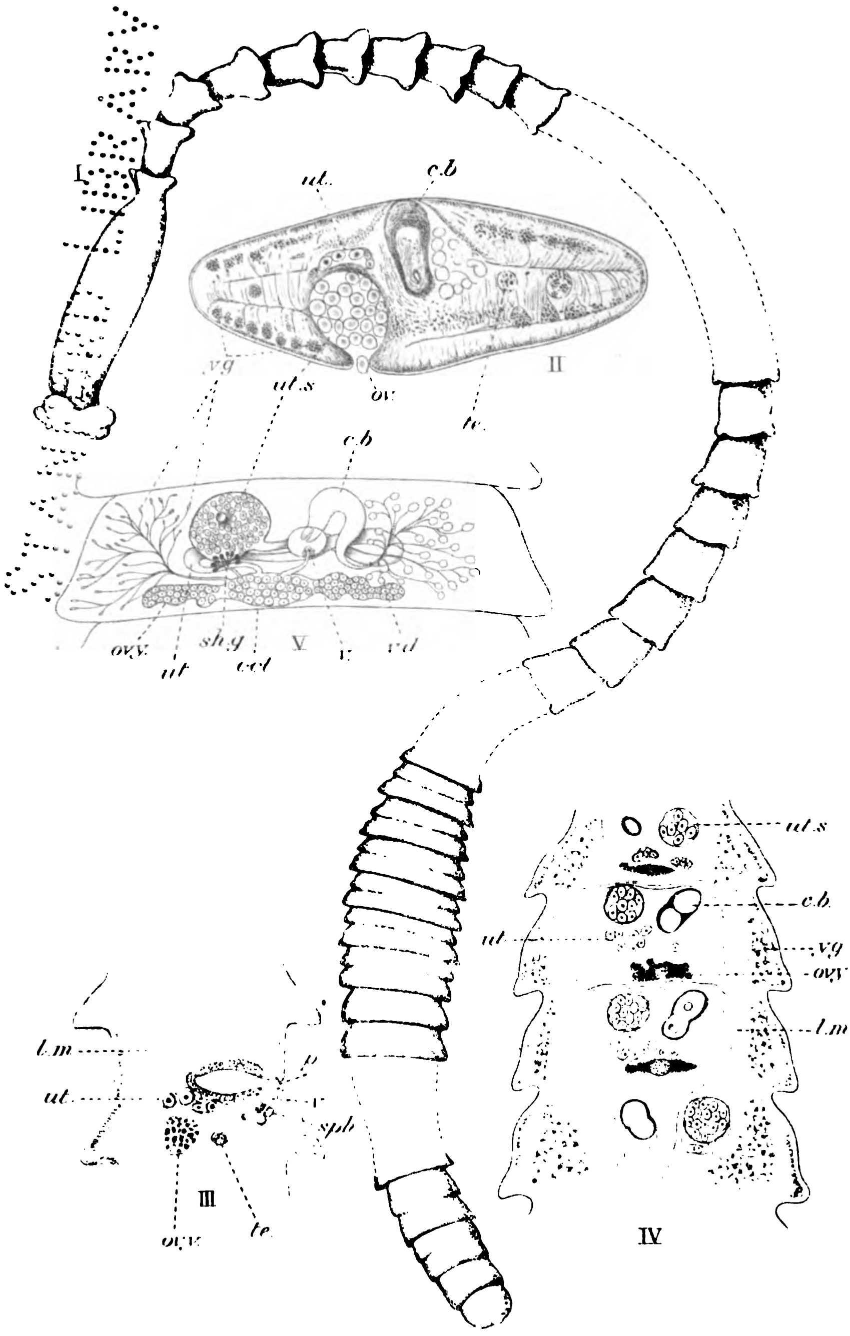
The testes are numerous, some 50 to 70, they lie in the inner core of parenchyma, and in every stage of development. The cirrus-bulb is very large and muscular. It extends from the opening of the penis right across the body to the ventral surface, lying in the median plane. The external opening of the penis lies slightly behind the level of the uterus-opening, near the posterior rim of the proglottis in the middle line. The cirrus-bulb stretches forward so that its inner end lies in a plane anterior to the outer. The inner end bends round for a short distance before fading away into the vas efferentia.

A single excretory canal ran down each side of the body. The distinction between the inner and the outer parenchyma is more clearly marked at the anterior and posterior ends of each proglottis than in the middle where it tends to be obliterated by the uterus-sac and other organs. In longitudinal sections the muscles which pass from proglottis to proglottis are conspicuous. The contour of the sections taken from the anterior end of a proglottis is smooth but that of those taken through the salient posterior angles is markedly crumpled or crenated.

Bothriocephalus histiophorus, n. sp.

Scolex unarmed; with dorsal and ventral longitudinal slit-like suckers, and a flat four lobed 'cap'; length 1·5; constricted near posterior end. No neck. Proglottides funnel-shaped with markedly salient angles especially anteriorly; ripe proglottis measures ·5 mm. broad by ·16 mm. in length. Cirrus-bulb large stretching from dorsal to ventral surface; penis opens medianly and dorsally (Lühe) close behind it opens the vagina which is surrounded by a well-marked sphincter muscle; uterus-sac pushed irregularly to the right or left and its pore is distinctly not median; yolk-glands in outer parenchyma close to the surface; testes 50 to 75 in inner parenchyma; ovaries paired, lobed, uniting in a conspicuous ootype; no receptaculum seminis; ova in uterus-sac ·045 × ·035 mm.

Habitat. The alimentary canal of *Histiophorus* sp. captured in the Indian Ocean.



E Wilson, Cambridge

B ,US HISTIOPHORUS

DESCRIPTION OF PLATE III, ILLUSTRATING MR A. E. SHIPLEY'S PAPER ON A NEW SPECIES OF *Bothriocephalus.*

Explanation of Lettering.

c.b.	= cirrus-bulb.	*sph.*	= sphincter muscle of vagina.
l.m.	= longitudinal muscles.	*te.*	= testis.
oot.	= ootype.	*ut.*	= uterus.
ov.	= ova.	*ut.s.*	= uterus-sac.
ovy.	= ovary.	*v.*	= vagina.
p.	= penis.	*v.d.*	= vas deferens.
sh.g.	= shell-gland.	*y.g.*	= yolk-glands.

1. *B. histiophorus*, showing the head, the shape of the proglottides in different parts of the body.

2. A transverse section through a ripe proglottis, showing the eggs issuing from the uterus-sac.

3. A longitudinal vertical section, showing the openings of the penis and vagina.

4. A longitudinal horizontal section, showing the irregular relations of the uterus-sac to the cirrus-bulb.

5. A diagram to show the arrangement of the reproductive organs in a ripe proglottis. The yolk-glands are omitted on the right side, and the testes on the left.

On the Rate of Growth of some Corals from Fiji. By J. STANLEY GARDINER, M.A., Balfour Student, and Fellow of Gonville and Caius College.

[*Read* 20 May 1901.]

About three years ago I received from Capt. W. W. Wilson, Harbour-master of Levuka, Fiji, five corals, which he had obtained off the chain of an anchorage buoy in Levuka harbour. I did not comment on these at the time owing to lack of some necessary information. I do so now in the hope that I may induce naturalists, who are studying this and kindred subjects, to carry out their work on a more precise and scientific basis. Capt. Wilson is accustomed to lift these buoys periodically, so as to remove weed, etc. and to repaint. He kindly carried out this operation with some during my visit to Fiji, and I subsequently recorded the weights of the specimens of coral obtained[1], but something more than this is needed for the proper study of the problem in relation to the formation of organically-built reefs.

Anyone accustomed to deal with anchorage buoys knows that such movement, as normally occurs owing to the tides, or due to a ship picking up its buoy, is not of a violent nature, nor likely to be seriously detrimental to coral, or other organic growths, as compared with surrounding areas. Such is especially the case in Levuka harbour; the anchors and chains are heavy enough for vessels of large tonnage, while ships of more than 300 tons only occasionally tie up to them. The fact that the chain from this cause varies little from the vertical is of importance, as the friction of the links on one another, owing to violent, horizontal movements, would naturally seriously impede the growth of the corals, or even destroy the colonies. The specimens, here especially under consideration, were all from a part of the chain between 1½ and 4 fathoms in depth, as indeed were those, which I previously recorded. There is never any coral growth on the buoys, nor on the chains immediately below them. They are indeed

[1] "The Coral Reefs of Funafuti, Rotuma and Fiji, etc." *Proc. Camb. Phil. Soc.* Vol. IX. Pt VIII. p. 487, 1898.

generally covered with green weed with a varying quantity of bivalve molluscs in great lumps; Polytrema and Lithothamnion are not found. The absence of corals here is probably due largely to the greater movement of the part near the surface of the water, destroying the colonies as well as causing difficulty of fixation to the larvae. Another cause is found in the lesser amount of light below the buoy, reef-building corals feeding mainly by means of their commensal algae. Other conditions being the same, the more intense the light the more vigorous is the growth of these corals, but this is not the case with the other organisms, which affix themselves to the buoys and chains. The green algae are of extremely rapid and vigorous growth as compared with corals, and in the waters of the tropics undoubtedly do better in partial shade, than where freely exposed to the sun's rays. Lamellibranch larvae affix themselves much more readily than coral larvae[1] and with apparent ease in places, where corals cannot grow. The increase in size of the shell depends mainly on the food supply, but a high temperature is likely to be at any stage of their existence fatal. The position under the buoy should be—and indeed is, to judge from the size of the shells and masses—extremely favourable, the surface movements bringing an ample supply of food, and the buoy itself giving shade from the vertical rays of the sun. Considering the competitors, it would hence appear probable that any corals, which might commence to grow under the buoy, would for a limited depth ultimately be killed and overgrown.

It is necessary to consider the physical conditions of the harbour, since the rapidity of growth of the corals, as in the case of the bivalves, depends very largely on the nature of these. The so-called harbour of Levuka is a stretch of the lagoon, opposite the town of the same name, within the great barrier reef of Ovalau, which joins on to that of Viti Levu, the largest island of the Fijies. There is here a small passage, caused mainly in all probability by the necessity for an outlet for the tidal waters; a small stream it is true comes down from the mountains of Ovalau about half a mile up the coast, but the size of this is due absolutely to the rains, and it can scarcely make any appreciable difference to the salinity of the water, even if this be an important factor. The town consists of a long straggling street by the sea, and has not probably more than 600 inhabitants, the shipping is inconsiderable, the shore is rocky, and the stream drains a very small, uncultivated, rocky area, so that the water is singularly clear even as compared with the lagoons of other

[1] This is well exemplified by the largest specimen of *Stylophora*, the base of which has almost completely enclosed a shell of some species of *Ostrea* (?), the larva of the coral probably in the first place affixing itself to the shell.

barrier reefs of similar form in the same islands. The bottom of the lagoon is hard sand or rock, little or no mud. As the buoy slowly rises and falls with the tide, the depth of any part of the chain is constant. The current is purely tidal, varying little during the year; it is never of sufficient force to cause the buoys to drag in any way. The harbour is very largely protected by the high island from the wind; outside the reef is broad, and there is not sufficient sea-room for rollers of any size to come up. Wind hence, except during hurricanes—and none such occurred during the growth of the specimens under consideration—is a negligible quantity. The depth of the lagoon, where the buoys are situated, is about 6 fathoms, so that the specimens were both well above any movement of sand or mud on the bottom and below any mere surface disturbance.

Considering that the tidal currents cause an ample change in the waters of the lagoon, and are almost continuously though not very appreciably felt in the harbour, together with the other conditions above described, I can only come to the conclusion that the situation of the specimens was one peculiarly favourable to a very vigorous and rapid growth. The same deduction would further seem to me amply justified alone by the numerous branchings and small size of the twigs of all the specimens, as well as the lightness of their different coralla.

The five specimens belong to the genera *Stylophora* and *Pocillopora*, both groups of extremely vigorous growth and general distribution throughout the Indian and Pacific Oceans. In Fiji they grow on the outer slopes, reef-flats and in the lagoons of probably every reef in the whole group. Their abundance is so great that, with the exception perhaps of *Madrepora* alone, no single coral genus can claim an equal importance with either in building up the reef limestone, with the ultimate fate of which I am not here concerned.

Two of the specimens I refer to *Stylophora raristella* (Defrance)[1], a fossil species from the miocene of Turin. The forms known as *S. danae* Ed. & H., *S. cellulosa* Quelch, and several others are probably only varieties of this species (if the descriptions of the latter are accurate), but the group of which it is the representative would appear probably to be quite distinct from the forms, of which *S. digitata* (Pallas) is the central species. The specimens of *Stylophora* in this country are not sufficiently numerous to examine the species question in the genus, nor have they been collected with this view. I cannot regard the differences in my specimens as more than varietal, and, as they differ from any of the so-called living species, I propose to give them the

[1] For references see "Histoire des Coralliaires" par Mm. H. Milne Edwardes et J. Haime, tome II. p. 138, 1857.

name *S. raristella*, var. *wilsoni*, after the gentleman, who besides sending the specimens is largely responsible for the information about Levuka harbour. The variety differs from the type in having the ring round the calicle less distinct, but crowned with the six large spines of the primary septa, generally six smaller intermediate spines, and often six or twelve of a third order, no trace of any corresponding septa however beyond the first order being ever visible. The calicles tend to be disposed in series, which do not lie necessarily in any determinable direction in respect to the branches; on the tips of the latter the series are ordinarily especially marked, crossing them in parallel rows. The cœnenchyma varies considerably, but is fairly well developed, and covered with low pointed spines; the calicles of a series at the tip of a twig generally have a common theca between, but below all tend to be separated by a breadth of about their own diameter (1 mm.) from one another. Often a row of spines divides the cœnenchyma markedly between two separate calicles, or two series. The septa have smooth edges, and fuse below with the columella. The latter is always distinct above, projecting freely generally to the level of the top of the "bourrelet," or rim round the calicle.

Two of the other specimens evidently belong to the same species, which I referred to *Pocillopora suffruticosa* Verrill, in describing my collections from the Pacific Ocean[1]. The identification down to the species is however uncertain at present, but it serves to indicate that the specimens belong to the finely branching division of the genus, noted also for its dense corallum. The last specimen may be provisionally referred to *Pocillopora plicata* (Dana); it is too small to determine properly its mode of growth, but it approaches close to var. *aspera* (Verrill)[2].

In any discussion of the rate of growth of corals the volume of the specimens must be considered rather than the weight, since according to all my observations the density of all forms of a single species varies inversely with the rate of growth; the slower the latter may be, the denser and heavier is the corallum, volume for volume. In the appended table I give the particulars of the rate of growth of the five specimens. In the third column I record the weights of the colonies, when they first arrived in this country, about three months after being obtained, to show by comparison with the fourth column the additional loss by thorough drying. In the last column I give the thicknesses of the coralla,

[1] *P. Z. S.*, 1897, p. 943.

[2] *P. Z. S.*, 1897, pp. 947—8. In looking up this reference I find that I fell into error in calling *P. aspera* the type. It is indeed the central form round which the varieties are grouped, but according to the recognised rules of nomenclature the type should be *P. plicata* and the three varieties var. *danae*, var. *ligulata*, and var. *aspera*.

imagined as flat plates, covering the same horizontal areas, as the specimens themselves naturally covered when alive.

Name of Coral	Time of Growth in Days	First Weight in Grams	Present Weight in Grams	Weight in Water in Grams	Specific Gravity	Volume in c.c.	Thickness as a Sheet in mm.
1. Stylophora raristella, var. wilsoni	1030	310	293	104	1·55	189	14·1
2. Stylophora raristella, var. wilsoni	1030	186	168	53	1·46	115	12·8
3. Pocillopora suffruticosa	1030	194	190	106	2·26	84	12·2
4. Pocillopora suffruticosa	1030	131	127	71	2·27	56	10·2
5. Pocillopora plicata, var. aspera	1030	195	177	57	1·47	121	24·7

In examining the table, the difference between the specific gravities and volumes of the two species of *Pocillopora* is most noticeable. Assuming that the corallum is an excretion, it would naturally be supposed that the greater the surface the quicker the rate of growth. Yet the reverse here is the case, *P. plicata* being a fairly massive species, while *P. suffruticosa* branches into fine twigs. It is not known, however, when the larvae affixed themselves, and the real time of growth is probably in every case very much less than the recorded time.

For comparison the last column is the most important, but it assumes that the position of growth, the vertical and horizontal axes, are known. It is necessary hence that these should be marked (by paint, or some other means), when the specimens are obtained. This is more especially the case with massive than branching species. All true reef corals grow towards the light, and with branching specimens of any size, remembering that the ends of the central larger branches will be approximately vertical and that the underside has only short blunt

branches without twigs, the position of growth may be nearly accurately fixed on the dry specimen. Massive species also show well-defined characters, but these vary with each genus, and can only be learnt by long experience on the reefs themselves.

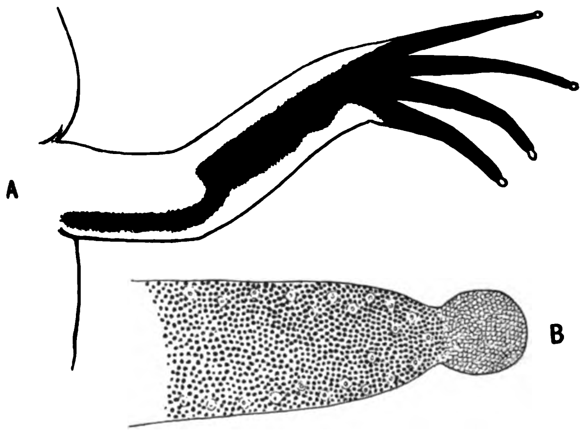
Finally I made a rough estimate of the number of polyps on some of these coralla, but the method is of little use, as the polyps of different species are not all of the same size, nor have they necessarily the same method of budding, nor indeed any connection with the volume of the skeleton. It is interesting however to note that, while No. 5 gives a sheet almost twice as thick as Nos. 1 and 3, it actually has only about half as many polyps, the calculated numbers of 1, 3 and 5 being respectively 25470, 23616 and 12672. Assuming an actual age of 1030 days and a regular geometrical progression of 2, periods of 71, 70, and 76 days would have elapsed in Nos. 1, 3 and 5 respectively between each set as it were of buds[1]. Examined in this way, the numbers of the polyps do not seem very large, considering the peculiarly favourable situation of the colonies. It is again suggested that the actual time of growth must have been much less than the observed time.

[1] I am indebted to Mr J. F. Cameron, Fellow of Gonville and Caius College, for this calculation.

On the breeding habits of Xenopus laevis *Daud.* By EDWARD J. BLES, B A., King's College.

[*Read* 20 May 1901.]

The tadpoles of the Cape Frog, known as Dactylethra larvae, are so remarkable and so little is known of their development and of the habits of the adult, that I gladly took the opportunity of getting four full-grown specimens from a dealer in December, 1896. There were two males and two females and they had been in his possession for two years. Their previous history could not be traced. They lived in an aquarium holding 2—3 gallons and seemed to thrive on a diet of earthworms and strips of raw meat or liver. The food is directed towards their mouths by their hands and they cram it in with jerky and clumsy movements of the arms, the back of the hand always directed towards the mouth. They invariably remain below the surface while taking



A. Left arm of *Xenopus laevis* ♂ (× 2) seen from below. To show the distribution of the nuptial asperities; they cover the black area.

B. Cast epidermis from the tip of digit I, dorsal surface (× 55). The markings on the expanded tip are the outlines of epidermis cells. The cuticular spines are indicated by black dots. Between them lie spaces occupied by skin glands with their median pores.

in their food. The aquarium stood in a sunny place in the summer and in a dark corner of a living-room during the winter. During January, 1899, the females refused food and the males took very little; this behaviour continued until Feb. 20th when one female died. On dissection her ovaries were found to contain quantities of mature ova ready to fall into the body-cavity. The three survivors were taken to the University Botanic Garden and through the kindness of Mr Lynch, the Curator, I was allowed to enclose a corner of the Tropical Lily Tank, about 10 ft. by 3 ft., with wire netting and put the frogs there (Feb. 23rd). Spawning commenced on the night of Feb. 28th.

When in the lily tank, in water at 22°—24° C., the frogs ate earthworms voraciously every day. The first sign that breeding is about to take place is the appearance of a dark patch on each hand and arm of the male. The darkening is caused by closely set, minute pointed spines which seem to be formed of thickened cuticle like the nuptial asperities on the hands of some European frogs. The extent of the patch is along the whole of the arm from the axilla to the tips of the fingers and it is in such a position that the whole inwardly directed surface of the arm and the back, not the palm, of the hand becomes roughened where it presses against the female during the pairing. No other nuptial changes were seen in the male, and in the female, the only alteration besides the greatly distended abdomen was an increased vascularisation and turgidity of the short cloacal spout.

The night before the eggs were laid, the male commenced croaking at dusk and, I believe, continued this for some hours. The sound produced is a continuous metallic rattle, almost exactly like the noise of winding a large clock with a key and with a similar alternating high and low note. This is distinctly audible ten yards off. The female is perfectly mute, so far as I know; like the females of other frogs and the difference is correlated with the dimorphism of the anatomy of the larynx, here more strongly marked than in any other Anura. The only way to identify the individual making sounds is to remove or disturb the others, as there is absolutely no outward movement discernible, not even in the gular and pectoral region.

The male continues croaking until within four or five minutes before pairing, while the female remains motionless at the surface of the water with her nostrils and eyes above the surface. He then swims quietly up behind the female and from a distance of about six inches makes a sudden dash at her. The amplexus is inguinal and lasts the whole of the time of spawning, that is from dusk until dawn. I followed the process of segmentation during one night and could therefore judge the length of time which had

elapsed since the last eggs were laid on the following morning. Pairing took place on the nights of Feb. 28th, March 3rd and March 5th and altogether between 400 and 500 eggs were laid.

Oviposition is carried out in the following manner. The pair swim about rapidly for a few minutes, then they stop and the female clasps a leaf of a water-plant between her feet and the egg which has been held in the cloacal tube is shot out against the weed. Each time this occurs a spasm passes over the body of the male, and most probably spermatozoa are then ejected, each egg being fertilised separately. There can be no doubt that there is nothing of the nature of a spermatophore and that fertilisation takes place in the water.

The egg when first laid has a strongly adhesive outer layer, which makes it fast to the first foreign body it touches. Before the tadpole hatches, which is 30—36 hours after spawning in water at 22°—24° C., this outer coat becomes hard and horny and very elastic. The embryo has to squeeze itself out through a narrow chink in this envelope.

In most of the above remarks I have extended the observations made by Mr J. M. Leslie[1] and there are only two important points to note on which I differ from him, one is regarding the extent of the nuptial markings on the arm of the male, and the other concerns the croak, which he stated to be absent.

It is noteworthy that these frogs bred after at least four years of captivity and not at the time of spawning at the Cape, which is in August.

I have much pleasure in thanking Mr Lynch and the indoor foreman at the Botanic Garden for their many kindnesses to me during these experiments.

On the Recovery of foliage Leaves from surgical Injuries. By F. F. Blackman and Miss G. L. C. Matthaei.

[*Read* 20 May 1901.]

It has been found that if definite areas of certain leaves be killed by heat or by physical means, the remaining sound tissues divide actively and form an absciss-layer which surrounds the dead cells and cuts out the area so that it drops away from the leaf. Specimens were exhibited showing the stages of this process which takes place with such precision that leaves may thus be shaped to any desired form.

[1] "Notes on the Habits and Oviposition of Xenopus laevis," by J. M. Leslie, F.Z.S., *Proc. Zool. Soc. Lond.*, 1890, p. 69.

Liquid Motion from a Single Source inside a Hollow Unlimited Boundary. By H. J. SHARPE, M.A., St John's College.

[*Read* 20 May 1901.]

PART I.

1. The problem expressed by the title of this Paper suggested itself to me as an analogue of the well-known and difficult one, that of a Paraboloidal Reflector of Sound, with a single source of sound in the focus, a problem which, as far as I know, has never been solved. It is proposed to consider the problem of Liquid Motion from a Single Source inside Hollow Material Boundaries,—surfaces limited in one direction, but unlimited in the opposite direction, surfaces (when the problem is considered in three dimensions) having a general resemblance either to a tube closed at one end, a hyperboloid, or paraboloid of revolution. It will be shewn that there are an infinite number of surfaces for which complete solutions can be found. I think the results may throw some light on the general phenomena of the reflection of liquid motion and perhaps of sound reflection at curved surfaces. The problem can also be solved in two dimensions, and with this case we will begin.

2. The liquid motion which is supposed to be in the plane of the paper is referred to two axes Ox, Oy, and is supposed to be symmetrical with regard to Ox. O is a single source of liquid supply. $DABA'D'$ is a rigid boundary whose possible shapes will presently be explained. Liquid issuing from O is reflected against the rigid boundary, and goes to infinity in the direction Ox. To find the motion.

3. ACA' is a circle with any radius a. Different expressions will be assumed for the liquid velocities inside and outside this circle, but such that these velocities are continuous at every point of the arc ACA' inside and outside. Let u_x, u_y be the liquid velocities parallel to Ox and Oy, expressed in the polar coordinates r and θ of any point.

Inside the circle, that is for $r < a$ put

$$u_x = c + \frac{\mu a \cos\theta}{r} + S\left(\frac{a_m r^m}{a^m}\cos m\theta\right) \qquad (1),$$

$$u_y = \frac{\mu a \sin\theta}{r} - S\left(\frac{a_m r^m}{a^m}\sin m\theta\right) \qquad (2).$$

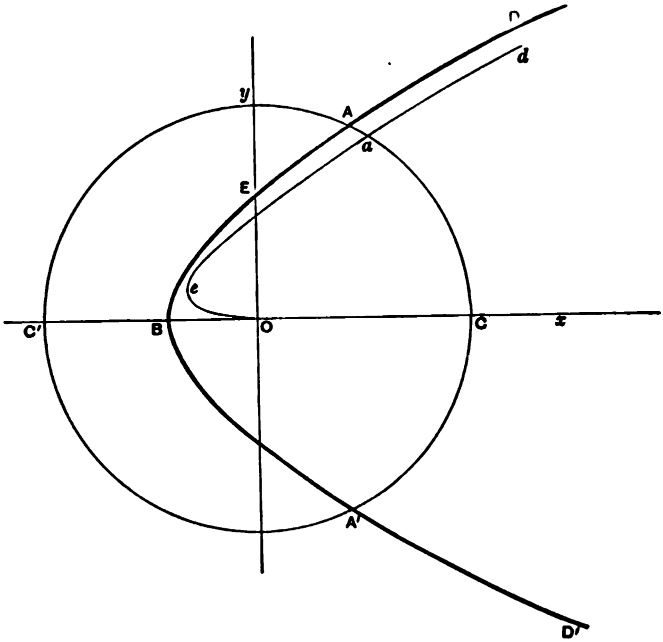


FIG. 1.

This makes $u_x + iu_y$ a function of $x - iy$, and therefore represents irrotational motion in two dimensions. m is integral and S represents the sum of a number of terms like that written down, but with different values of m.

Outside the circle, that is for $r > a$ put

$$u_x = c + \frac{\mu a \cos\theta}{r} + S\left(\frac{a_m a^m}{r^m}\cos m\theta\right) \qquad (3),$$

$$u_y = \frac{\mu a \sin\theta}{r} + S\left(\frac{a_m a^m}{r^m}\sin m\theta\right) \qquad (4)$$

which make $u_x + iu_y$ a function of $x - iy$.

Let the quantities m and a_m be so chosen that

$$S(a_m \sin m\theta) = 0 \dots\dots\dots\dots\dots\dots\dots (5),$$

for all values of θ that lie between certain fixed limits, let us say from 0 to $\pi/2$. We might here for $\pi/2$ substitute any aliquot part of π, but we begin with $\pi/2$ as the simplest supposition. We shall see presently that condition (5) can be satisfied in an infinite number of ways. Supposing it so satisfied, we see that when $r = a$, that is for points on the circumference of the circle from $\theta = 0$ to $\theta = \pi/2$, the velocities and pressures on each side of the arc ACA' are continuous. The general shape of the stream lines will be like $Oead$. If there is any stream line such as ABA' which cuts xO produced orthogonally, we shall take it for a rigid boundary. We shall directly shew that there is such an one. The equation to the stream lines, which is got either by integrating $r(u_x \cos\theta + u_y \sin\theta)$ with regard to θ, or $u_x \sin\theta - u_y \cos\theta$ with regard to r comes out

For points inside the circle, or for $r < a$

$$cr\sin\theta + \mu a\theta + S\left[\frac{a_m r^{m+1}}{a^m(m+1)}\sin(m+1)\theta\right] = ca\sin\alpha + \mu a\alpha$$
$$+ S\left[\frac{a_m a}{m+1}\sin(m+1)\alpha\right] \dots\dots\dots (6),$$

where a and α are the polar coordinates of the point a where the particular stream line we are considering cuts the circle $r = a$, so that α may be called the parameter of the system of stream lines.

For points outside the circle, or for $r > a$

$$cr\sin\theta + \mu a\theta + S\left[\frac{a_m a^m}{r^{m-1}(m-1)}\sin(m-1)\theta\right] = ca\sin\alpha + \mu a\alpha$$
$$+ S\left[\frac{a_m a}{m-1}\sin(m-1)\alpha\right] \dots\dots\dots (7).$$

These stream lines are of course continuous and touch each other at the point a. If there be such a stream line as ABA' and if for this particular stream line $\alpha = \alpha_1$ then farther supposing the constants so chosen that the point B is within the circle $r = a$ (and we shall presently see that they can be), (6) must be satisfied by $r = OB$ and $\theta = \pi$, and we shall get from (6)

$$\mu a\pi = ca\sin\alpha_1 + \mu a\alpha_1 + S\left[\frac{a_m a}{m+1}\sin(m+1)\alpha_1\right] \dots\dots (8).$$

This equation determines α_1 or the angle AOx. If we put $\alpha = \alpha_1$ in (6) it becomes the equation to ABA'. Substituting in

this equation from (8) so as to get rid of α_1, after dividing out by $(\theta - \pi)$ we shall get the following result,

$$0 = c - \frac{\mu a}{r} + S\left(\frac{a_m r^m}{a^m}\cos m\pi\right) \text{..............(9)},$$

which gives us the value of OB. Comparing (9) with (1) we see that at the point B $u_x = 0$ as we should expect, so that ABA' cuts xO produced orthogonally. If E be the point where the boundary BA cuts the axis of y we see from (6) and (8) that OE is determined by the following equation,

$$cr + S\left[\frac{a_m r^{m+1}}{a^m(m+1)}\sin(m+1)\frac{\pi}{2}\right] = \frac{\mu a\pi}{2} \text{.......(10)}.$$

As we shall always suppose $\alpha_1 < \pi/2$ and as the points B and E must be within the circle $r = a$ the equations (8), (9) and (10) are very useful to determine the relations and limits of the constants employed. It is true it is conceivable that the boundary BAD might cut the circle in more than one point. Such cases *could* be treated by the present method, but probably they would be very complicated. We shall therefore exclude them at any rate at first, and suppose the constants so chosen that equation (8) in α_1 shall have one and only one solution.

4. It will presently (Arts. 5 &c.) be proved that there are an infinite number of cases where the least value of m in (5) is unity. For a moment assuming this, we see that at points at a great distance from O the terms depending on this unity value of m are the most important. From (7) the form of the boundary at such points can be inferred from the following equation,

$$cr\sin\theta + \mu a\theta + a_1 a\theta = ca\sin\alpha_1 + \mu a\alpha_1 + a_1 a\alpha_1 \text{......(11)}.$$

If c is finite, we have an asymptote parallel to Ox and the shape generally resembles a canal closed at one end. If $c = 0$ and the right-hand side of (11) is finite, $\theta = \alpha_1$ and there will be an

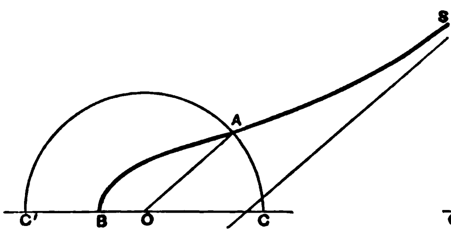


Fig. 2.

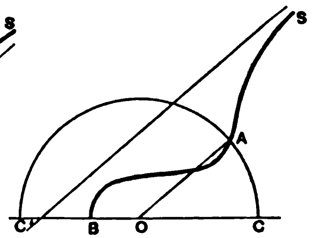


Fig. 3.

oblique asymptote which is parallel to OA, and the shape may be

roughly described as hyperbolic (see figs. 2 and 3). In (11) if $c=0$ we cannot make θ vanish unless a_1 vanishes, which we do not suppose, so in the two-dimensional part of the problem there is no parabolic form in those cases where the least value of m in (5) is unity.

Next, from (3) and (4) the liquid velocities at distant points are given by

$$\left.\begin{aligned} u_x &= c + \frac{\mu a \cos\theta}{r} + \frac{a_1 a \cos\theta}{r} + \&c. \\ v_y &= \frac{\mu a \sin\theta}{r} + \frac{a_1 a \sin\theta}{r} + \&c. \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (12).$$

We see thus that in all cases where the least value of m in (5) is unity the effect of the rigid boundary on the velocity at distant points is as it were to multiply the effectiveness of the source in the ratio of $(\mu + a_1) : \mu$. The strength of the original source is $2\pi\mu a$. The strength of the additional fictitious source which represents the effect of the boundary is $2\pi a_1 a$. May it not be that in some cases, at any rate, of the Reflection of Sound at curved surfaces a similar explanation would apply, if only the exact solution of the problem could be found?

[It may be remarked, but the remark will not be pursued into detail, that equations (1), (2), (3), (4) are, with a certain modification, applicable to the motion through liquid of a solid whose boundary is $DABA'D'$ (Fig. 1). The modification is that equation (5) instead of holding over the arc ACA' must now hold over the arc $AC'A'$. This condition is more easily satisfied if A is in the quadrant yOC'.]

5. We have now to shew how the condition (5) can be satisfied. Probably the simplest way is the following. It is shewn on page 607 of De Morgan's *Diff. and Int. Calculus*, that if θ lie between 0 and $\pi/2$ (but excluding the actual limit $\pi/2$)

$$\sin\theta - \sin 3\theta + \sin 5\theta - \&c. = 0 \quad \ldots\ldots\ldots\ldots (13),$$

provided that (13) may be regarded as the limit of the series

$$\rho \sin\theta - \rho^3 \sin 3\theta + \rho^5 \sin 5\theta - \&c.,$$

and in that way we shall use it, so that in (5) we can take

$$a_m = (-1)^{\frac{1}{2}(m-1)} \times b,$$

where m has the successive *odd* values 1, 3, 5, &c. to ∞, and b is a constant, or what is the same thing and more convenient, we can take $S(a_m \sin m\theta) \equiv bS(-1)^{n+1} \sin(2n-1)\theta$ and give n all integer values from 1 to ∞. Another way of satisfying equation (5) is to

expand (say) θ in a series of sines true from 0 to $\pi/2$, then in another series of sines true from 0 to π and subtract one from the other. We shall thus get zero, and we can take

$$S(a_m \sin m\theta) \equiv \beta \left[2\Sigma(-1)^{n+1} \frac{\sin n\theta}{n} + \Sigma(-1)^n \frac{\sin 2n\theta}{n} \right] \ldots\ldots (14).$$

This will be zero from 0 to $\pi/2$ excluding $\pi/2$ itself. Instead of expanding θ in a series of sines &c. we could of course similarly expand any function of θ. Again, by Fourier's Theorem it can be shewn that between the limits $\theta = 0$ and $\theta = \pi/2$ (but excluding the actual limit $\pi/2$)

$$\sin(2p+1)\theta - \frac{2}{\pi} \sum_1^\infty \sin 2n\theta \,.$$

$$\times \left[\frac{\sin(2n-2p-1)\frac{\pi}{2}}{2n-2p-1} - \frac{\sin(2n+2p+1)\frac{\pi}{2}}{2n+2p+1} \right] = 0 \ldots\ldots (15),$$

where p is an integer and Σ signifies summation with regard to n from 1 to ∞. By this means, between the given limits, the sine of any *odd* multiple of θ, or any finite expression of the form

$$A \sin\theta + B \sin 3\theta + C \sin 5\theta,$$

can be expanded in series of sines of *even* multiples of θ. We thus might use the single series (15) with a particular value of p between $\theta = 0$ and $\theta = a_1$ where $a_1 < \pi/2$, or if we wish to use expansions that would be true even *at* the limit $\theta = \pi/2$, we might use expansions of expressions like $A \sin\theta + B \sin 3\theta + C \sin 5\theta$ with the condition $A - B + C = 0$. Again, to satisfy equation (5) we might take any linear combination of the expressions (13), (14) and (15) multiplied by constants. (5) can therefore be satisfied in an infinite number of ways, so the problem proposed admits of an infinite number of solutions.

6. We will now proceed to examine in some detail the simplest case of equation (5) viz.

Suppose

$$S(a_m \sin m\theta) \equiv b\Sigma(-1)^{n+1} \sin(2n-1)\theta = 0 \ldots\ldots (16),$$

n having all integer values from 1 to ∞, and (16) being true from $\theta = 0$ to $\theta = \pi/2$ ($\pi/2$ being excluded). Equation (8) for finding α_1 will then become

$$\mu\pi = c \sin\alpha_1 + \mu\alpha_1 + b \left\{ \frac{\sin 2\alpha_1}{2} - \frac{\sin 4\alpha_1}{4} + \frac{\sin 6\alpha_1}{6} - \&c. \right\} \ldots\ldots (17).$$

But by De Morgan's *Diff. Calculus*, p. 608, it is known that the series in brackets is equal to $\alpha_1/2$, if $\alpha_1 < \pi/2$ and (17) becomes

$$\mu\pi = c \sin \alpha_1 + \left(\mu + \frac{b}{2}\right) \alpha_1 \ldots\ldots\ldots\ldots\ldots\ldots (18).$$

If we put for shortness $r/a = \rho$, (9) the equation for finding OB becomes

$$0 = c - \frac{\mu}{\rho} - \frac{b\rho}{1+\rho^2} \ldots\ldots\ldots\ldots\ldots\ldots (19).$$

From (6), (8) and (16) the equation to the inner part of the boundary ABA' is

$$c\rho \sin \theta + \mu\theta + b\Sigma \left[(-1)^{n+1} \times \frac{\rho^{2n}}{2n} \sin 2n\theta\right] = \frac{\mu\pi}{2} \ldots (20).$$

From this we at once get $OE = \mu\pi a/2c$, whence we must have

$$\mu\pi < 2c \ldots\ldots\ldots\ldots\ldots\ldots (21).$$

If we could suppose $c = 0$ it would greatly simplify (18) and (19), but then it would be impossible to determine the value of OE from (20). The difficulty arises from our having chosen a special case of equation (5), where only *odd* multiples of θ are involved, but we see from Art. 5 that there are an infinite number of possible forms for (5) where this difficulty does not occur. Summing the series in (20) we get for the equation of ABA',

$$c\rho \sin \theta + \mu (\theta - \pi) + \frac{b}{2} \left[\theta - \tan^{-1} \left(\frac{1-\rho^2}{1+\rho^2} \tan \theta\right)\right] = 0 \ldots (22).$$

So from (7) the equation of AD will be found to be

$$c\rho \sin \theta + \mu (\theta - \pi) + \frac{b}{2} \left[\theta + \tan^{-1} \left(\frac{\rho^2-1}{\rho^2+1} \tan \theta\right)\right] = 0 \ldots (23).$$

(18) and (19) contain 2 arbitrary quantities μ/c and b/c. α_1 found from (18) must be

$$< \pi/2 \ldots\ldots\ldots\ldots\ldots\ldots (24).$$

From (19) as ρ must be < 1 we must have

$$\mu/c < 1 \ldots\ldots\ldots\ldots\ldots\ldots (25),$$

so μ/c and b/c have to satisfy the 3 conditions (21), (24) and (25), but it will be found that these are perfectly compatible, and that so we get an infinite number of boundaries of a canal-like form (see Figs. 4 and 5) to which the remarks in Art. 4 apply. On account of the comparative simplicity of equations (18) and (19) and the other conditions it may be well to point out an interesting result that readily comes from them. The case is illustrated in

Figs. 4 and 5, in which $OE = \mu\pi a/2c$ accurately, $OS = \mu\pi a/c$

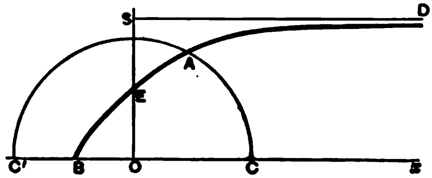


Fig. 4.

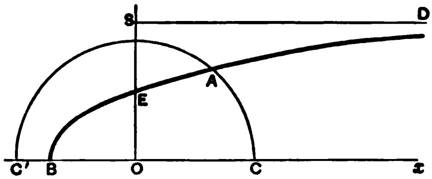


Fig. 5.

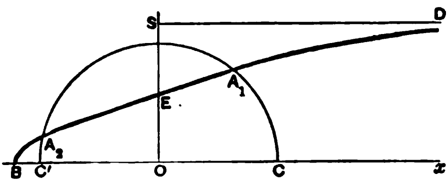


Fig. 6.

accurately, and from (19) $OB = \mu a/c$ to a first approximation, the second approximation being given by

$$OB = \frac{\mu a}{c} + \frac{(\mu + b)\,\mu^2 a}{c^3} \quad \text{..................(26).}$$

Now suppose we keep μ, c and a constant but increase b, we see from (26) that OB is increased, and from (18) that a_1 is decreased, and because b has been increased, by Art. 4 the reflecting power of the boundary has been increased. In Fig. 5 therefore the velocity at a distance is greater than in Fig. 4. This is exactly analogous to a remark made by Lord Rayleigh in his *Sound*, Art. 280, where in treating of conical pipes with a source of sound at the vertex of the cone, he says that when the angle of the cone is decreased the intensity is increased. It will be noticed in Fig. 5 that the boundary is more cramped in the neighbourhood of the source than in Fig. 4. See also *Sound*,

Art. 300. The advantage of the present method is that with a given change in the boundary, we are enabled to calculate the resulting effects on the velocity at a distance.

7. But another interesting extension of the preceding Article may be readily obtained. In Figs. 4 and 5 the idea naturally occurs to one. Supposing B, instead of being within the circle, were outside of it on the left of C', so that BED now cuts the circle in *two* points. Let us call them A_2 and A_1. Can we get a solution with the line BA_2A_1D as boundary? It will be found we can without very much trouble. Let angle $A_1Ox = \alpha_1$ and angle $A_2Ox = \alpha_2$. We shall again begin with the form (16) for equation (5). From (7) the equation of BA_2 will be

$$c\rho \sin\theta + \mu\theta + b\Sigma\left[(-1)^{n+1} \times \frac{\sin(2n-2)\theta}{\rho^{2n-2}(2n-2)}\right]$$

$$= c\sin\alpha_2 + \mu\alpha_2 + b\Sigma\left[(-1)^{n+1} \times \frac{\sin(2n-2)\alpha_2}{2n-2}\right] \ldots (27).$$

But if, and only if, BA_2 be that stream-line which cuts OC' produced orthogonally (27) must be satisfied by $\theta = \pi$ and $\rho = OB/a$, so we must have

$$\mu\pi = c\sin\alpha_2 + \mu\alpha_2 + b\Sigma\left[(-1)^{n+1} \times \frac{\sin(2n-2)\alpha_2}{2n-2}\right] \ldots (28).$$

But by De Morgan's *Diff. Calculus*, p. 608, when $\pi/2 < \phi < \pi$,

$$\frac{1}{2}\sin 2\phi - \frac{1}{4}\sin 4\phi + \frac{1}{6}\sin 6\phi - \&c. = \frac{\phi}{2} - \frac{\pi}{2}.$$

Applying this to (28) it becomes

$$\mu\pi = c\sin\alpha_2 + \mu\alpha_2 + b\left(\frac{\alpha_2}{2} + \frac{\pi}{2}\right) \ldots\ldots\ldots\ldots (29),$$

which gives us α_2 in terms of μ/c and b/c.

Putting $u_x = 0$ at B in (3), or directly from (27), we shall get to determine OB,

$$0 = c - \frac{\mu}{\rho} - \frac{b\rho}{1+\rho^2} \ldots\ldots\ldots\ldots\ldots\ldots (30),$$

which is the same form as (19), but as here we must have $\rho > 1$ we shall find that its first approximate value is $(\mu + b)/c$ which is therefore

$$> 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (31).$$

Next summing the series in (27) we shall get for the equation of BA_2,

$$c\rho\sin\theta + \mu\theta + \frac{b}{2}\left[\theta + \tan^{-1}\left(\frac{\rho^2-1}{\rho^2+1}\tan\theta\right)\right] = \mu\pi \ldots (31\,a).$$

We notice that if in this we put $\rho = 1$ and $\theta = \alpha_2$ the result is exactly identical with (29). The rest of the solution is now plain. It will be found that the equation of A_2A_1 is

$$c\rho \sin \theta + \mu\theta + \frac{b}{2}\left[\theta - \tan^{-1}\left(\frac{1-\rho^2}{1+\rho^2}\tan\theta\right)\right] = \mu\pi \quad \text{...(32)}.$$

Here of course $\rho < 1$. Putting $\rho = 1$ and $\theta = \alpha_1$ we get for determining α_1 in terms of μ/c and b/c,

$$\mu\pi = c \sin \alpha_1 + \mu\alpha_1 + \frac{b\alpha_1}{2} \quad \text{..................(33)},$$

which is the same form as (18). Finally the equation to A_1D is

$$c\rho \sin \theta + \mu\theta + \frac{b}{2}\left[\theta + \tan^{-1}\left(\frac{\rho^2-1}{\rho^2+1}\tan\theta\right)\right] = \mu\pi \quad \text{...(34)}.$$

I believe it will be found that the equations (29) and (33) are perfectly compatible. As a particular case we may have $\alpha_1 = \pi/3$, $\alpha_2 = 5\pi/6$. We shall then get $\mu/c = 3{\cdot}0531$ and $b/c = {\cdot}3814$, and these values satisfy all the conditions to which these constants are subject.

Part II.

8. We will next consider the problem in three dimensions. Suppose a single source of liquid supply at O (Fig. 1) inside a hollow boundary $DABA'D'$ which is a surface of revolution round Ox. To find the possible forms of such surfaces. The liquid motion is supposed to be symmetrical round the axis of x and in planes through that axis. As before we shall use different expressions for the velocities within and without a sphere of radius a, but such that when $r = a$ they are continuous. Let ϕ_1 be the velocity-potential inside the sphere, then we may take for $r < a$

$$\phi_1 = \frac{-\mu a}{r} + \sum_0^\infty \left(\frac{a_n r^n}{a^n} P_n\right) \quad \text{..................(35)}.$$

The 1st term represents the source at O. In the 2nd term a_n is an arbitrary constant, P_n is a Legendre coefficient of θ of the nth order and n has a set of integral values from 0 to ∞, but exactly what set will be shewn presently. Then

$$\frac{d\phi_1}{dr} = \frac{\mu a}{r^2} + \sum_0^\infty \left(a_n P_n \frac{n r^{n-1}}{a^n}\right) \quad \text{.............. (36)},$$

$$\frac{1}{r}\frac{d\phi_1}{d\theta} = \sum_0^\infty \left(\frac{a_n r^{n-1}}{a^n}\frac{dP_n}{d\theta}\right) \quad \text{.................. (37)}.$$

Outside the sphere or for $r > a$ we will take for velocity-potential

$$\phi_2 = Vr\cos\theta - \frac{b_0 a}{r} + \sum_1^\infty \left(\frac{b_n a^{n+1}}{r^{n+1}} P_n\right) \dots\dots\dots (38).$$

In case the boundary is tubular at infinity, the 1st term gives us V the constant velocity at infinity. If we want the boundary to resemble a hyperboloid or paraboloid at infinity, V will be 0. The 2nd term in (38) represents the fictitious source at O, which will ultimately be seen to represent the effect of the boundary on the velocity at a distance. In the 3rd term in (38) n is supposed to have the same values as in (35) excepting zero. Then

$$\frac{d\phi_2}{dr} = V\cos\theta + \frac{b_0 a}{r^2} - \sum_1^\infty \left\{b_n (n+1) \frac{a^{n+1}}{r^{n+2}} P_n\right\} \dots\dots (39),$$

$$\frac{1}{r}\frac{d\phi_2}{d\theta} = -V\sin\theta + \sum_1^\infty \left(\frac{b_n a^{n+1}}{r^{n+2}} \frac{dP_n}{d\theta}\right) \dots\dots\dots (40).$$

It will be seen either from (35) and (38) or from (37) and (40) that the motion is symmetrical with respect to Ox.

On the surface of the sphere $r = a$ we must have

$$\frac{\mu}{a} + \sum_0^\infty \left(\frac{n a_n P_n}{a}\right) = V\cos\theta + \frac{b_0}{a} - \sum_1^\infty \left\{b_n (n+1) \frac{P_n}{a}\right\} \dots (41),$$

$$\sum_0^\infty \left(\frac{a_n}{a} \frac{dP_n}{d\theta}\right) = -V\sin\theta + \sum_1^\infty \left(\frac{b_n}{a} \frac{dP_n}{d\theta}\right) \dots\dots\dots (42).$$

These must be true for all values of θ from 0 up to a certain definite angle, which may be $\pi/2$ or any less angle.

In (42) we must have generally $a_n = b_n$ except when $n = 1$, in which case remembering that $-\sin\theta$ is $dP_1/d\theta$, we must have $b_1 = a_1 - aV$. Since $dP_0/d\theta$ is zero, a_0 is arbitrary. Putting these values in (41) it will become

$$\frac{\mu}{a} + \frac{a_1 P_1}{a} + \frac{1}{a}\sum_2^\infty (n a_n P_n)$$
$$= VP_1 + \frac{b_0}{a} - \frac{2}{a}(a_1 - aV) P_1 - \frac{1}{a}\sum_2^\infty (n+1) a_n P_n$$

or $$(\mu - b_0) + 3P_1 (a_1 - aV) = -\sum_2^\infty (2n+1) a_n P_n \dots\dots (43).$$

Whether V is finite or zero (43) can be satisfied in an infinite number of ways, so that we get an infinite number of solutions of the problem proposed. The method is as follows.

9. By Todhunter's *Laplace's Functions* (*T.L.F.*) Arts. 28 and 62 it can be shewn that between the limits $\theta = 0$ and $\pi/2$ any

Legendre coefficient of odd order can be expanded in a series of the same coefficients of even order, and of course any expression of the form (say) $AP_1 + BP_3 + CP_5$ (where A, B, C are arbitrary) could be similarly expanded. Applying this to (43) and remembering that the constants a_n for $n = 2$ upwards are disposable, it is easy to see how (43) can be satisfied.

Probably the simplest case is the following. In (43) put for P_1 its expansion. Then (43) becomes

$$(\mu - b_0) + 3(a_1 - aV)\left[\frac{1}{2} + \frac{5}{8}P_2 + \&c. + (-1)^{1+n/2} \times\right.$$

$$\left.\times \frac{3.5 \ldots (n-3)}{2.4 \ldots (n+2)}(2n+1)P_n + \&c.\right] = -\sum_2^\infty (2n+1)a_n P_n \quad (44).$$

On both sides n is now supposed even, but on the left-hand side the least value is supposed to be 4. (44) is satisfied if

$$\mu - b_0 + \frac{3}{2}(a_1 - aV) = 0 \ldots\ldots\ldots\ldots\ldots\ldots(45),$$

$$-\frac{3}{8}(a_1 - aV) = a_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots(46),$$

and $$3(a_1 - aV)(-1)^{n/2} \times \frac{3.5 \ldots (n-3)}{2.4 \ldots (n+2)} = a_n \ldots\ldots(47),$$

in the last equation n being supposed 4 or some higher even number.

From (35), (45), and (46) we shall get

$$\phi_1 = -\frac{\mu a}{r} + a_0 + \left\{aV - \frac{2}{3}(\mu - b_0)\right\}\frac{r}{a}P_1 +$$

$$+ \frac{1}{4}(\mu - b_0)\frac{r^2}{a^2}P_2 + \sum_4^\infty\left(\frac{a_n r^n}{a^n}P_n\right) \ldots(48).$$

If ψ be any stream-function, and ϕ its corresponding velocity-potential, it can be shewn (Rayleigh *on Sound*, Art. 238) that ψ and ϕ are connected by the relations

$$\frac{d\psi}{dr} = -\frac{d\phi}{d\theta}\sin\theta \quad \text{and} \quad \frac{d\psi}{d\theta} = \frac{d\phi}{dr}r^2\sin\theta \ldots\ldots\ldots(49).$$

Putting here for ϕ the value of ϕ_1 from (48) and taking account of the equation which P_n satisfies, viz.

$$\frac{d}{d\theta}\left(\sin\theta\frac{dP_n}{d\theta}\right) + n(n+1)P_n\sin\theta = 0 \ldots\ldots\ldots(50),$$

we get on integration

$$\psi_1 = C_1 - \mu a \cos\theta - \sin\theta\left[\left\{aV - \frac{2}{3}(\mu - b_0)\right\}\frac{r^2}{2a}P_1' + \right.$$

$$\left. + \frac{1}{4}(\mu - b_0)\frac{r^3}{3a^2}P_2' + \sum_4^\infty\left\{\frac{a_n r^{n+1}}{a^n(n+1)}P_n'\right\}\right] \ldots (51),$$

where P_n' stands for $dP/d\theta$. (48) and (51) belong to points inside the sphere $r = a$. For points outside this sphere (38) becomes

$$\phi_2 = Vr\cos\theta - \frac{b_0 a}{r} - \frac{2}{3}(\mu - b_0)\frac{a^2}{r^2}P_1 + \frac{1}{4}(\mu - b_0)\frac{a^3}{r^3}P_2 +$$

$$+ \sum_4^\infty\left(\frac{a_n a^{n+1}}{r^{n+1}}P_n\right) \ldots (52).$$

Putting in (49) for ϕ the value of ϕ_2 from (52) and integrating we get

$$\psi_2 = C_2 - b_0 a\cos\theta - \sin\theta\left[-\frac{Vr^2}{2}\sin\theta + \right.$$

$$\left. + \frac{2}{3}(\mu - b_0)\frac{a^2}{r}P_1' - \frac{1}{4}(\mu - b_0)\frac{a^3}{2r^2}P_2' - \sum_4^\infty\left(\frac{a_n a^{n+1}}{nr^n}P_n'\right)\right] \ldots\ldots (53).$$

10. If in (51) $\psi_1 = 0$ is to be the equation of that particular stream line which cuts OC' (fig. 1) orthogonally, it must be satisfied by $\theta = \pi$ and $r = OB$, so that we must have

$$C_1 = -\mu a \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (54).$$

OB is most easily got from (36) by putting in it $d\phi_1/dr = 0$ and $\theta = \pi$. We thus get from (36), (45) and (46) for finding OB

$$0 = \frac{\mu a}{r^2} - \frac{1}{a}\left\{aV - \frac{2}{3}(\mu - b_0)\right\} + \frac{r}{2a^2}(\mu - b_0) - \&c. \ldots (55).$$

Remembering (*T.L.F.* Art. 14) that P_n' vanishes when n is even and $\theta = \pi/2$ we get from (51) for finding OE

$$0 = -\mu a + \left\{aV - \frac{2}{3}(\mu - b_0)\right\}\frac{r^2}{2a} \ldots\ldots\ldots\ldots (56)$$

accurately. We see from this that if $V = 0$ we *must* for OE to be real have $b_0 > \mu$. We have next to get the value of C_2 in (53). This may be done in two ways. We can either suppose V to be finite and remembering that from (35) $4\mu a\pi$ is the quantity of liquid given out by the source in the unit of time, and making the

same quantity to be discharged in the same time through the asymptotic cylinder, we shall get

$$C_2 - b_0 a = -2\mu a \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (57).$$

Or we may suppose $V=0$, then from (52) it is easy to shew that the quantity of fluid discharged through the asymptotic cone at infinity in unit of time $= 2\pi b_0 a(1-\cos\theta_1)$. θ_1 being the semi-angle of the asymptotic cone, therefore we must have

$$2\pi b_0 a(1-\cos\theta_1) = 4\mu a\pi \;\ldots\ldots\ldots\ldots\ldots (58),$$

and remembering from (53) that $\cos\theta_1 = C_2/b_0 a$ we get for C_2 the same value as is given by (57). From (58) we see that as θ_1 cannot vanish without μ vanishing, which we do not suppose, *the boundary cannot be parabolic at infinity.*

11. Looking at (45) and (47) let us put for brevity a_n for $a_n/(\mu - b_0)$. Then (51) may be written, being the equation to BA (Fig. 1),

$$0 = -\mu a - b_0 a\cos\theta + \frac{1}{2}Vr^2\sin^2\theta - (\mu - b_0)a\cos\theta -$$

$$-(\mu - b_0)\sin\theta\left[-\frac{r^2}{3a}P_1' + \frac{r^3}{12a^2}P_2' + \sum_4^\infty\left\{\frac{a_n r^{n+1}}{a^n(n+1)}P_n'\right\}\right]\ldots(59).$$

(53) may be written, being the equation to AD (Fig. 1),

$$0 = -\mu a - b_0 a\cos\theta + \frac{1}{2}Vr^2\sin^2\theta - (\mu - b_0)a -$$

$$-(\mu - b_0)\sin\theta\left[\frac{2a^2}{3r}P_1' - \frac{a^3}{8r^2}P_2' - \sum_4^\infty\left(\frac{a_n a^{n+1}}{nr^n}P_n'\right)\right]\ldots\;(60).$$

As the curves BA, AD meet in the same point A on the circle ACA' it is evident that when $r=a$ the two equations (59) and (60) should give us the same value for θ, therefore the coefficients of $(\mu - b_0)$ in both equations must be equal. We must therefore establish the truth of the following equation,

$$\sin\theta\left[\frac{5}{24}P_2' - \sum_4^\infty\frac{2n+1}{n(n+1)}a_n P_n'\right] = 1 - \cos\theta - \sin^2\theta\ldots(61).$$

This can be done thus. From (50)

$$\frac{\sin\theta\,.\,P_n'}{n(n+1)} = -\int_0^\theta P_n\sin\theta\,d\theta.$$

Applying this to (61) it becomes

$$\int_0^\theta\sin\theta\left\{-\frac{5}{4}P_2 + \sum_4^\infty(2n+1)a_n P_n\right\}d\theta = 1 - \cos\theta - \sin^2\theta.$$

Differentiating this and dividing by $\sin\theta$, we should get

$$-\frac{5}{4}P_2+\sum_4^\infty(2n+1)\,a_nP_n=1-2\cos\theta.$$

But this is true, for it is the expansion of $\cos\theta$ or P_1 between the limits 0 and $\pi/2$ in terms of Legendre Coefficients of even order, in fact the same series that occurs in (44) remembering that

$$a_n=2(-1)^{1+n/2}\times\frac{3.5\ldots(n-3)}{2.4\ldots(n+2)}.$$

If using the notation of Arts. 1—7 we call α_1 the $\angle AOx$ (Fig. 1) we can get its value by putting $r=a$ either in (59) or (60).

12. The supposition $b_0=\mu$ gives an extremely simple solution of the problem. We thus get a tubular surface closed at one end whose equation is from (59) and (60)

$$Vr^2\sin^2\theta=2\mu a\,(1+\cos\theta)\ \ldots\ldots\ldots\ldots\ldots(62).$$

The general shape is given by Fig. 4. We see that in this case the equation to the boundary is the same inside and outside the circle $r=a$.

13. The next simplest case is got by supposing $V=0$ and the $\angle AOx$ (Fig. 1) to be $\pi/2$. We may do this because the expansion for P_1 which we have made use of in (44) is true even *at* both limits 0 and $\pi/2$. Moreover this expansion has never been differentiated. Accordingly putting $r=a$ and $\theta=\pi/2$ in (59) or (60) we shall get since P_n' vanishes when $\theta=\pi/2$ if n be even

$$b_0=4\mu\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(63),$$

and this satisfies the condition derived from (56). If θ_1 be the semi-angle of the asymptotic cone, we get from (60) $\theta_1=\pi/3$.

From (55) we get for determining OB (Fig. 1), putting $OB/a=\rho$

$$\frac{1}{\rho^2}-2-\frac{3\rho}{2}+\&c.=0\ \ldots\ldots\ldots\ldots\ldots\ldots(64)$$

from which we get $OB/a=\cdot5726$ nearly.

From (60) we get for the approximate form of AD (Fig. 1) at infinity the equation

$$\frac{r}{a}=\frac{\sin^2\theta}{1-2\cos\theta}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(65)$$

from which we see, comparing it with a hyperbola, that the boundary crosses and ultimately lies within its asymptotic cone.

The shape of the curve is given in Fig. 7. If OY be the perpendicular on the asymptote it can be shewn from (65) that OY is about $\frac{1}{10}$th of the radius, so that the boundary strongly resembles

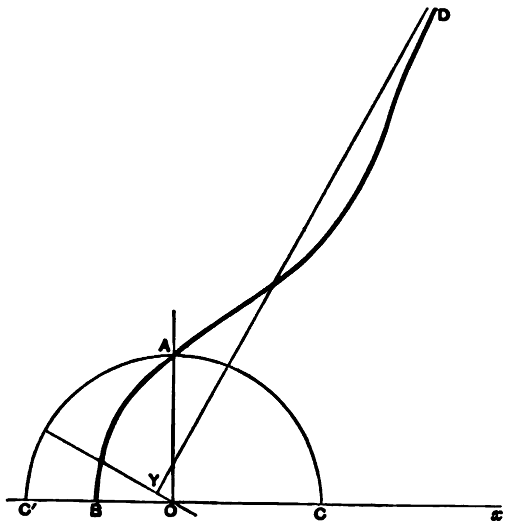


Fig. 7.

a cone of semiangle $\pi/3$ with source at the vertex. It is interesting to notice that if we had such a conical boundary, we might see by only considering the inflow of the liquid at O and its outflow at infinity, that equation (63) would hold, and the same would be the case for a boundary having a general likeness to BAD (Fig. 7), but this would give us only the approximate velocity of the liquid *at infinity*. The above investigation however gives us the *whole of the fluid motion* from the source outwards. We can also get an infinite number of boundaries having a given reflective power—say for instance defined by equation (63).

Liveingite, a new mineral from the Binnenthal. By R. H. Solly, M.A., Downing College, with analysis by H. Jackson, M.A., Downing College.

[*Abstract, read* 6 May 1901.]

This new mineral, to which I have given the name "Liveingite" in honour of the Professor of Chemistry, G. D. Liveing, F.R.S., is a new member of the group of sulpharsenites of lead which comprise Sartorite $PbS + As_2S_3$, Rathite $3PbS + 2As_2S_3$, Dufrenoysite $2PbS + As_2S_3$ and Jordanite $4PbS + As_2S_3$.

It crystallizes in the oblique system with a pseudo-rhombic habit, $\beta = 89° \, 45\frac{1}{2}'$.

The crystals consist of a twin aggregation, twinned about (100).

The twinning is beautifully shown by the iridescent tarnish they exhibit; one crystal has a green shade of colour, the other a red.

This difference in colour of two similar twinned crystals is well shown in Jordanite.

Through the kindness of Professor Liveing I have lately been able to make a few experiments on the artificial tarnishing of twinned crystals. The experiments made on Bournonite and Redruthite confirm the arrangement of tarnish observed in Liveingite and Jordanite.

The crystal habit is partially like Rathite and Sartorite.

The development of the dome zone resembles Rathite, while in the prism it resembles Sartorite. The pyramid zone has planes not found on either Rathite or Sartorite.

The prism zone exhibits oblique symmetry. I had only two crystals and the greater portion has been used by Mr Jackson in his analysis.

I am hoping to obtain this summer more material so as to be able to fully describe the crystallographic characters of this new mineral.

R. H. S.

The crystals were finely powdered and then examined in the manner described in a former publication (*Min. Mag.* XII. 289).

The amount of substance placed at my disposal was ·5 gram and this was used in the single estimation. The results obtained were

$$\begin{aligned} Pb &= 47{\cdot}58 \\ S &= 24{\cdot}91 \\ As &= \underline{26{\cdot}93} \\ & \quad 99{\cdot}42 \end{aligned}$$

A small trace of Iron was present, but could not be estimated. The percentages required for a mineral having the formula $4PbS\ 3As_2S_3$ are

$$\begin{aligned} Pb &= 48{\cdot}75 \\ S &= 24{\cdot}61 \\ As &= \underline{26{\cdot}64} \\ & \quad 100{\cdot}00 \end{aligned}$$

H. J.

PROCEEDINGS AT THE MEETINGS HELD DURING THE SESSION 1900--1901.

ANNUAL GENERAL MEETING,

October 29th, 1900.

MR J. LARMOR, PRESIDENT, IN THE CHAIR.

The following were elected officers for the ensuing year:

President:

Professor A. Macalister.

Vice-Presidents:

Mr J. Larmor.
Mr W. Bateson.
Dr D. Sharp.

Treasurer:

Mr H. F. Newall.

Secretaries:

Mr H. F. Baker.
Mr A. E. Shipley.
Mr S. Skinner.

Other Members of the Council:

Dr H. Gadow.
Professor J. J. Thomson.
Mr A. Berry.
Sir G. G. Stokes.
Mr A. C. Seward.
Mr G. T. Walker.
Professor Liveing.
Mr F. Darwin.
Dr E. W. Hobson.
Mr A. Hutchinson.
Mr C. T. R. Wilson.
Mr J. Graham Kerr.

The President, Professor A. Macalister, then took the chair.

The names of the Benefactors were recited.

The following were elected Fellows of the Society:

G. Elliot-Smith, B.A., St John's College.
F. W. B. Frankland, B.A., Clare College.
W. Myers, M.A., Gonville and Caius College.
W. H. R. Rivers, M.A., St John's College.
Professor G. S. Woodhead, M.A., Trinity Hall.

The following Communications were made:

1. On the structure and classification of the Cheilostomatous Polyzoa. By Dr S. F. HARMER, King's College.

2. Observations on the minute structure of the surface ice of glaciers. By S. SKINNER, M.A., Christ's College.

November 12th, 1900.

In the Lecture Room of Human Anatomy and Physiology.

PROFESSOR MACALISTER, PRESIDENT, IN THE CHAIR.

The following was elected a Fellow of the Society:

L. Cobbett, M.D., Trinity College.

The following Communications were made:

1. The Natives of the Maldives. By J. STANLEY GARDINER, M.A., Gonville and Caius College.

2. The Atoll of Minikoi. By J. STANLEY GARDINER, M.A., Gonville and Caius College.

November 26th, 1900.

In the Cavendish Laboratory.

PROFESSOR MACALISTER, PRESIDENT, IN THE CHAIR.

The following was elected a Fellow of the Society:

G. H. A. Wilson, M.A., Clare College.

The following Communications were made:

1. Some Experiments on the electrical properties of a mixture of Hydrogen and Chlorine when exposed to light. By Professor J. J. THOMSON.

2. On the leakage of electricity through dust-free air. By C. T. R. WILSON, M.A., Sidney Sussex College.

3. On a solar Calorimeter used in Egypt at the total solar eclipse of 1882. By J. Y. BUCHANAN, M.A., Christ's College.

4. Some Theorems in regard to matrices. By T. J. I'A. BROMWICH, M.A., St John's College.

5. On the rational space curves of the fourth order. By J. H. GRACE, M.A., St Peter's College.

6. On *Trifolium pratense*, var. *parviflorum*. By I. H. BURKILL, M.A., Gonville and Caius College.

A paper on the relations of Radiation to Temperature, by J. LARMOR, M.A., St John's College, was postponed.

January 21*st*, 1901.

In the Optical Lecture Room.

SIR G. G. STOKES IN THE CHAIR.

Sir George Stokes proposed from the chair that in consequence of the very serious illness of Her Majesty the Queen the Society do now adjourn out of respect for our beloved Sovereign without transacting the business of the meeting.

This was unanimously resolved.

Note added February 18*th*, 1901.

The Society here records with profound regret the death of Her Most Gracious Majesty Queen Victoria on 22 January 1901.

February 4th, 1901.

In the Cavendish Laboratory.

PROFESSOR MACALISTER, PRESIDENT, IN THE CHAIR.

The following were elected Fellows of the Society:

G. T. Bennett, M.A., Emmanuel College.
J. S. Budgett, M.A., Trinity College.
I. H. Burkill, M.A., Gonville and Caius College.
F. G. Hopkins, M.A., Emmanuel College.
R. W. H. T. Hudson, B.A., St John's College.
H. Jackson, B.A., Downing College.
W. T. N. Spivey, M.A., Trinity College.
T. B. Wood, M.A., Gonville and Caius College.

The following Communications were made:

1. Geometrical Notes. By C. TAYLOR, D.D., Master of St John's College.

2. On the interference bands produced by a thin wedge. By H. C. POCKLINGTON, M.A., St John's College.

3. On some rare and interesting Fungi collected during the past year. By Professor H. MARSHALL WARD.

4. On Geotropism. By F. DARWIN, M.A., Christ's College.

5. Notes on artificial Cultures of Xylaria. By Miss E. DALE (communicated by Professor H. Marshall Ward).

6. The Habits and Development of some West African Fishes. By J. S. BUDGETT, M.A., Trinity College.

7. On a new Form of Microtome. By H. M. LEAKE, B.A., Christ's College (communicated by Mr A. E. Shipley).

8. The Ignoration of Co-ordinates. By T. J. I'A. BROMWICH, M.A., St John's College.

9. A Theorem on curves belonging to a linear complex. By J. H. GRACE, M.A., St Peter's College.

February 18*th*, 1901.

In the Chemical Laboratory.

SIR G. G. STOKES IN THE CHAIR.

On the motion of Mr J. Larmor, seconded by Professor Thomson, it was agreed that inasmuch as the Philosophical Society is included in the larger body of the University its congratulations to our new Sovereign King Edward VII. on his accession to the Throne be deemed to be included in the address presented by the University.

The following Communications were made:

1. On the most volatile gases of atmospheric air. By Professor LIVEING and Professor DEWAR.

2. On a method of comparing the affinity-values of acids. By H. J. H. FENTON, M.A., Christ's College, and H. O. JONES, B.A., Clare College.

3. On isomeric esters of di-oxymaleic acid. By H. J. H. FENTON, M.A., Christ's College, and J. H. RYFFEL, B.A., St Peter's College.

4. Note on the constitution of cellulose. By H. J. H. FENTON, M.A., Christ's College, and Miss M. GOSTLING.

5. Some substituted ammonium compounds of the type $NR'R''R_2'''X$. By H. O. JONES, B.A., Clare College.

6. On the molecular weight of glycogen. By H. JACKSON, B.A., Downing College.

7. On the condensation of formaldehyde and the formation of β-acrose. By H. JACKSON, B.A., Downing College.

March 4*th*, 1901.

In the Optical Lecture Room.

PROFESSOR MACALISTER, PRESIDENT, IN THE CHAIR.

The following were elected Fellows of the Society:

J. B. B. Burke, B.A., Trinity College.
B. Cookson, B.A., Trinity College.
C. E. Inglis, M.A., King's College.

The following Communications were made:

1. The ossification and varieties of the occipital bone. By Professor A. MACALISTER, St John's College.

2. On the fifth book of Euclid's Elements. By Dr M. J. M. HILL.

3. Exhibition of Mr Graham Kerr's method of reconstructing objects from thin sections. By J. S. BUDGETT, M.A., Trinity College.

4. Note on the colour vision of the Eskimo. By W. H. R. RIVERS, M.A., St John's College.

5. Note on the influence of external conditions on the spore-formation of Acrospeira (Berk. and Br.). By R. H. BIFFEN, B.A., Emmanuel College.

6. On a reserve Carbohydrate which produces Mannose from the bulb of Lilium. By J. PARKIN, M.A., Trinity College.

7. Notes on new and interesting plants from the Malay Peninsula. By R. H. YAPP, B.A., Gonville and Caius College.

8. The prevention of Malaria. By Dr J. W. W. STEPHENS.

9. On the effect of a magnetic field on the resistance of thin metallic films. By J. PATTERSON, B.A. (communicated by Professor J. J. Thomson).

10. On the theory of electric conduction through thin metallic films. By Professor J. J. Thomson.

May 6*th*, 1901.

In the Cavendish Laboratory.

PROFESSOR MACALISTER, PRESIDENT, IN THE CHAIR.

The following Communications were made:

1. The oscillations of a Fluid in an annular trough. By B. COOKSON, B.A., Trinity College.

2. Some Experiments upon Beams under endlong compression. By H. E. WIMPERIS, B.A., Gonville and Caius College.

3. Note on the magnetic deflection of cathode rays. By H. A. WILSON, B.A., Trinity College.

4. On an attempt to detect radiation from the surface of wires carrying currents of high frequency. By O. W. RICHARDSON, B.A., Trinity College.

5. On the diminution of the potential difference between the electrodes of a vacuum tube produced by a magnetic force at the cathode. By J. E. ALMY, University of Nebraska.

6. Liveingite, a new mineral from the Binnenthal. By R. H. SOLLY, M.A., Downing College, and H. JACKSON, B.A., Downing College.

May 20th, 1901.

In the Optical Lecture Room.

PROFESSOR MACALISTER, PRESIDENT, IN THE CHAIR.

The following were elected Fellows of the Society:

Hon. R. J. Strutt, B.A., Trinity College.
G. H. F. Nuttall, M.A., Christ's College.
G. Birtwistle, B.A., Pembroke College.

The following Communications were made:

1. On the rate of growth of certain Corals. By J. STANLEY GARDINER, M.A., Gonville and Caius.

2. On the Recovery of foliage Leaves from surgical Injuries. By F. F. BLACKMAN, M.A., St John's College, and Miss G. L. C. MATTHAEI.

3. On the breeding habits of *Xenopus laevis* Daud. By E. J. BLES, B.A., King's College.

4. On a new species of *Bothriocephalus*. By A. E. SHIPLEY, M.A., Christ's College.

5. On a class of matrices of infinite order and on the existence of matricial functions on a Riemann surface. By Dr A. C. DIXON.

6. On liquid motion from a single source. By Rev. H. J. SHARPE.

A paper on Some remarks on the notion of number by Dr HOBSON was postponed.

PROCEEDINGS

OF THE

Cambridge Philosophical Society.

On the Hall Effect in Gases at Low Pressures. By HAROLD A. WILSON, B.A., Clerk Maxwell Student, Fellow of Trinity College.

[*Read* 28 October 1901.]

The experiments described in this paper were undertaken with the object of detecting and investigating the Hall effect in the positive column of the ordinary electric discharge at low pressures.

The existence of a Hall effect in the positive column was to be expected because many phenomena connected with the discharge indicate that the negative ions have a much greater velocity than the positive ions due to the electric field, and it is well known that a difference between the velocities of the two kinds of ions is the condition theoretically necessary for the existence of a Hall effect.

The present investigation has shown that a very large Hall effect can easily be obtained in the positive column which indicates a very great difference between the velocities of the two sorts of ions. After some unsuccessful trials with other forms of apparatus the apparatus shown in the figure was constructed and found to work satisfactorily.

It consisted of a glass 'vacuum tube' *AB*, having an aluminium disc electrode at each end. The distance between the electrodes was 20 cms. and the diameter of the tube 21 mms. Half-way between the electrodes a side tube was joined on, into the upper end of which a conical tube *K* was fitted by grinding in with emery. This ground joint, which permitted the tube *K* to be rotated, was made air-tight by means of mercury at *H*.

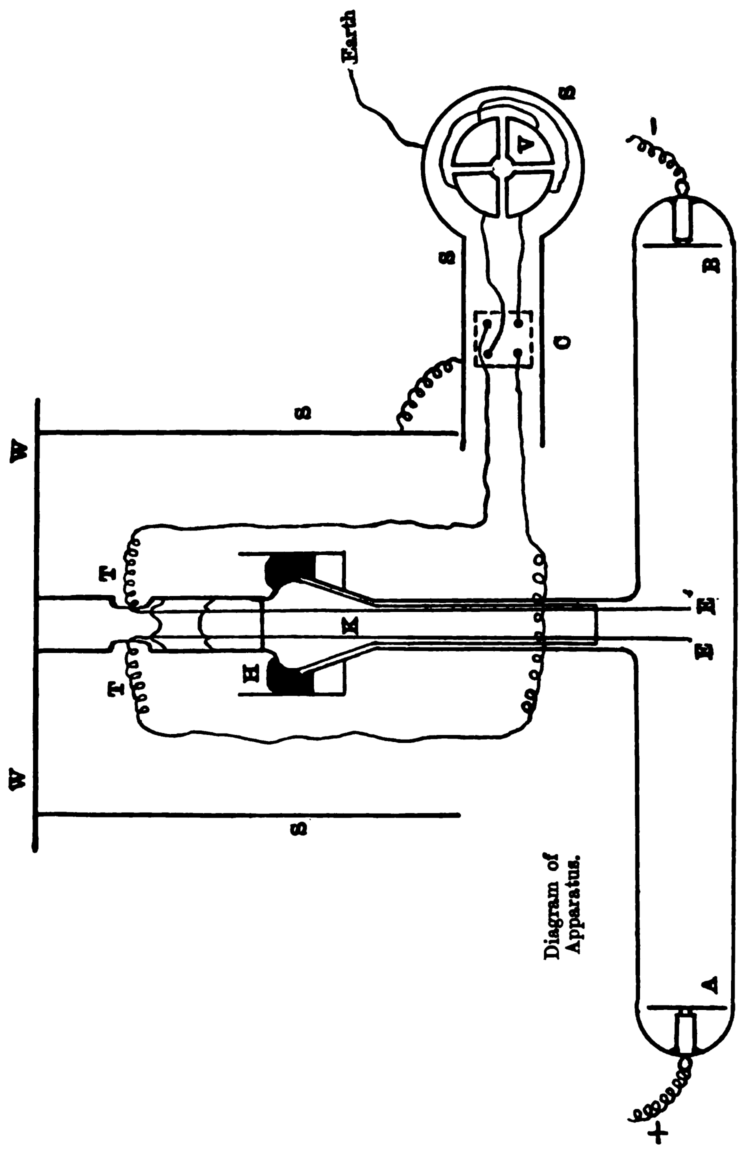


Diagram of Apparatus.

Two small electrodes E and E' were fixed in the tube K by means of sealing-wax at its upper end. These electrodes were made of platinum wire about 0·1 mm. in diameter, enclosed in fine glass tubes 0·5 mm. in external diameter. At EE' the ends of the wires were left bare for two millimetres. The small glass tubes were fixed to the inside of the tube K near its lower end with sealing-wax at opposite ends of a diameter of its bore. The bare wires at E and E' were fixed in this way 6·5 mms. apart and extended to the centre of the discharge tube AB. By turning K the angle between the line joining E to E' and the axis of the tube AB could be made to have any desired value.

The upper part of the tube K was fixed with wax into a brass tube which carried a graduated circle WW 15 cms. in diameter. A vernier reading to minutes of arc on this circle was held in a fixed position relatively to the tube AB so that the angular position of K and consequently of the line EE' could be read off on it.

The small glass tubes enclosing the wires E and E' were embedded at their upper ends in sealing-wax, and the wires were brought out through holes in the brass tube at TT. These wires were connected through a reversing commutator C to an insulated quadrant electrometer V. The electrometer and connecting wires were enclosed in metallic screens $S\,S\,S\,S$ all well soldered together and connected to a 'good earth.' In this way all variations in the electrometer readings due to outside influences were completely stopped.

The electrodes E and E' were found to be very well insulated from each other and from the screens. Either would hold a charge for several minutes without appreciable loss as indicated by the electrometer. The electrometer was extremely well insulated in all parts. One pair of the quadrants was connected to the case, which latter was insulated by paraffin blocks. The quadrants were supported on ebonite legs coated with sulphur, and the electrometer 'jar' was made of a brass plate coated with a thin layer of sulphur on which another brass plate was placed. All four quadrants were provided with adjustable supports so that each could be separately levelled and raised or lowered. The quadrants were all very carefully levelled and arranged symmetrically about the needle, and when the needle was charged no appreciable deflection of the needle occurred. The sensibility of the instrument usually only fell off 2 or 3 per cent. in 48 hours. If the needle was charged up to 2000 volts a sensibility of about 250 millimetre divisions (scale at one metre distance) for one volt was obtained. When measuring the Hall effect a sensibility of about 70 mms. per volt was generally used.

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uniform column can only be obtained with the very smallest current that will maintain the discharge. At any pressure below about 2 mms. as the current is increased from the smallest possible steady value the column is at first uniform, but after a time begins to develope faint striae, which rapidly become more and more distinct as the current rises in value. At pressures of several millimetres the current begins to oscillate when it is increased much, but striae do not usually appear.

The electrodes E and E' although made as small as possible nearly always produced some disturbance of the luminosity of the column. Usually they appear surrounded by a faint dark space, and followed on the side nearest the anode by a faint stria. This effect was generally greatest at pressures near one millimetre. At higher or lower pressures it was possible to nearly get rid of it by carefully adjusting the current. If this disturbance was very marked the results obtained did not agree very well with those obtained when it was not present, but when it was slight they were in good agreement. It will be seen in some of the tables of results given below that the discrepancies which the observations show from the regular laws which they nearly follow are rather greater at pressures near one millimetre than at higher or lower pressures. This is no doubt due to this disturbance of the discharge by the small electrodes being especially marked at these pressures. When an observation was being taken if this disturbance was slight, then the results were always sure to be in good accord with the others of the same kind. All the observations given in this paper were taken when the disturbance in question was very small.

The application of the magnetic field always produces a transverse motion of the positive column so that it becomes brighter along one side of the tube than along the other. This shift is in the direction in which a flexible conductor carrying a current moves. The electrodes E and E' were sufficiently near the axis of the discharge to be still well immersed in the bright part of the column after the field was applied.

This concentration of the discharge on one side by the field is theoretically a necessary consequence of the Hall effect.

Table I. shows the variation of the P.D. between the electrodes E and E' with the angle between EE' and a plane perpendicular to AB. Evidently if the fall of potential along the discharge is uniform this P.D. should vary as the sine of the angle in question. The results given in Table I. show that this is approximately the case.

TABLE I.

Pressure 0·30 mm. Current $1·56 \times 10^{-4}$ ampere					
Angle	P.D.	P.D. ÷ Sine	Angle	P.D.	P.D. ÷ Sine
90°	122	122	40°	75	117
80°	120	122	30°	59	118
70°	114	121	20°	39·5	116
60°	103	119	10°	21	121
50°	90	118	0°	3	—

Table II. shows the variation of the Hall effect (that is the electrometer deflection) with the magnetic field. When measuring the Hall effect the field was always reversed and the sum of the two deflections obtained taken as the Hall effect due to double the field employed.

The results in Table II. show that the Hall effect is proportional to the magnetic field. Similar results were obtained at other pressures.

TABLE II.

Pressure 1·56 mm. Current $5·4 \times 10^{-3}$ ampere		
Hall Effect (Electrometer deflection)	Magnetic Field	Ratio
79	113	1·43
48	69	1·44
24	35	1·46
14	21	1·50

The results in Table III. show that the Hall effect is very nearly independent of the current producing the discharge. At pressures below one millimetre the range of current which could be employed at any one pressure was too small to allow this independence to be satisfactorily verified for each pressure separately, but the results on the variation of the Hall effect with the pressure make it extremely probable that the Hall effect is nearly independent of the current at pressures down to 0·26 mm.

TABLE III.

Pressure 0·98 mm. Mag. Field 45·1		Pressure 1·85 mm. Mag. Field 45·1	
Hall Effect (Volts)	**Current**	**Hall Effect (Volts)**	**Current**
0·806	$6{\cdot}65 \times 10^{-4}$	0·38	$2{\cdot}16 \times 10^{-3}$
0·791	8·54 „	0·36	2·79 „
		0·40	4·41 „
0·828	13·20 „	0·36	5·31 „
0·814	20·60 „	0·40	7·11 „
Mean 0·810		Mean 0·38	

Table IV. gives the results obtained for the Hall effect at different pressures from 2·90 to 0·266 mms.

These results show that the Hall effect varies inversely as the pressure. If Z is the transverse fall of potential or Hall effect, p the pressure in millims. of mercury, and H the magnetic field, then

$$Z = 0{\cdot}0248\,\frac{H}{p}.$$

TABLE IV.

Pressure (p)	Hall Effect (Volts per cm.)	Mag. Field	$\frac{Zp}{H}$
2·91	0·356	45·2	$2{\cdot}29 \times 10^{-2}$
2·23	0·475	45·8	2·32 „
1·85	0·597	46·3	2·38 „
1·22	0·915	44·7	2·49 „
0·983	1·20	46·3	2·54 „
0·650	1·85	46·8	2·56 „
0·508	2·42	46·3	2·76 „
0·333	3·24	46·8	2·31 „
0·266	4·60	46·4	2·64 „
			Mean 2·48 „

Table V. contains some of the results obtained on the variation of the electric intensity along the discharge with the current. These results were obtained, by turning EE' until the line joining E to E' coincided with the axis of the tube AB and then measuring the P.D. between E and E'.

At 2·81 mms. pressure the electric intensity X can be represented approximately by the formula

$$X = 60{\cdot}5 - 860\iota,$$

where ι is the current in amperes.

At 2·08 mms. the formula for X is

$$X = 51{\cdot}6 - 924\iota.$$

TABLE V.

Pressure 2·81 mm.	
Electric Intensity (Volts per cm.)	Current
57·0	$4{\cdot}05 \times 10^{-3}$
56·1	5·05 ,,
54·0	7·75 ,,
50·6	11·50 ,,

Pressure 1·52 mm.	
Electric Intensity (Volts per cm.)	Current
40·2	$1{\cdot}17 \times 10^{-3}$
40·2	1·35 ,,
40·4	1·98 ,,
40·9	2·80 ,,
40·7	3·25 ,,
39·9	4·96 ,,
38·2	7·30 ,,
37·4	9·20 ,,

Pressure 2·08 mm.	
Electric Intensity (Volts per cm.)	Current
50·8	$0{\cdot}90 \times 10^{-3}$
49·8	1·26 ,,
49·4	2·08 ,,
47·6	3·88 ,,
45·4	7·04 ,,
44·3	7·94 ,,

Pressure 1·125 mm.	
Electric Intensity (Volts per cm.)	Current
31·7	$5{\cdot}3 \times 10^{-4}$
31·1	6·03 ,,
30·8	7·80 ,,
30·8	8·84 ,,
31·3	10·20 ,,
30·8	14·35 ,,
30·3	22·60 ,,

At pressures below 1 millimetre X is nearly independent of the current for the small range of current which can be employed. For the purpose of comparing the values of X at different pressures I have used the value of X corresponding to the smallest possible steady current at each pressure. When the current is nearly as small as possible X is nearly independent of the current except at the higher pressures, where it always diminishes slowly as the current is increased.

Table VI. shows the way in which X varies when the pressure (p) is changed. The results show that $\frac{X}{\sqrt{p}}$ is nearly a constant for pressures from 0·20 to 2·82 mms.

$$X = 34{\cdot}9\sqrt{p}.$$

TABLE VI.

Pressure	Electric Intensity	$\frac{X}{\sqrt{p}}$
0·200	16·05	35·9
0·275	19·7	37·6
0·310	20·5	36·9
0·383	22·7	36·7
0·550	26·3	35·5
0·583	26·3	34·5
0·792	28·4	32·0
1·08	33·4	32·3
1·13	31·8	33·6
1·52	42·9	34·9
2·05	49·2	34·5
2·08	50·8	35·3
2·82	57·0	34·0
		Mean 34·9

There is another way in which the apparatus described can be used to measure the Hall effect. After applying the magnetic field in one direction the electrodes EE' are rotated until they are both at the same potential. The field is then reversed and the electrodes again rotated till their potentials are equal. The

angle between the two positions of zero potential difference is evidently the rotation of the equipotential surfaces in the discharge due to the Hall effect. This angle is given by the equation

$$\frac{Z}{X} = \tan\frac{\theta}{2}.$$

Substituting for Z and X the values given above, we have

$$\tan\frac{\theta}{2} = \frac{0{\cdot}0248H}{34{\cdot}9p^{1{\cdot}4}}$$

$$= 7{\cdot}1 \times 10^{-4} Hp^{-1{\cdot}4}.$$

Measurements were made by this method, using a magnetic field of 23 C.G.S., which was reversed as already described.

Unfortunately it was found that this method was not capable of giving at all accurate results. When p is small θ varies very rapidly with p, and when p is large θ is very small. Also the process of adjusting the electrodes until they are at the same potential requires considerable time, so that the discharge is liable to change during a measurement of θ. Successive measurements of the angle varied sometimes by as much as 1 degree. Although this method was not found capable of giving accurate results still it afforded a means of confirming the results obtained by the first method.

Table VII. gives the values of θ observed at a number of different pressures and the corresponding angles given by the formula

$$\tan\frac{\theta}{2} = 7{\cdot}1 \times 10^{-4} Hp^{-1{\cdot}4}.$$

TABLE VII.

Pressure	Angle (Observed)	Angle (Calculated)
0·30 mm.	10·1°	11·2°
0·36	7·2	8·5
0·55	5·5	4·6
0·62	4·8	3·8
0·90	3·2	2·2
1·87	0·3	0·6
2·02	0·9	0·5

The theory of the Hall effect in salt solutions has been worked out by Donnan (*Phil. Mag.*, Nov. 1898), and Larmor (*Aether and Matter*, p. 301) gives the theory for a completely dissociated salt solution.

In air at low pressures the amount of dissociation is relatively very small and the ionisation does not depend merely on the concentration as in salt solutions.

Nevertheless it is easy to show that, provided the influence of the walls of the discharge tube is neglected, Larmor's theory for completely dissociated salt solutions applies without modification to the uniform positive column.

Let $v_1\,(=k_1X)$ be the velocity along the tube of the positive ions, and $v_2\,(=k_2X)$ that of the negative ions.

Then if i is the current density at any point

$$i = Xe\,(k_1n_1 + k_2n_2),$$

where n_1 and n_2 are the numbers of positive and negative ions in unit volume respectively, and e is the charge carried by an ion.

Let the axis of the discharge tube AB be denoted by x, and the perpendicular horizontal direction of the Hall effect by z.

Then $\dfrac{dX}{dx} = 0$, and since Z is small and probably uniform, we may put $\dfrac{dZ}{dz} = 0$, whence $n_1 = n_2$, and $i = Xne\,(k_1 + k_2)$.

In the uniform positive column the ionisation is everywhere equal to the recombination or $q = \alpha n^2$ where q is the rate of ionisation and α the constant of recombination. Now the most important peculiarity of the positive column is that X is independent or nearly so of i, so that

$$q = \alpha n^2 = \alpha\,\frac{i^2}{X^2e^2\,(k_1 + k_2)^2}$$

is simply proportional to the square of the current density. Hence if owing to the Hall effect the transverse distribution of the current is changed the ionisation will adjust itself automatically to the new conditions and still be everywhere equal to the recombination.

The ionisation and recombination may therefore be neglected and the calculation becomes identical with that for a completely dissociated salt solution. The result for this case as Larmor shows (*loc. cit.*) is

$$Z = \frac{1}{2}\,HX\,(k_2 - k_1).$$

If z is reckoned positive in the direction in which a current going

in the positive direction along x tends to move, then Z is positive when the velocity of the negative ions is greater than that of the positive ions. That is, Z helps on the positive ions in the positive direction of z but retards the negative ions.

When a steady state is reached evidently

$$n_1 k_1 (Z + k_1 XH) - K_1 \frac{du_1}{dz} = 0,$$

where K_1 is a coefficient of diffusion of the positive ions. Similarly

$$n_2 k_2 (-Z + k_2 XH) - K_2 \frac{du_2}{dz} = 0.$$

But $n_1 = n_2 = n$ (say) so that

$$\frac{K_1}{k_1} \cdot \frac{du}{dz} = n (Z + k_1 XH),$$

$$\frac{K_2}{k_2} \cdot \frac{du}{dz} = n (-Z + k_2 XH).$$

But $\frac{K_2}{k_2} = \frac{K_1}{k_1}$ according to the kinetic theory of gases, so that

$$Z + k_1 XH = -Z + k_2 XH;$$

$$\therefore\ Z = \frac{1}{2} HX (k_2 - k_1).$$

In this expression for Z the change in the transverse distribution of the discharge produced by the magnetic field is taken into account so that no error is introduced into this investigation by this effect.

In the above calculation the influence of the walls of the discharge tube is left out of account and it is possible that this influence may not be unimportant in some cases, especially at low pressures. It is well known that when a gaseous ion strikes a solid body such as glass it remains stuck to it (or at least its charge remains stuck), thus for example it is easy to remove all the ions from a gas by passing it through a glass wool plug. Consequently when a discharge is passing through a glass tube the walls of the tube ought to be regarded as perfect absorbers of the ions so that close to the glass $n_1 = 0$ and $n_2 = 0$.

The glass must therefore get charged up sufficiently to make the number of negative ions striking it in any time equal to the number of positive ions. Since the negative ions diffuse quicker than the positive ions the glass will get a negative charge.

In the same way the electrodes E and E' ought to be regarded

as perfect absorbers of the ions and they must consequently take up a negative charge in the same way as the glass walls of the tube. This is no doubt the explanation of the disturbance which the electrodes produce in the discharge. Near the walls of the tube the luminosity appears to be less intense than at the centre of the tube, but this appearance is probably partly due to the greater thickness of glass through which the discharge near the walls is seen.

The fact that the Hall effect was found to be proportional to the magnetic field and independent of the current shows that the charges which the walls of the tube and the electrodes no doubt take up are too small to influence the measurements made. For since the mean value of n is proportional to the current and (as is shown below) the ratio of n at E to n at E' is independent of the current, hence the difference between n at E and n at E' must be proportional to the current, so that increasing the current ought to have increased the difference between the charges on the two electrodes necessary to make the numbers of positive and negative ions striking then equal, if these charges had been appreciable. According to these considerations the smaller the currents used the more reliable the results obtained should be, and this is exactly what was found experimentally to be the case.

When the magnetic field is being applied it is easy to see that the luminosity increases in intensity as one passes horizontally from one side of the tube to the other.

The above theory of the Hall effect shows that this should be the case, for we have

$$\frac{K_1}{k_1} \cdot \frac{dn}{dz} = n(Z + k_1 HX),$$

$$\frac{K_2}{k_2} \cdot \frac{dn}{dz} = n(-Z + k_2 HX).$$

Putting $\frac{K_1}{k_1} = \frac{K_2}{k_2} = \beta$ and adding these equations give

$$2\beta \frac{dn}{dz} = n HX (k_1 + k_2).$$

Hence $$\log n = \frac{HX}{2\beta} (k_1 + k_2) z + A.$$

For a tube of square cross section with its sides respectively parallel and perpendicular to the magnetic field and sides of length a,

$$I = a \int_0^a i dz,$$

where I is the total current through the tube.

But $$i = Xne\,(k_1 + k_2)$$

$$= Xe\,(k_1 + k_2)\,\epsilon^{\left\{\frac{HX}{2\beta}(k_1+k_2)z+A\right\}}.$$

Therefore $$I = aeX\,(k_1 + k_2)\,\epsilon^A \int_0^a \epsilon^{\frac{HX}{2\beta}(k_1+k_2)z}\,dz$$

$$= \frac{2\beta ae\,\epsilon^A}{H}\,\{\epsilon^{\frac{HX}{2\beta}(k_1+k_2)a} - 1\}.$$

Hence $$A = \log\frac{IH}{2\beta ae\,\{\epsilon^{\frac{HX}{2\beta}(k_1+k_2)a} - 1\}};$$

$$\therefore\ \log n = \frac{HX}{2\beta}(k_1 + k_2)\,z + \log\frac{IH}{2\beta\, ae\,\{\epsilon^{\frac{HX}{2\beta}(k_1+k_2)a} - 1\}}.$$

Thus as we move across the tube $\log n$ increases uniformly with z. If n_1 is the value of n at z_1 and n_2 at z_2, then

$$\log\frac{n_1}{n_2} = C\,(z_1 - z_2),$$

where $$C = \frac{HX}{2\beta}(k_1 + k_2).$$

Thus if n_1 and n_2 denote the values of n at E and E' respectively we see that $\log\frac{n_1}{n_2}$ increases proportionally to H. If therefore there were an appreciable charge on E or E' due to the negative ions diffusing more rapidly than the positive ions, as explained above, then the Hall effect could not have been found proportional to the magnetic field.

Substituting the values of Z and X found experimentally in the formula

$$Z = \frac{1}{2}HX\,(k_2 - k_1),$$

we get, since $Z = \dfrac{0{\cdot}0248H}{p}$ and $X = 34{\cdot}9\sqrt{p}$,

$$k_2 - k_1 = \frac{2Z}{HX} = 1{\cdot}42 \times 10^{-3}\,p^{-1\cdot5}.$$

This must be multiplied by 10^8 to get $k_2 - k_1$ in cms. per second, so that finally

$$k_2 - k_1 = 1{\cdot}42 \times 10^5\,p^{-1\cdot5}.$$

At one millimetre pressure this gives

$$k_2 - k_1 = 1\cdot42 \times 10^5 \frac{\text{cms.}}{\text{sec.}}.$$

The actual difference between the velocities of the negative and positive ions in the positive column is given by

$$v_2 - v_1 = X(k_2 - k_1) = 34\cdot9\sqrt{p} \times 1\cdot42 \times 10^5 p^{-1\cdot5}$$
$$= \frac{4\cdot95 \times 10^6}{p}.$$

These experiments were carried out in the Cavendish Laboratory and my best thanks are due to Prof. J. J. Thomson for his kindly interest and advice throughout the course of the work.

On some Phenomena connected with the Combination of Hydrogen and Chlorine under the influence of Light. By P. V. BEVAN, B.A., Trinity College.

[*Read* 28 October 1901.]

The present note is to give an account of the results of experiments on the initial expansion observed on illuminating a mixture of Hydrogen and Chlorine when the gases are damp. This expansion was discovered by Draper[1] and studied more carefully by Pringsheim[2]. Pringsheim concluded that the expansion took place before any combination to form HCl occurred, and he attributed the expansion to dissociation of Hydrogen and Chlorine molecules giving rise to a larger number of systems in the gas mixture than before illumination.

The apparatus used for the experiments from which the results in this note were obtained was essentially that of Bunsen and Roscoe[3]. The gas mixture to be illuminated is confined in a glass bulb about one-eighth filled with water saturated with chlorine. The bulb communicates with a capillary tube in which a water index defines the volume of the gas mixture in the bulb.

The bulbs in these experiments were considerably larger than those used by Bunsen and Roscoe, admitting of more accurate observation of small percentage changes of volume.

In the insolation bulb is placed a fine platinum wire (·001 inch) with its ends sealed through the bulb. By observing the resistance of this wire the change in temperature of the gas mixture can be determined when combination takes place, or when the initial expansion takes place. This apparatus was well suited for observing the expansion of chlorine alone under the influence of light. This expansion was found to be associated with a rise in temperature equal, within the limits of accuracy of the experiment, to that required to produce the expansion under constant pressure. The change of resistance of the platinum wire could be observed as accurately as desired with a sensitive galvanometer. It was found that the chlorine had no sensible action on the platinum even after

[1] *Phil. Mag.* XXIII. 1843, p. 415.
[2] *Wied. Ann.* 1887, XXXII. p. 384.
[3] *Phil. Trans.* 1857, p. 359.

several weeks of constant use of the apparatus, so that the method of measuring the change in temperature was all that could be desired.

For experiments on hydrogen and chlorine the mixture of gases was prepared by electrolysis of pure aqueous hydrochloric acid saturated in the cold with HCl gas. This method gave a mixture which yielded very constant results affording thus a sufficient guarantee of the purity of the mixture.

The results to which I desire here to call attention are in relation to the connection between the initial expansion, the rise in temperature of the mixture associated with the expansion, and the amount of hydrochloric acid formed when the pure mixture is submitted to the action of light.

Pringsheim states that the initial expansion for a given quantity of light is independent of the state of the induction, that is, independent of the previous history of the gas mixture in respect of exposure to light and consequent combination. My experiments however have shewn that the expansion depends in amount on the extent to which the gas mixture has been insolated previously to the illumination producing the expansion. Bunsen and Roscoe found that after the gases had been combining under the influence of light, if the mixture were darkened for a short time and again illuminated the rate of combination attained its maximum sooner than if the gas had remained in the dark for a long time before illumination. Some effect then of the previous illumination remained after darkening the mixture. It was not till about half-an-hour of darkening that all of this after-effect passed off. A similar effect was found from my experiments on the initial expansion. The expansion was found to be greater if there had been previous illumination than if the mixture had remained in the dark for a long time.

With regard to the relations between the initial expansion, hydrochloric acid formed, and the rise in temperature of the mixture, three series of experiments were made investigating the expansion and quantity of hydrochloric acid formed, the expansion and the rise in temperature, and the amount of hydrochloric acid formed. It was found that the initial expansion was always followed by a contraction to a volume less than the original volume, shewing the formation, and subsequent absorption by the water in the instrument, of hydrochloric acid. This was found to be the case with illuminations of different intensities and durations. With constant intensity of illumination the ratio of the expansion to the hydrochloric acid formed measured by the final contraction from the original volume increased with decreasing duration of illumination; but the value of this ratio was never greater than 5, the illumination for this case being by means of a single electric

spark. With constant duration the ratio increased, with increase of intensity of illumination.

The expansion was also always accompanied by a rise in temperature as indicated by the increase in the resistance of the platinum wire in the insolation bulb. This rise in temperature was always sufficient to account for the expansion. And finally the experiments on the rise in temperature and the hydrochloric acid formed shewed that for very short illuminations the rise in temperature was proportional to the hydrochloric acid formed, and further that the heat of formation of the hydrochloric acid was always rather more than that required to produce the observed rise in temperature of the whole gas mixture, and therefore that the heat of formation of the hydrochloric acid formed is more than sufficient to account completely for the initial expansion.

The final conclusion is therefore that the initial expansion occurs only when combination of the hydrogen and chlorine takes place, and is due to the heat which is liberated by this combination.

Other experiments shewed that the contraction which occurs after the illumination of the gases is stopped is due to the gas mixture cooling to the temperature of the surrounding medium after being heated by the combination, so that the action ceases at once on cutting off the light.

In conclusion I wish to thank Prof. J. J. Thomson for having suggested the investigation and for his advice on the experiments as far as they have as yet been carried.

Notes on the Development of Sagitta. By L. DONCASTER, B.A., King's College.

[*Read* 11 November 1901.]

O. Hertwig's account of the embryonic development was confirmed in every way except that in *Sagitta bipunctata* head-cavities are formed just as Bütschli described in 1873. In *S. enflata* however these cavities are so small as to be seen with difficulty, and Hertwig probably studied a similar species. Sections of the embryo show that in its early stages the nuclei lie at the free ends of the cells, but as development proceeds those of the ventral ectoderm sink into their bases, and in the ventrolateral areas a great proliferation of nuclei takes place, giving rise to the lateral nuclear bands of the ventral ganglion. The cavities of the embryo disappear entirely, and the endoderm becomes reduced to a thin septum, the mesoderm to two solid strands, in which most of the nuclei become aggregated dorsally and ventrally, and the cell-protoplasm beneath them becomes converted into the longitudinal muscles. The larva is as described by Hertwig, but he failed to observe the mode of formation of the posterior transverse septum, which arises between the genital cells of each side as they migrate from the splanchnic mesoderm across the body-cavity to the body-wall. This migration takes place at the time of the reappearance of the cœlom, and the septum is probably formed from the mesodermal envelopes of the genital cells. The ectoderm of the larva in the neck region and at the front end of the fins is thickened and consists of vacuolated cells like those composing the epidermis of *Spadella draco*. No trace of excretory organs nor of genital ducts was found in the larva; the latter appear only as maturity approaches. There is no cœlomic epithelium, but the muscles are formed from the basal ends of the cells which line the cœlom, as in the Nematoda. This fact, combined with the mode of origin of the transverse septa and the absence of many Annelid characters, supports the view that the Chætognatha are not related to the Annelida.

On the Unit of Classification for Systematic Biology. By HENRY M. BERNARD, M.A.

(*Communicated by* MR A. E. SHIPLEY.)

[*Read* 11 November 1901.]

My object in appearing here to-day is not merely to awaken an interest in the purely abstract problem of ideal classification, but to awaken an interest which will, I hope, sooner or later result in a practical reform of method. So far as I can see, zoologists are face to face with an ever increasingly serious practical difficulty. This difficulty is at present felt in very varying degrees, which depend upon the stability of the forms on which we are engaged. Those working with constant forms know nothing about it, whereas in my own case it actually brought my work on the stony Corals to a standstill. To state very briefly the difficulty as it affects my own work. 'Species' as distinct genetic groups, or indeed as anything even approaching distinct genetic groups, are not discoverable. However striking the form-differences within a genus may be, their variations are so great and so numerous and intergraded that no trustworthy conclusion can be arrived at as to their value for the purposes of genetic classification, at least until we have a far wider survey of forms than any at our disposal to-day. And yet we have no other formula for our attempts at classification than that supplied by the binominal species name of Linnaeus, which compels us either to group the specimens into 'species' or to leave them alone.

My experience during the past year, during which I have spent a great deal of time in working at this problem, this paper being about the fifteenth attempt I have made to clarify the subject, while at the same time I have been discussing it both publicly and privately, has led me to see that though the immediate difficulty is a practical one and requires a practical solution, it must be attacked primarily from its philosophical side. I see quite clearly that had I confined myself merely to stating the difficulty I should have gained a good deal of sympathy; but I did not do so, I went further and made a definite constructive proposal involving a reform of our methods of naming specimens. We all know that, provided we only stick to the work, a practical way out of a practical difficulty is always sooner or later suggested by the work itself; and that happened in this case. After a seven years' hopeless attempt to give scientific precision to what was,

in the nature of the case, necessarily mere conjecture, I gradually realised exactly what it was possible to do with the specimens so that the work which had been expended on them could be recorded, and, at the same time, a start made *towards* a natural classification. I found it, however, almost useless to suggest this practical solution to my fellow workers. One here and there, who has found exactly the same kind of difficulty as I have with the Corals, expressed approval, but the great mass of modern systematists say they do not want it. For their worst difficulties, they say, a little patching of the old system will amply suffice and so on. I perceived further that this attitude is not solely due to the fact that, with regard to the more stable forms of life, it is still possible, by means of the old formula of naming, to do accurate scientific work, preparatory towards a natural classification, but also largely to the fact that the prevailing ideas as to the aims of classification are not sufficiently defined. Discussions have been all about nomenclature and not about classification, that is, not about the *principles* that should inspire nomenclature.

It became quite clear, then, that before any better method of naming is demanded, a need for it must be established. The actual formulae of classification are purely secondary; they are only the symbols which we agree to use to express an order of ideas. Before any reform in our methods of designating specimens can be accepted, it is essential that the ideal aims of classification should be very clearly understood. When that is once achieved, it will be psychologically impossible to revert to any system which totally fails to express them. I propose, then, after describing the aims of classification as the theory of Evolution has silently advanced them, to show that the present method of classification, which has survived from pre-Darwinian days, fails to express these aims, and fails even in those cases in which, owing to the stability of the group, the work done is as exact as it could be, while, in cases where the forms are very inconstant, the old formula, as we are at present forced to use it, positively hinders the attainment of any good object whatever. That being established, I shall sketch the line of reform of method which the exigencies of my own work suggested to me, and which, the longer I work with it becomes more practical and promising. I am aware that to attempt to alter our time-honoured methods of classification may be a daring thing to do, but the strength of my position lies in this, that I am simply appealing to the first principles of scientific work, and maintaining that, in classification as in all other departments, we must have a system of work which allows us to begin by collating the facts and nothing but the facts; and further that the time has come when what all evolutionists

have been thinking and saying shall find practical expression in our methods.

I should like to see the question put to all candidates for Natural History appointments, "What are the aims of classification?" The man who is a museum official and nothing more would say: "In order that persons might bring their collections and get them named." A more frequent answer, however, is, "Classification is solely for the purposes of reference." Now the last answer is sometimes given by men who fully appreciate the vast field of research which the theory of Evolution has opened up. "Continue the old fashion of naming," say they, "but don't call it classification in any high sense. The gradual discovery of genetic affinities by the construction of evolutionary series must be undertaken separately with special formulae for that kind of work." This is the attitude which Dr Sharp took up in a pamphlet published nearly thirty years ago[1], and, if I understand him correctly, it is that which Professor Lankester seemed inclined to endorse at one of the meetings of the Linnean Society given up to the discussion of my proposals.

But, the more I think about it, the less do I find it possible to fall into line. Dr Sharp is of course perfectly correct in maintaining that we can make no satisfactory classification of forms which we take up, perhaps for the very first time, in order to describe them and give them names for reference. Their real positions in the evolutionary series must be left to future generations of zoologists to find out. But at the same time it seems to me hopelessly unpractical, and indeed very undesirable, to separate the naming of animal forms for reference from at least some attempt at classification. We should have no means of knowing what forms had been named, and what not, but for the assistance which a classification gives us of running them down.

This, however, does not completely overthrow the position of those who are inclined to advocate that naming and classification should be kept quite apart. For it is still possible to recommend the continuance of the present rough and ready method of classification for the purposes of mere naming for reference fully aware all the while that the classification in no way corresponds with what we now mean by natural classification. Such advice can hardly be defended. It says: Let us start our work on lines which we know to be faulty, in order that we may do it later all over again on a better principle.

This of course is the very last thing that Dr Sharp meant. The real strength of his position lies in the fact that the only available formula for classification implies, by the very terms

[1] *Object and Method of Zoological Nomenclature.* London, 1873.

used, a definitive natural classification, which we know to be unattainable at the present stage of our knowledge. Hence as long as this *is* the only available formula, the advice not to put our very first attempts into final form must be endorsed by every thinking man. This is just the point where my proposition comes to the rescue. I shall try to show that it is possible to have a formula which shall not mean definitive classification but shall classify only so far as the facts allow and no further. With such a formula we can both name, and at the same time lay a solid foundation which time will complete into a natural classification, no matter whether the groups are as stable as the Mammals or as unstable as the stony Corals.

Now, on the face of it, one would think that if only we know what our aims are it cannot be impossible to begin our work with those aims in view, in such a way that all good work should help towards their attainment. What then is the ultimate aim of classification? Surely it is to arrange the organic kingdom in the order of its evolution. This has long been accepted as the *ultimate aim* of our attempts at classification, and the wiser of our systematic workers acknowledge that their work is at its best but a stumbling along in what appears to them to be the right direction. But the question then is, Why do we continue to work blindly, laying all sorts of fanciful foundations, which the next worker roots up for another almost equally fanciful, burdening himself at the same time by having to keep a faithful record of all former attempts, however worthless they have been? There seems to me to be only one explanation, viz., that the old formula of classification maintains too strong a mechanical hold over us, and we have never seen our way to remodel it. This is hardly to be wondered at when we think of the length of time the Linnean method has been in use, of the indispensable services it has rendered to science, and also of the fact that its inadequacy to meet the needs of the new evolutionary philosophy is felt chiefly in work with very inconstant groups and, even when felt most acutely, and understood most clearly, does not immediately suggest any remedy. It is further to be noted that the whole formula as an ideal terminology for classification appears to be quite adequate to the demands of the evolutionist, it being doubtful whether we shall ever want other divisions than the tribes, orders, families, genera, species and varieties. These are all powerful reasons for retaining the Linnean formula for naming the divisions of the organic kingdom. But that of course is not the question under discussion, which is, How shall we discover the divisions which we want to name? It is quite possible that *when* we have done the necessary preliminary sorting out of the forms of life the divisions supplied by a Linnean

formula will be found adequate. But it is now a question of how best to carry out this indispensable preliminary sorting. The higher orders are apparently fairly easy to sort out, and being comparatively few in number, errors are lightly corrected. But it is when we come to finer subdivisions, and have to disentangle the ultimate twigs of the *tree* of life, that the difficulties become almost insuperable, and if we are to attempt it, it behoves us to work according to the most accurate and methodical system we can devise.

Now in all sorting-out processes it is obvious that we must fix upon a unit. We must then select these units and lay them together in groups, and these groups again into groups of a higher order, and so on. Hence the first question which confronts us is, What unit shall we adopt for our attempts to classify the organic world? The case is of course a specially difficult one because the variations are so multitudinous, and of so many different degrees of value, vast numbers being excessively minute. But the more difficult it is the more perfect our technique must be. The very first step we can take towards this perfection of our technique is to see that our units have some actual demonstrable existence, and the next that they can be fairly accurately defined. Unless these conditions are fulfilled one would fancy that all our attempts at sorting were but waste labour.

Now with this insight into the needs of the case it is not difficult to see why the Linnean system, however excellent it may be as a formula for a natural classification when we have worked such a classification out, is useless as a technique for the work itself. Its unit is no longer even approximately definable, it is in fact one of the very divisions which we wish to discover, and of all the divisions that one which is the most difficult to discover. I refer to the ideal genetic group called 'species,' which is the unit of classification under our present system. Borrowed at first as the fixed created species of the theologian and of common observation, these species have served as the most natural units to be marshalled into genera and so on. It is true that varieties early came in as difficulties, so slight however as to be negligible. So long as we believed in fixed species, they could be regarded as accidental appendages of one kind or another which did not affect the existence of the unit. The species was the fundamental fact.

It is not uninteresting to enquire why, so long after we all recognise that there is no such thing as a fixed species, and that all is in a state of flux, we have not altered our unit. I explain this as due to the fact that universal variability has been regarded too theoretically. In practice it may be argued we see only what our fathers saw, that is, fixed species. In practice, therefore, there

seemed to be no need to alter our methods. But is it true—that we *still* only see apparently fixed species? Is it not a fact that, as our collections grow and as our work becomes more exact and our comparisons closer, variability is being revealed to us all along the line until it is folly to continue to work as if we could ignore it. Only continue your collection—say of Lepidoptera—and you will find, as the collection of my friend Mr Elwes has amply demonstrated, that the 'species' melt away into apparently endless 'varieties.' In the stony Corals, the variation is so great that any attempt at genetic grouping into species can only be the purest guesswork. No two workers would do it alike. Our types have become ridiculous. They are not the types of anything in nature: their special value is purely historical; they were the forms accidentally first described. In the great majority of animal groups, then, variability is being revealed in some cases but slowly, but in others the moment any competent person undertakes to describe a really large collection. On all sides, indeed, we are hearing demands for some more elastic system of work than that supplied us by the Linnean binominal species name. The complexities of the organic world due to the procession of life through time are clearly too great to be investigated by so clumsy and indefinable a unit.

It is I think, then, obvious that we must cease to use a purely ideal genetic group such as the Linnean 'species' as a unit for work. We must have one which more nearly fulfils the requirements already laid down, it must at least be an ascertainable fact of Nature. One such unit we have and as far as I can see only one, viz. that supplied us by the form, each form being an aggregate of structural characters regarded in the abstract. The different forms assumed by living matter are the units with which we must work.

Now this conclusion that the form is the only possible unit for accurate scientific work is not only what my practical work with the stony Corals resulted in but it is what we might have arrived at theoretically. For only the form can be the unit in evolutionary classification. Organic evolution means nothing more than the gradual modification of relatively simpler forms in various directions resulting in the production of relatively more complex and specialised forms. The forms as such, that is, in the abstract, are the only important things for the evolutionist and morphologist. It is not easy to keep this clear and to convince others of it. The fact that the individual concrete forms of life possess the power of almost exactly reproducing themselves, sometimes through many generations and over great areas, somewhat dazzles us. These great armies of living beings, reproducing themselves so far as we can see exactly, have natur-

ally imposed upon our forefathers and still impose upon us, whereas it is the form in the abstract which they reproduce which alone concerns the evolutionist and it is a matter of absolute indifference to him, however interesting it may be to the biologist, whether any particular form has been reproduced only twice or millions of times. Evolutionary classification has to endeavour to arrange each particular form in its order of development above the forms from which it can be derived and below those to which it has itself given rise.

And here in passing I should like to remark that I am only developing the teaching of my honoured friend and teacher Prof. Ernst Haeckel, who 30 years ago in his *Biologie der Kalk Schwämme* insisted that classification was worthless unless based upon profound morphological study. It is the neglect of this teaching which has made modern Systematic Zoology what Dr Dohrn calls it[1], an Augean stable.

Now some may maintain that they really mean the abstract form when they say the species, thereby maintaining that the two can be practically treated as one and the same thing. This, however, is only true in the few accidental cases of exceptionable stability combined with complete morphological isolation, so that the form features of any single individual actually represent those of the whole group. But we cannot allow these rare cases to supply us with a rule of work for all the rest of the organic world. We know that no genetic group is absolutely stable and that, therefore, the form of no single individual can be arbitrarily selected as a type of the rest without covering up and hiding exactly what it is the aim of our classification eventually to reveal to us, viz. the variations in the form features and their evolutionary interrelationships. The present plan of grouping a number of individuals which appear only slightly to differ from one another round a type would be vicious enough even if our types were selected after a careful survey of all the known facts,—at least until the available facts are very much more numerous than they now are. But, instead of our types being even selected, they are purely arbitrary; the specimen which is accidentally first described becomes a type. This type is given special prominence and other specimens are more or less indiscriminately and solely according to the subjective feelings—or better, the morphological insight—of the individual worker associated with it as mere varieties. In the case of the more stable groups and of those easier to examine, the mischief done is not so great as it is in the case of the less stable and more difficult groups. There, the resulting confusion can not be described; it has to be experienced[2].

[1] In a private letter to Mr F. Jeffrey Bell.

[2] I need hardly remind the reader that the description of every apparently varying

I do not know who is the author of the saying, but systematic work has been described as "the taking of snapshots at the procession of life." That is exactly my ideal, but I deny that our 'species' *are* snapshots at any procession of Nature. Let us by all means snapshot the forms of life which come within our range and leave it to future generations to arrange the 'procession' as our labour shall slowly reconstruct it. But when we place one form called a 'type' at the head, and trail others anyhow after it as varieties, our "snapshot of the procession" becomes a 'fake' unpardonable in the domain of science.

I can, however, understand a philosophical difficulty being here thrown in my way. It is this. We cannot possibly deal with all the finer shades of variation; we have neither eyes to see them nor instruments to measure, nor means to test their value or to unravel the complexities of concomitant variations in the more specialized organisms. We shall still have to 'lump' the forms together and our method will after all be the same as that which is now adopted. While this may be verbally true, for our most perfect method can only be an approximation, it is practically false. I am contending that the doctrine of evolution demands that we should take the varying forms assumed by living organisms as the units of our classification. It is useless to say, "But there are shades and complexity of variation which our powers of observation are not exact enough to enable us to appreciate." All we have to do is to describe and designate those forms whose differences we can now appreciate. We do not really know what powers of observation we may not acquire so soon as we deliberately adopt this as our method of work. We already distinguish forms which our forefathers 'lumped' together and our systematists are describing thousands of apparently new forms almost every year, although unfortunately they continue to group them blindly into genetic 'species' with their varieties, thereby making assumptions which are not only useless but even seriously impede progress.

The different forms, then, which we *can* distinguish must be our units, and we want a formula which will enable us to designate them. We need not be appalled at the idea of having to try to group the almost infinite number of different forms assumed by living matter into evolutionary series. For unless our collections are large enough to reveal to us series, we have nothing to do but to record the forms and what little fragments of series we

form as the type of a new "species" is universally denounced as a useless multiplication of "species." And yet if the "species" is to continue to be the unit of work I cannot see what else a really conscientious worker can do. By conscientious worker I mean one who will not guess, and lays no claim to having any special "feeling" for species.

happen to discover. Time and further work will show us the completed chains and reveal to us where forms are diverging, where special structures having apparently reached a climax beyond which they cannot go remain stationary until they die down. And these completed chains will show us how the forms can be genetically grouped and for the first time tell us what a 'species' really is.

The difficulty which we have just been considering, arising from the existence of shades of variation so fine that the units I am advocating will be in comparison as coarse and clumsy as the present Linnean types of species, naturally leads us to notice a method of dealing with fine variations which is daily becoming more popular. I refer to the mathematical treatment of fine variations in the sizes of definite selected structures. This method of work is so far removed from the present rough and ready use of hypothetical genetic groups as units with which to start our classification that we are not surprised to hear Professor Davenport declare, from its standpoint, that the Linnean system of naming is doomed[1]. The mass of systematists simply stand aghast; that the work is excellent few, I suppose, doubt, but where can it be made to hinge on with the present work? It seems almost to belong to another sphere. I think, however, after what has been said, that we can easily see where this work will fall into its place. It also takes the 'form' for its unit; every single specimen, indeed, no matter how slightly it differ from its neighbours, counts as one of equal value for the work as any other. But the method belongs to a different department of research from the system of sorting forms for the purposes of arranging them into series which I am advocating. It is in reality a collateral study, which may or may not be helpful to classification. The arrangement of the varying forms of life as so many distinct units into evolutionary series is the objective of the systematist of the future. I do not see how this will need the aid of mathematical formulae. It seems to me only to require careful comparisons of structure, that is, of the relative dispositions and forms of the component parts, rather than of mere quantitative differences in parts otherwise similar. The mathematician, as far as I can see, has no part nor lot in the matter, it belongs solely to the comparative anatomist or morphologist[2]. But as soon as the new unit of classification has been adopted, then the

[1] See *Nature*, Oct. 5, 1901.

[2] I have already endeavoured to give samples of analyses of structure with the object of elucidating lines of evolution. They are not, alas, models but poor samples; e.g. in my '*Apodidae*' (Nature Series, 1892), but more philosophically in the *Comparative Morphology of the Galeodidae* (Trans. Linnean Society, VI. 1896), and I have a third work on the *Evolution of the Coral Skeleton* which is in course of preparation.

method of work we are referring to will discover its true place and receive new life and stimulus from the discovery. This particular study does not deal with our series of units as series but works within the individual unit, *i.e.* with its numberless reproductions; and it may be that a mathematical study of the fine quantitative variations among the representatives of a unit may reveal to us the laws which link one unit with the next above it and the next below it, or explain the divergence of a unit into two or three new ones. We shall only *know* when we can compare its results with those obtained by the comparative method. I feel confident, indeed, that all along the line, biological study will be both controlled and stimulated to new efforts, and to new enquiries, as soon as our systematic work supplies us only with facts, and facts arranged ready for further research. For instance, as soon as we can leave the 'species' to take care of themselves and have forms as the main objects of systematic work, and collect and arrange and study these, observing the surroundings in which they are produced to get all the knowledge we can about them and about the causes of their differences, we shall, it seems to me, be laying the best foundations for the study of evolution and of the laws and causes of variation. Without wishing to make any rash prophecy, it really seems to me as if the change of the unit of classification here advocated by confining the work of the systematist strictly to the facts of Nature, would stimulate Biology almost as much as it was stimulated a century and a half ago by the original adoption of the Linnean system itself.

But, leaving these ideal advantages to be gained by the system advocated on one side, I only wish to emphasise the advance which the science of systematic biology must make as soon as we have a real instead of an ideal unit of classification. It is evident that with a symbol for the designation of each varying form, all our systematic work can, from its very first step to its last, be made, relatively speaking, exact and, so far as it goes, constructive. We shall be simply accumulating the facts out of which definitive classifications can be slowly built up. We need establish no more hypothetical 'species' for the perplexing of the next student; as already stated the facts will themselves reveal the true species in process of time. And then, but not till then, we shall be able as far as I see to name such species in the usual way with the Linnean binominal formula.

It is, then, primarily to this demand for a change in the unit of classification for the purposes of work that I wish specially to draw attention. The question as to what symbol shall be used for the new unit is quite a different one. I have described below a system which, with the assistance of my friend Mr Jeffrey Bell, I have already elaborated for the purpose of working out the

stony corals. I have now worked long enough with it to be convinced not only that it is a workable symbol but that it is one which promises to be an instrument of work of real value. But my experience has taught me that it is almost useless at this stage to discuss, and quite useless to wrangle over, mere suggestions. The first thing is to establish the need for the change; it will then not be long before we have several earnest attempts made to discover a satisfactory formula. Time will select the best, the best being that which, in practice, satisfies most completely the new demands.

Supplement.

(*A brief description of the method of designating the varying forms suggested by the author; for some important details he is indebted to Professor Jeffrey Bell, of the British Museum, and to Professor Weldon of Oxford.*)

A few examples will serve best to explain the system.

Goniopora New-Guinea $\frac{1}{2}$.

[Port of Doreh, north-west corner of Great Geelvink Bay, Dutch New-Guinea; voy. de L'Astrolabe; Paris Mus.]

Syn. Goniopora pedunculata Q. & G. etc., etc.

Goniopora Tonga Islands $\frac{2}{3}$.

[Tongatabu. Tonga Islands; coll. J. J. Lister; Brit. Mus.]

Goniopora Vicenza $\frac{1}{11}$.

[S. Urbano, near Montecchio-Maggiore, Vicenza; Eocene "Calcaria Grossolana".]

Syn. Porites ramosa, Catullo [non Reuss] Corals of the Venetian Alps etc., etc.

The first line in each case is the *designation*, for the purposes of naming and reference. It consists of (1) the genus name, (2) the district, in which the locality occurs, (3) numbers arranged like a fraction.

The genus name is quite suitable for the corals, at least in my own case, because as I am practically monographing the whole group I can satisfy myself that the genus is fairly stable. I find it is the recently established genera which cannot be relied upon, and which have to be suppressed as our collections increase. An old genus, if still accepted, and from which other new genera have been separated, is generally safe. As a rule, however, if there were great doubts, we might adopt some higher divisional name, for instance the family, with the genus suggested with a

note of interrogation. Our aim is simply to record the form with a designation which expresses only what we can ascertain about the facts.

To this generic name [or the first, *i.e.* lowest divisional name of which we feel sure] is added the district in which the locality occurs. This we shall see when we come to describe the method of work is a most important point. In the meantime we may note that a geographical distinguishing name is better than a morphological name, which is nearly always misleading, for it emphasises a character the value of which we know nothing about; while purely trivial names would be, as I shall presently show, mere encumbrances.

The Numerator of the fraction means the distinguishing number of the form from the district. It might have been A, B, C, running on after 26 to α, β, γ. This number (or letter) never changes and forms part of the fixed designation of the *form*, so long as we need to treat it as an isolated unit.

The Denominator is not a necessary part of the designation but is added to give a little more information. It indicates the number of forms known from the district up to date. This will have many uses, but it need not be regarded as essential for the purposes of reference. We may regard the designation of the form here proposed as practically the same as that adopted by a good collector who is not yet in a position to classify his finds; this is exactly the position we are in, with regard at least to the difficult and little known groups.

In the second line, and not forming any part of the name or designation, come the exact locality, the geological formation etc. if the specimen is fossil, and other information such as name of Collector, or Expedition, or Museum where the specimen or specimens are preserved.

In the third line come the references to earlier descriptions either of the same, or of what appear to have been the same, form.

Then would follow the detailed description.

These descriptions with the figures will form the bulk of the work, and in the case of my work on the corals they will constitute the *Catalogue* of the Collection. They will be arranged geographically, that is, as the forms appear to be distributed over the surface of the planet. This at any rate cannot be wrong, whereas the usual plan of arranging them according to variation of structure may be. For instance, I arranged the Turbinarians in the second vol. of the Madreporarian Catalogue according to their growth form. I am now told by Mr Pace, that his observations on a reef where Turbinarians were specially plentiful, convinced him that the form depended largely upon the presence or absence

of mud or sand. While of course I do not agree that this can be the only factor determining the form, I have taken the warning to heart and recognize that it is safer in this matter of arrangement to assume nothing but to adhere only to the ascertainable facts. The geographical arrangement then is all that is left to us.

So far then in such a Catalogue or Monograph we have only recorded the facts, and now we come to the first line of work upon these facts. This consists in making Tables. The first table would be a simple list of the recorded forms, this owing to the geographical component of the designations, gives at a glance the distribution, and a few other important facts such as the relative abundance or scarcity of the genus in certain parts of the world, or at least so far as our knowledge extends up to date.

Then should follow a series of tables none of them really much more difficult to construct than the first. These would take *all the important structural features in succession* and the forms should be arranged according to each one in turn. In this way we should gradually get on the track of many important truths, we should for instance at once begin to see whether certain definite morphological features followed definite geographical lines. Indeed with such a geographical chart of the genus, we should have at our disposal a permanent basis on which to lay down any experimental method of classification we like, even one suggested by characters drawn from internal anatomy hitherto necessarily ignored by the systematists. No new line suggested by variations hitherto unnoticed need be subversive as they too often are now; all good methods of comparison can only be contributory to the final result. Along these lines there can be little doubt but that interesting and instructive series will come to light gradually of themselves, the interpretation of which will be a new stimulus to research. We can now I think see how necessary it is to have geographical designations, any others would multiply the labour uselessly.

This system is obviously not intended to be a rival to the Linnean system of *naming the genetic divisions* of the organic kingdom. The Linnean system stands for the names of the orders, families, genera, and it will stand also for the species when we know what they are. What we are suggesting is not therefore a new system of naming *divisions* at all, but a new technique for systematic work to enable us to discover the divisions, which we can then name according to the Linnean formula. The present exclusive use of the Linnean system which is only applicable to a final definitive classification compels us now to guess blindly at what we can only discover by patient research.

Notes on the Binney Collection of Coal-Measure Plants. Part III. *The Type-Specimens of Lyginodendron Oldhamium* (BINNEY). By E. A. NEWELL ARBER, B.A., Trinity College, University Demonstrator in Palæobotany.

[*Received* 25 November 1901.]

In a paper published in the *Proceedings* of the Manchester Literary and Philosophical Society for 1866, the late E. W. Binney gave a brief account of the structure of a new fossil-plant, which he named *Dadoxylon Oldhamium*. Binney's description will be found quoted at some length by Williamson[1], in his fourth memoir on fossil plants. In the same memoir, Williamson gives reasons for transferring Binney's plant to the genus *Lyginodendron*. The structure of this plant has been worked out in great detail by Williamson[2], and also by Williamson and Scott[3]; and to these authors we are indebted for a singularly complete knowledge of one of the most important among Palæobotanical types.

Binney did not figure any of his sections of this plant. His collection was presented to the Woodwardian Museum, Cambridge, in 1892. Unfortunately very few of the sections have any record as to their nature, or origin, and the often difficult task of identifying type, and figured specimens, has gone on ever since. The collection contains four sections, two transverse and two longitudinal, which are undoubtedly those named by Binney *Dadoxylon Oldhamium*; they are in fact sections of the type specimen (apparently not now in the collection) of the plant generally known as *Lyginodendron Oldhamium* (Binney), or by some authors as *Lyginopteris Oldhamia*. The two transverse sections[4], cut from the same specimen at different heights, agree exactly with Binney's description. They are, as he states, ½ inch in diameter, and show an apparent line of separation between the medullary region and the wood. There are only three other transverse sections in the collection, of which one does not show all the tissues described by Binney, and the other two are much larger (1 inch or more in diameter), and do not show "the intervening spaces vertically" between the medulla and the wood. These three specimens were probably acquired later than the type, for Binney says that, at that time, he had only one specimen of

[1] Williamson, *Phil. Trans.* p. 377, 1873.
[2] Williamson, *ibid.* and Part XVII. p. 89, 1890.
[3] Williamson and Scott, Part III. *Phil. Trans.* p. 703, 1896.
[4] These were recently thinned, and covered, by Mr Lomax.

the plant. The corresponding radial, and tangential longitudinal sections, of the type, were easily distinguished by measurements, in comparison with the transverse sections.

The structure of *Lyginodendron Oldhamium* is now so well known, that it would be difficult to add anything to our knowledge of this plant. The opportunity has however been taken here to figure some of Binney's sections, and, at the same time, to point out some of the more interesting features in the structure of these important types.

Lyginodendron Oldhamium (Binney). Type specimens. Woodwardian Museum, Cambridge. Binney Collection. Nos. 179 and 180 (T. S.), No. 181 (R. L. S.), and No. 182 (T. L. S.).

Locality. Calcareous nodule in the Upper Foot Coal, 15 yds. above the Gannister, at Moorside, near Oldham, Lancashire.

1866. *Dadoxylon Oldhamium.* Binney, Proc. Lit. and Phil. Soc. Manchester, 1866.

1873. *Dictyoxylon Oldhamium.* Williamson, Part IV. Phil. Trans., 1873.

1873. *Lyginodendron Oldhamium.* Williamson, ibid.

1890. *Lyginodendron Oldhamium.* Williamson, Part XVII. Phil. Trans., 1890.

1896. *Lyginodendron Oldhamium.* Williamson and Scott, Part III. Phil. Trans., 1896.

1899. *Lyginopteris Oldhamiana.* Potoniè, "Lehrbuch der Pflanzenpaleontologie," Berlin, p. 170.

1900. *Lyginopteris Oldhamia.* Zeiller, "Éléments de Paléobotanique." Text fig. 96, p. 127.

1900. *Lyginodendron Oldhamium.* Scott, "Studies in Fossil Botany." London, p. 308.

Description of the Specimens.

Slide 179. *Text-figure* I.

A transverse section cut from the upper portion of the specimen, as the position of the leaf-traces, compared with that which they occupy in Slide 180, shows. The stem has a diameter of 1·3 cm., and is almost circular.

The medulla is 1·5 mm. in diameter, and the tissues are much decomposed. Most of the thin-walled elements have disappeared, forming a number of spaces, especially near the periphery of the medulla, as is clearly seen in the figure. From the occurrence of these spaces, Binney concluded that the medulla was similar to that of Dadoxylon. It is hardly necessary to add that this apparent separation between the medulla and the wood is entirely due to bad preservation. Sclerotic nests with thick walls, and darkly coloured contents, are numerous in this region.

There are five primary mesarch bundles in which, as usual, the centripedal xylem is more developed than the centrifugal.

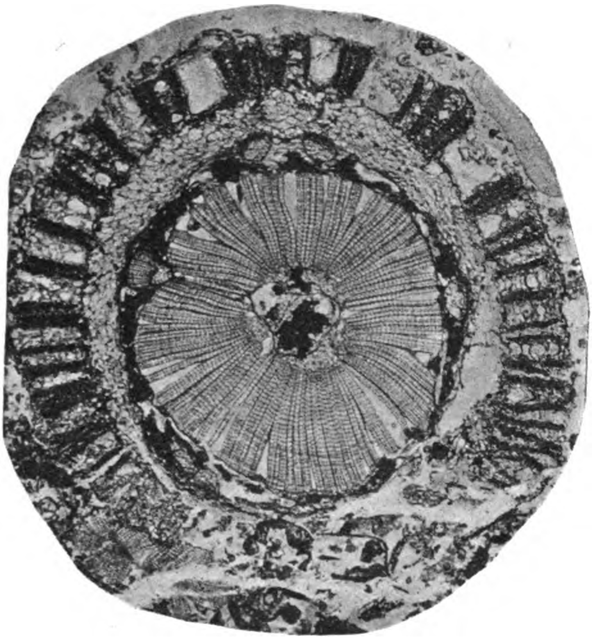


FIGURE I. *Lyginodendron Oldhamium* (Binney). Transverse section of the stem × 5½. Photo. by Tams, Cambridge.

The broad ring of secondary wood has a diameter of 2·5—3 mm., with about 30 elements in the ray. The tissue of the medullary rays has largely perished. External to the secondary wood, traces of a cambium, and of phloem, may here and there be found. The pericycle and inner cortex are badly preserved.

There are five cortical leaf-traces, three of which have divided, or begun to divide. Another, to which reference will be made in the next slide, is seen passing through the external margin of the wood. It still retains much of its secondary wood. Some tangential elongation of the parenchyma of the pericycle has taken place, for where the trace has divided, the two parts are separated. The pericycle is remarkable as containing unusually large groups of sclerotic nests.

The outer cortex has the usual dictyoxyloid structure, but much of the thin-walled tissue has disappeared, and secondary crystallisation taken place. The sclerenchymatous strands con-

sist of irregular rows of cells, and are distinctly broader than in many specimens. The external layer is almost entirely absent.
The structure, as a whole, is typical of *Lyginodendron Oldhamium* (Binney), but the preservation is not of the best.

Slide 180. *Text-figure* II.

This section was probably cut less than an inch, *i.e.* less than the length of an internode below section 179. The structure is essentially similar to that of the former slide, but is interesting as showing a leaf-trace passing through the secondary wood.

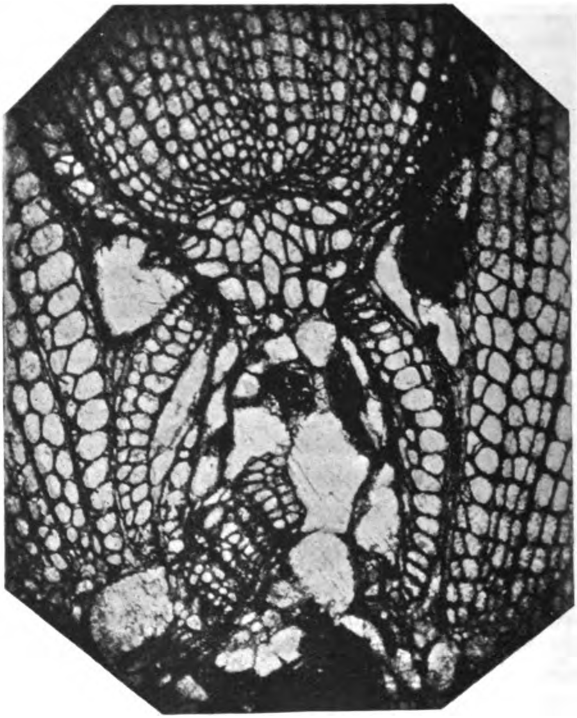


FIGURE II. *Lyginodendron Oldhamium* (Binney). Transverse section of the stem, showing a leaf-trace passing up through the trace-gap in the secondary wood. ×40. Photo. by Tams, Cambridge.

This trace, which is shown considerably enlarged in figure II, seems to have had some difficulty in forcing its way up through the trace-gap, owing to the pressure of the secondary wood. It has left a portion of its own secondary wood behind it on each

side. That this secondary wood belongs to the trace is shown by the length of the row of woody elements, the small tracheides towards the periphery, and the signs of rupture at the base of the trace.

In this section an adventitious root is also seen passing through the cortex.

Slide 181.

A radial longitudinal section, showing the structure of the medulla and the wood. In the medulla, the sclerotic nests are seen to be several cells thick. The section also passes through a primary bundle. The medullary rays are muriform, and of considerable height. The tracheides of the secondary wood show the usual multiseriate reticulation of bordered pits on the radial walls. Binney[1] speaks of these, as being "divided by oblique and transverse dissepiments placed at great distances." These divisions are apparently due to cracks, or other imperfections of the preservation, and I have not observed in the specimen any certain indication of definite transverse walls. The section is bounded at one end by a portion of ill-preserved periderm, and at the other by a series of sclerotic nests in the pericycle.

Slide 182.

The tangential longitudinal section passes through the secondary wood, and a parenchymatous band of the outer cortex. The tracheides are somewhat flexuous, and bear here and there reticulate pittings. Numerous transverse partitions are seen, as in the radial section, but these again are due to imperfect preservation. The medullary rays consist of 1—4 rows of small rounded cells. Occasionally a chain of much larger, oval, or rounded cavities, is found occupying the position of the ray, and often surrounded by the ordinary ray cells. These "much larger cellules," which Binney notices, are no doubt due to the disruption of more than one series of the cells of the ray.

In the pericycle, the section has passed through a series of sclerotic nests. The inner cortex, and the parenchymatous zone of the outer cortex, through which the section passes, are very badly preserved, only occasional fragments of the cells being seen.

[1] Binney, in Williamson, *Phil. Trans.* 1873, p. 377.

On the Negative Radiation from Hot Platinum. By O. W. RICHARDSON, B.A., Coutts Trotter Student, Trinity College.

[*Read* 25 November 1901.]

Ever since the original discovery by Messrs Elster and Geitel[1] that the air in the neighbourhood of a hot metal discharged electricity, several physicists have investigated the laws of this phenomenon. Most of these investigations have been concerned with the effect in gases at pressures approximately atmospheric, when the charge on the gas may be either positive or negative, according to the gas used. Professor McClelland[2], however, was able to show that platinum at high temperatures produced a negative charge in all surrounding gases. The effect was more marked the lower the pressure; the discharge also increases very rapidly with the temperature of the wire, as the figures given by Professor McClelland indicate. The fact that the carriers of the negative electrification are charged particles was proved by Professor Thomson[3], who showed that they were deflected by a magnetic field.

The present investigation was undertaken with the idea that in the negative radiation at high temperatures the phenomenon of conductivity produced by metals took its simplest form. This idea seemed legitimate since only in this case is the conductivity to any extent independent of the surrounding gas.

The following experiments were made to determine the saturation current from the wire at various temperatures, since this appeared to be the most fruitful method of attack.

With regard to the theory of the phenomena, the results can best be explained on the corpuscular theory of conduction in metals. According to that view a metal is to be considered as a sponge-like structure of atoms and positive ions with negative ions or corpuscles moving freely throughout the mass. The mean free path of the corpuscles varies from about 10^{-4} cms. in the case of bismuth to 10^{-6} cms. in the case of most other metals. It is thus of the same order as that of a molecule in air at atmospheric pressure. The time during which the corpuscles are colliding is

[1] Elster and Geitel, *Wied. Ann.* xvi. 1882, p. 193, and later papers.
[2] J. A. McClelland, *Proc. Camb. Phil. Soc.* x. 1900, p. 241.
[3] J. J. Thomson, *Phil. Mag.* xliv. 1897, p. 203.

therefore small compared with that in which they move freely, so that to determine their equilibrium state we can apply the methods of the kinetic theory of gases. In this way we find the corpuscles have a distribution of velocity which follows the Boltzmann-Maxwell Law, and their average energy is the same as that of a molecule of gas at the same temperature. If we consider what happens at the surface of the metal we must suppose there is here a discontinuity in the potential which prevents the negative ions escaping. It is conceivable that as the temperature is raised, some of the corpuscles will acquire sufficient velocity to enable them to overcome this discontinuity in the potential. Since the number of corpuscles with velocity components between u, v, w and $u+du$, $v+dv$, $w+dw$ in unit volume is

$$n\left(\frac{km}{\pi}\right)^{\frac{3}{2}} e^{-km(u^2+v^2+w^2)}\,du\,dv\,dw,$$

where n is the total number of corpuscles in unit volume, $3/4k$ is the energy of a corpuscle and m is its mass; the number having these velocity components which strike unit surface perpendicular to u per second is

$$n\left(\frac{km}{\pi}\right)^{\frac{3}{2}} u e^{-km(u^2+v^2+w^2)}\,du\,dv\,dw.$$

If Φ is the work done by a corpuscle in passing through the surface layer, the number which escape from unit area of the metal surface per second is given by

$$N=\int_{\sqrt{\frac{2}{m}\Phi}}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} n\left(\frac{km}{\pi}\right)^{\frac{3}{2}} u e^{-km(u^2+v^2+w^2)}\,du\,dv\,dw$$

$$=\frac{n}{2}(km\pi)^{-\frac{1}{2}}e^{-2k\Phi}=n\sqrt{\frac{R\theta}{2m\pi}}\,e^{-\Phi/R\theta},$$

since k is connected with θ the absolute temperature by the relation $k=(2R\theta)^{-1}$, R being the gas constant for a single corpuscle. If then the negative radiation is due to the corpuscles coming out of the metal, the saturation current (s) should obey the law $s=A'\theta^{\frac{1}{2}}e^{-\frac{b}{\theta}}$. This law is fully confirmed by the experiments to be described.

In the experiments the current was measured which passed from a thin platinum wire to an aluminium cylinder surrounding it. To measure the current the cylinder could be put to earth through a sensitive Thomson galvanometer but was, otherwise, insulated. The wire was heated by a steady current of not more

than 1·5 ampères; it was found that the temperature of the wire was practically constant during each observation; the variation (about 5°) being due to gas given off from the wire. The temperature of the wire therefore steadily decreased during each observation, but it was held that by taking the mean of readings immediately before and after each measurement of current from the wire the true temperature would be obtained. The resistance was measured by placing the wire in one arm of a Wheatstone's bridge, the corresponding arm being a thick German silver resistance of 1·7 ohms. This resistance was not sensibly heated by the current which passed through it. The resistance of the other two arms was of the order of 1000 ohms, so that practically the whole of the heating current passed through the wire and the German silver resistance. The heated platinum wire passed axially through the aluminium cylinder, being fixed to two electrodes in the bulb which contained the cylinder. The bulb was connected with a drying apparatus, pump and McLeod gauge. In most of the experiments the pressure was about ·02 mm. but it varied from ·01 to ·16 mm. It was very difficult to keep the pressure down at the higher temperatures owing to the gas given off by the hot wire.

The same Thomson galvanometer was used both to measure the leak and for the Wheatstone's bridge. It had a resistance of 4058 ohms and gave 1 scale division for a current of 7×10^{-10} ampères. In order to use it for both purposes, suitable shunts had to be inserted. The whole of the measuring apparatus was insulated in paraffin so that the potential of the hot wire could be raised to any multiple of 40 volts up to 400. It was found that with 400 volts positive on the wire, there was no observable deflexion of the galvanometer, whereas quite big currents were obtained when the wire was negative. In all cases the saturation current, *i.e.*, the total number of ions given off by the wire, was measured.

To reduce the determinations of resistance of the platinum wire to temperatures use was made of the measurements of the melting points of potassium and sodium sulphates by Messrs Heycock and Neville[1]. The wire was set up in air and its resistance determined first at the ordinary temperature, and afterwards when the smallest possible grain of potassium sulphate placed on it just melted. In this way the resistance for two temperatures differing by about 1000 degrees was obtained, and the temperature corresponding to any other resistance reading could be got by interpolation from the curves given by Professor Callendar[2]. To test the method, the melting point of sodium

[1] Heycock and Neville, *Chem. Soc. Journal*, LXVII. 1895, p. 160.
[2] H. L. Callendar, *Phil. Mag.* XLVIII. 1899, p. 519.

sulphate was determined and no determination was more than 20 degrees from the true value. This agreement was held to be quite good enough for the purpose.

The following observations were made to see how the current from the wire varied with the potential to which it was raised and if a saturating current could be obtained. The potentials were measured with a Weston voltmeter. A constant current was used to heat the wire and it was sought to keep its temperature constant by pumping out the gas as fast as it was given off. The pressure was thus kept at about ·008 mm.; the resistance of the wire is given to show the extent to which the temperature varied. The maximum current was roughly 2×10^{-7} ampères and is given in scale divisions of the galvanometer.

The slightly smaller current at the higher voltages is doubtless due to the fall in the temperature of the wire as shown by

Volts on Circuit	Current in Scale Divisions	Pressure of gas in mm. of mercury	Resistance of wire
− 40	205	·0076	8·752 ohms
− 80	318	·0083	8·767 „
− 120	306	·0072	8·722 „
− 160	315	·0086	8·745 „
− 256	275	·0074	8·696 „
− 370	268	·0088	8·700 „

the resistance column. The table clearly shows that the saturating potential is somewhere between 40 and 80 volts since there is no increase of the current with higher potentials. No current was obtained under these conditions when the wire was positive.

In measuring the variation of the current from the wire with the temperature, a potential of − 120 volts was always put on the wire so as to make sure of the current being saturated. In general the deflexion of the galvanometer decreased with the time, but not more than might be explained by the lowering of temperature produced by the gas which seemed always to be given off from the hot wire. At temperatures below 1400° very little gas was given off and the readings of the galvanometer were quite steady, while at higher temperatures this was not the case. The curves for lower temperatures are therefore much better than

those for higher temperatures. The accompanying table gives a series of observations of the temperature and the current, the results of which are plotted in Diagram I.

Negative Potential of hot wire in volts	Pressure of gas in mm. of mercury	Resistance of hot wire in ohms	Current from wire to cylinder in ampères × 10^{-9}	Temperature of wire in degrees centigrade
123	·023	(1) 8·338 (2) 8·335	2·52	1031
121	·025	(1) 8·438 (2) 8·430	8·28	1058
120	·021	(1) 8·642 (2) 8·625	30·6	1105
120	·025	(1) 8·795 (2) 8·782	100·5	1146
120	·024	(1) 8·894 (2) 8·875	188	1170
120	·028	(1) 8·969 (2) 8·950	300	1190
120	·028	(1) 9·106 (2) 9·088	728	1224
120	·032	(1) 9·163 (2) 9·131	858	1243
120	·032	(1) 9·263 (2) 9·230	1414	1269
120	·037	(1) 9·381 (2) 9·350	2600	1298
120	·044	(1) 9·472 (2) 9·445	4025	1323
120	·063	(1) 9·603 (2) 9·574	11,320[1]	1354
120	·063	(1) 9·925 (2) 9·883	16,740[1]	1445

[1] The current began to be very unsteady here.

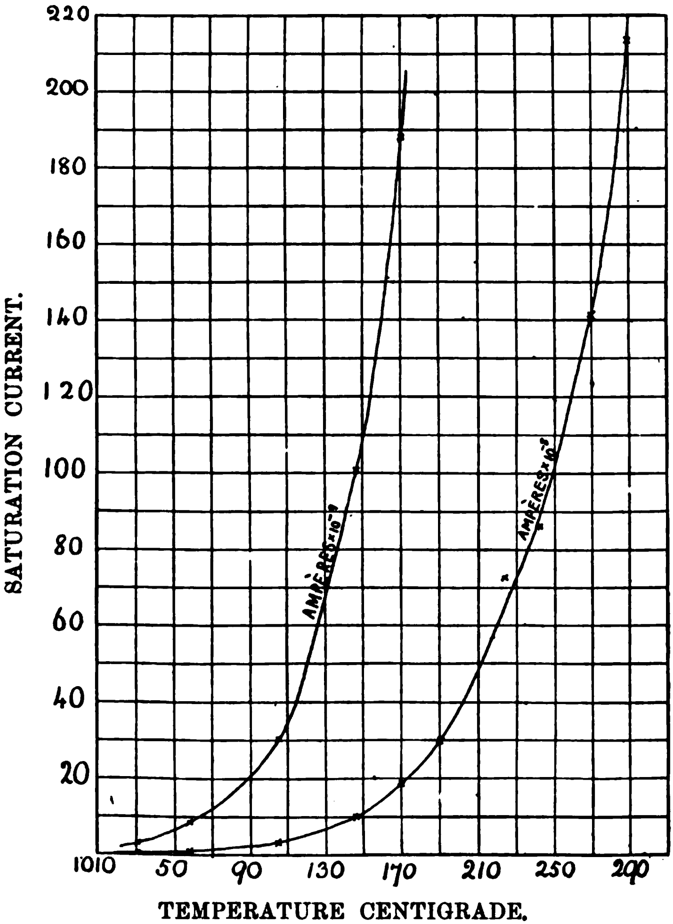


DIAGRAM I.

The subjoined table gives a further series of observations for somewhat higher temperatures. The agreement here is not so good, the individual observations being liable to be 20 % off the curve. Shortly above the highest temperature (1600°) the wire gave so that it was not possible to take the observations further. The most important thing is the relatively enormous current registered, viz., $4{\cdot}18 \times 10^{-4}$ ampères or $1{\cdot}03 \times 10^{-3}$ ampères per sq. cm. of surface.

It remains to consider the relation between the experimental results and the expression obtained theoretically for the number

N of corpuscles shot off from unit area of the wire. This expression is

$$N = n\sqrt{\frac{R}{2m\pi}}\,\theta^{\frac{1}{2}}\,e^{-\phi/R\theta} = A\theta^{\frac{1}{2}}e^{-b/\theta}.$$

NEGATIVE POTENTIAL IN VOLTS	Pressure in mm. of mercury	Resistance of wire in ohms	Saturation Current in ampères $\times 10^{-7}$	Temperature in degrees centigrade
111	·024	9·725 9·718	1·04	1194
111	·044	10·16 10·14	13·62	1298
111	·091	10·63 10·61	116	1419
111	·106	10·79 10·75	578	1449
111	·152	10·95 10·91	1370	1490
111	·18	11·13 11·07	1730	1533
110	·162	11·35 11·355	4180	1599

The number N of corpuscles from unit area is connected with the saturating current in electrostatic units (C) by the relation $C = N\epsilon S$, ϵ being the charge on an ion and S the superficial area of the wire. The simplest way of testing the proposed formula is to take logs, when we obtain the equation

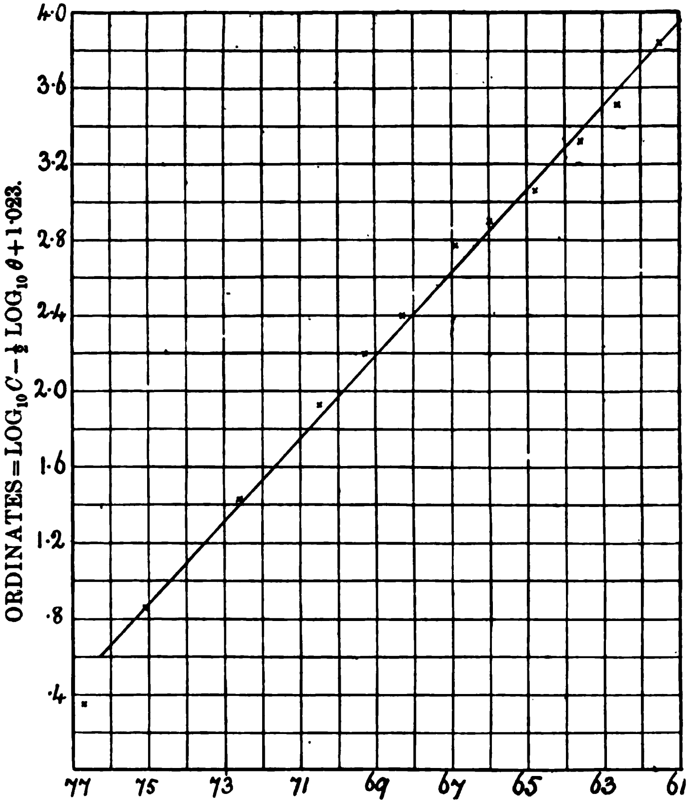
$$\log_{10} C - \log_{10} \epsilon S = \log_{10} A + \tfrac{1}{2}\log_{10}\theta - \frac{b}{2\cdot 303\theta}.$$

If we put, for convenience, $\log_{10} C - \frac{1}{2}\log_{10}\theta - \log_{10} 3 + 1\cdot 5 = y$ and $1/\theta = x_0$ we may write our equation

$$y = a - b_0 x_0,$$

so that plotting the values of y against those of $1/\theta$ should give a straight line. In the accompanying graph the ordinates are the values of $\log_{10} C - \frac{1}{2}\log_{10}\theta + 1\cdot 023$, the abscissae being $1/\theta \times 10^5$. The curve got is very approximately indeed a straight line;

though any variation from strict rectilinearity might be explained by the variation with the temperature of n the number of cor-



ABSCISSAE $= 1/\theta \times 10^5$. $\theta =$ ABSOLUTE TEMPERATURE.

DIAGRAM II.

puscles per c.c. of platinum. We may therefore say with certainty that the main features of the phenomenon are to be represented by a formula of the type $A\theta^{\frac{1}{2}}e^{-b/\theta}$.

Interesting conclusions are also to be drawn from the actual values of the constants themselves. From the constant A we obtain the number n of free corpuscles in a cubic centimetre of solid platinum, since we have the relation $n = \left(\frac{2m\pi}{R}\right)^{\frac{1}{2}} A$. A is obtained by putting corresponding values θ and C in the equation

$$\log_{10} A = \log_{10} C - \tfrac{1}{2} \log_{10} \theta - \log_{10} \cdot 788 + 9\cdot523 + \frac{2\cdot24 \times 10^4}{\theta}.$$

At θ (absolute) $= 1542°$ this gives $A = 1{\cdot}51 \times 10^{26}$. The various constants in the logarithmic equation come from the area of the wire which was ·394 sq. cm. and the value of the charge on an ion which was taken to be 6×10^{-10} electrostatic units. The value of m/R [m being the mass of, and R the gas constant for, one corpuscle] was found to be $= 1{\cdot}204 \times 10^{-11}$. Putting this in the expression for n we find $1{\cdot}3 \times 10^{-21}$ free negative ions in a cubic centimetre of platinum at 1542° absolute. An independent value of n has been obtained by Mr Patterson from experiments made in the Cavendish Laboratory on the change of resistance of platinum in a magnetic field. This when calculated by the method given by Professor Thomson[1] yields $n = 1{\cdot}37 \times 10^{22}$. The agreement of the value found above with this is really very good when one considers the numerous sources of error to which the measurements are liable and that an error of 7 % in the absolute temperature, among other things, would multiply the value of n by ten.

It was thought possible that some regular change of n with the temperature might be observed if the values of n at different temperatures were calculated. The deviations from the mean however seem to be to a great extent purely irregular as is shown by the following data, calculated from the first table:

No. of ions per c.c. of platinum	Absolute Temperature
$1{\cdot}2 \times 10^{21}$	1304
$1{\cdot}7 \times 10^{21}$	1331
$1{\cdot}8 \times 10^{21}$	1378
$1{\cdot}8 \times 10^{21}$	1419
$1{\cdot}95 \times 10^{21}$	1443
$1{\cdot}65 \times 10^{21}$	1463
$2{\cdot}0 \times 10^{21}$	1497
$1{\cdot}5 \times 10^{21}$	1516
$1{\cdot}3 \times 10^{21}$	1542
$1{\cdot}25 \times 10^{21}$	1571
$1{\cdot}2 \times 10^{21}$	1596

The numbers in the second table yield similarly:

$\cdot 43 \times 10^{21}$	1467
$\cdot 58 \times 10^{21}$	1571
$\cdot 48 \times 10^{21}$	1692
$1{\cdot}45 \times 10^{21}$	1722
$1{\cdot}55 \times 10^{21}$	1763
$1{\cdot}1 \times 10^{21}$	1806
$\cdot 98 \times 10^{21}$	1872

[1] J. J. Thomson, *Rapports présentés au Congrès International de Physique*, III. p. 138, Paris, 1900.

These figures are at any rate interesting as indicating the order of agreement between the different observations. As a matter of fact the probable error of any observation is much greater than its deviation from the mean, owing to sources of error which affect all the observations, so that one has to be very careful in drawing conclusions from them.

The signification of the constant $b = \frac{\Phi}{R}$ which occurs in the exponential factor is equally important, since Φ is the work done by an ion in overcoming the discontinuity in the potential at the surface layer. We obtain b from the equation

$$b = \frac{\log_e \frac{C}{C'} - \frac{1}{2}\log_e \frac{\theta}{\theta'}}{\frac{1}{\theta'} - \frac{1}{\theta}},$$

where C, C' and θ, θ' are corresponding currents and absolute temperatures. Substituting the values of C and C' for $\theta = 1571$, $\theta' = 1378$ respectively we get the average value of b from 1378 to 1571 absolute as $4{\cdot}93 \times 10^4$. Since R is equal to $\left(\frac{2}{1{\cdot}204}\right) \times 10^{-16}$ we have $\Phi = 4{\cdot}93 \times \frac{2}{1{\cdot}204} \times 10^{-12} = \epsilon\delta\phi$, where ϵ is the charge on an ion and $\delta\phi$ is the discontinuity in the potential at the surface of the metal. From this we obtain $\delta\phi = 1{\cdot}365 \times 10^{-2}$ electrostatic units

$$= 4{\cdot}1 \text{ volts}.$$

This is therefore the discontinuity in the potential at a platinum-vacuum surface and it is of the right order to give the contact E.M.F. as the difference of its value for different metals. The author intends to make further experiments on other conductors, notably iron and carbon, with the hope of confirming this part of the theory.

The preceding theory only claims to represent the main features of the radiation from a hot platinum wire. In the first place it is evident that the number of ions emitted per unit area cannot be regarded in the strict sense as an exact function of the temperature. This number will evidently be altered by anything which changes the state of the surface, so that we should expect the current to depend on the previous treatment of the wire. Variations under this head do certainly seem to occur, but the amount of disturbance produced by them remains to be investigated.

In conclusion I wish to thank Professor Thomson for numerous suggestions during the course of this work.

On the Action of Incandescent Metals in producing Electric Conductivity in Gases. By J. A. McClelland, M.A., Trinity College; Fellow of the Royal University of Ireland; Professor of Natural Philosophy in University College, Dublin.

[*Read* 25 November 1901.]

1. In this paper an account is given of some experiments on the nature of the conductivity produced in gases by the action of incandescent metals. The conductivity can be shown to be due to ionisation produced by the hot metal, and in the paper some properties of the carriers of the electricity are studied in air at atmospheric and at lower pressures.

A statement is first given of the results obtained when the gas surrounding the incandescent wire is at atmospheric pressure; the paper then deals with experiments in air at reduced pressures, and here it may be mentioned that when the pressure is sufficiently reduced many important changes occur in the phenomena observed.

Experiments with the Incandescent Wire in Air at Atmospheric Pressure.

2. Many of the results obtained with the incandescent platinum wire in air at atmospheric pressure have already been given in a previous paper "On the Conductivity of Gases from an Arc and from Incandescent Metals[1]," and we shall merely mention them here.

It was shown that the conductivity produced by the incandescent wire is of the nature produced by the ionisation of a gas; the current varies with the E.M.F. in the usual way, and the gas loses all its conductivity when passed between terminals kept at a sufficiently great difference of potential.

The gas taken from near the hot wire discharges a negatively charged body but not a positively charged body if the wire is at a dull red heat, in fact a negative charge is discharged if the wire is only just luminous. The wire must be raised to a higher temperature before the gas from it can discharge a positively charged body.

This shows that as the temperature of the wire is increased we get in the gas from near it positive carriers before we get negative

[1] *Proceedings of the Cambridge Philosophical Society*, Vol. x. Pt. iv.

carriers. When the temperature is sufficiently high we get carriers of both signs, the amount of negative being at first much less than that of positive.

The amount of both positive and negative carriers increases rapidly with the temperature of the wire, and at very high temperatures we get in the gas approximately the same amount of each.

The preponderance of the positive at low temperatures may be due to the ionisation being produced by the hot wire only in a very thin layer of gas close to itself, and the negative ions owing to their very small initial mass are discharged to the wire, the positive coming off to some extent in the gas. When the layer in which ionisation takes place gets thicker as the temperature of the wire is increased, the negative carriers are no longer all discharged to the wire, and when the temperature is sufficiently high the amount of negative discharged in this way is small compared with the total ionisation.

A similar excess of positive at low temperatures is observed with other incandescent metals, as with platinum; iron, German silver, and brass wires gave the same result. Also CO_2 gave a result similar to air.

The velocity under electric force of the negative and positive carriers was determined as described in the paper referred to above. The velocity was found to vary greatly with the temperature of the wire, the velocity diminishing as the temperature of the wire increased. The greater disintegration of the wire at the higher temperatures rendered the conditions more favourable for uncharged masses collecting round the carrier. It was also found that the negative carrier had a greater velocity under a given electric force than the positive, the excess in air being about 20 %.

Ionisation produced by an Incandescent Wire in Air at Reduced Pressure.

3. All the results mentioned above refer to the case where the gas surrounding the wire is at atmospheric pressure.

To experiment at reduced pressures the following apparatus was used:

AA is a glass tube in connection with a mercury air-pump so that the pressure in the tube can be reduced as desired. Inside the tube an aluminium cylinder *C*, 3·8 cms. long and 2 cms. diameter, is placed and supported by a thick wire passing through an ebonite plug at *D* so that the cylinder *C* can be insulated when desired. A fine platinum wire *W* is stretched along the axis of

the cylinder C between two thick copper wires G and G' which pass through ebonite plugs at F and F'. The wires G and G' are connected to the terminals of a battery B of a few large storage cells through a rheostat R, so that any required current can be used to heat the wire W to the desired temperature. The battery B and rheostat R are placed on blocks of paraffin so that the whole circuit in connection with the wire W can be insulated. A

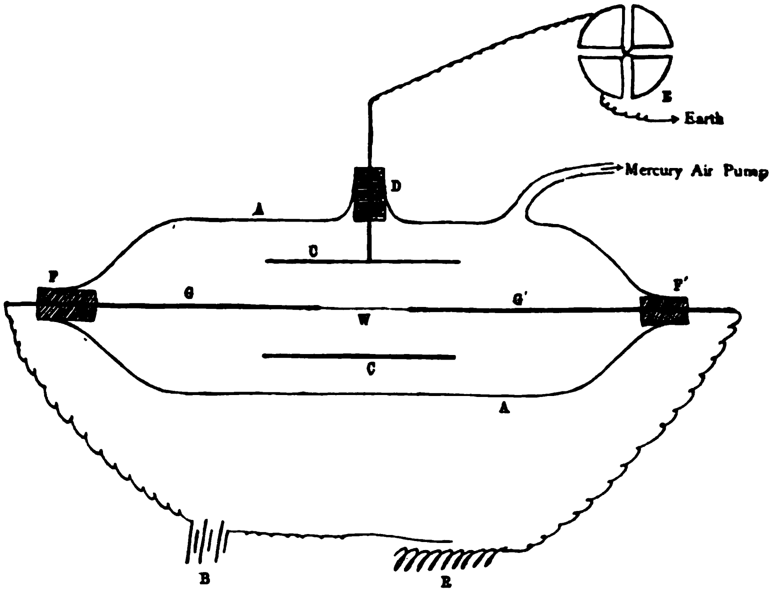


FIG. 1.

second battery of a large number of small storage cells is used to raise this circuit to any required potential. One point of the battery of small cells is put to earth and another point connected to the circuit containing the wire W, so that W is heated by the current from B and at the same time is raised to any potential required. The potential difference between the ends of W is very small compared with its mean potential. The cylinder C is connected to one pair of quadrants of an electrometer, the other pair of quadrants being permanently to earth. The quadrants connected to C are first earthed, then insulated and the time observed until C, with whatever capacity is connected to it, is charged to some definite potential. This gives a measure of the current through the gas between the charged wire W and the cylinder C.

Relation between Current and E.M.F.

4. As the results at moderate pressures are more or less similar to those at atmospheric pressure, we shall go at once to the effects at low pressures. The numbers below show the relation between the current and the E.M.F. at a pressure of two-thirds of a millimetre, the wire being charged negatively.

Potential of the wire in volts	Current in arbitrary units
40	19
80	38
160	61
240	78
280	99
320	135

The curve in Fig. 2 is drawn from these numbers.

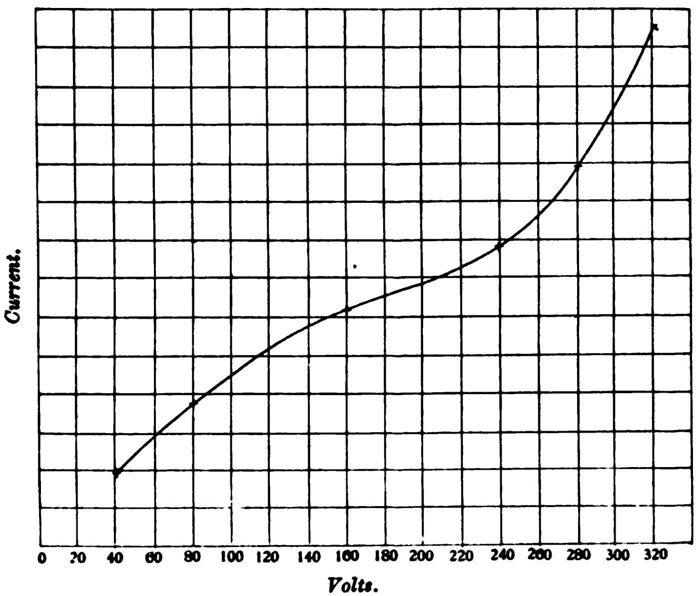


Fig. 2.

The current does not reach a maximum as it does when the

pressure of the gas is atmospheric[1], but after a certain potential difference between the terminals has been reached the current increases rapidly for further increases of potential. The curve shows that the current at first increases proportionately with the E.M.F., then shows signs of reaching a maximum and afterwards increases rapidly.

This second rapid increase of current can easily be explained if we assume that the negative ions travelling from the negatively charged wire to the cylinder C have the power of producing ionisation when they collide with molecules, provided the velocity of the ion is sufficiently great. In the case we are dealing with the E.M.F. has been sufficient to produce such a velocity in the air at a pressure of $\frac{3}{8}$ mm. When the potential difference is sufficiently great to produce the required velocity the secondary ionisation begins and then we have the rapid increase of current. At a pressure of $\frac{3}{8}$ mm. the negative ion has not the large mass travelling with it that it has at atmospheric pressure and again it is moving in a rare medium, so that it acquires a high velocity. Prof. Townsend[2] has used the above theory to explain similar results obtained with the ionisation produced by Röntgen radiation in a gas at low pressure. The numbers given above refer to an experiment in air at a pressure of $\frac{3}{8}$ mm. We have selected the results at that pressure because the secondary ionisation is then more marked. At higher pressures the ions do not so easily attain the necessary velocity, and at very much lower pressures the secondary ionisation is smaller compared with the initial ionisation produced direct by the wire.

If we use a smaller current to heat the wire the secondary ionisation is even more apparent than in the curve given above. The negative ions have then a greater velocity, as we have seen above.

The following numbers refer to such an experiment, the wire being at a lower temperature and a much smaller capacity joined up to the electrometer.

Potential of the wire in volts	Current in arbitrary units
40	12
80	25
120	45
160	91
200	145
280	430

[1] *Proceedings Cambridge Phil. Soc.* Vol. x. Pt. iv.
[2] *Phil. Mag.* Feb. 1901.

The curve in Fig. 3 is plotted from these numbers.

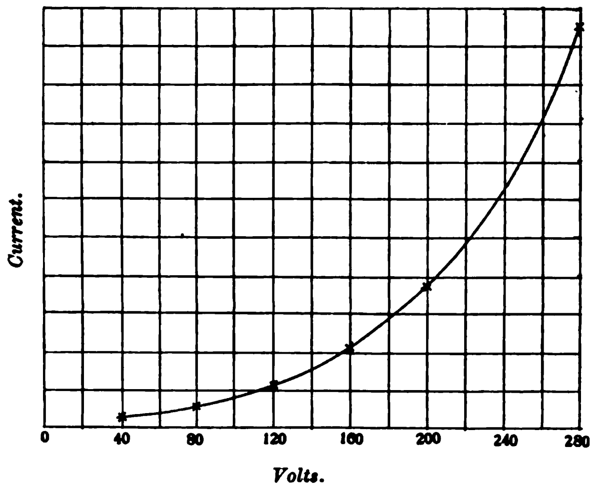


FIG. 3.

Care is taken in the above experiments that the highest E.M.F. used is not in itself sufficient to produce discharge or even to maintain it when started.

We have above ascribed the secondary ionisation to the negative ions alone because the wire probably only produces ionisation close to itself, in which case only negative ions would have moved through the body of the gas. Of course when the secondary ionisation has started positive ions will be travelling through the gas towards the wire. But to account for the above results it would be sufficient to assume that the negative ions by collisions with molecules can produce ionisation, and we know that in a gas at such a pressure the velocity of the negative ion is great, its mass being small.

5. It is found, however, that when the wire is charged positively we get a similar effect. The form of the curve giving the relation between current and E.M.F. in this case again shows a rapid increase of current with E.M.F. after the E.M.F. has reached a certain value.

The following numbers give the result of such an experiment, the wire being charged positively.

Potential of wire in volts	Current in arbitrary units
8	19
18	22
40	24
120	25
240	27
280	29
320	37
360	50

The curve in Fig. 4 is plotted from these numbers.

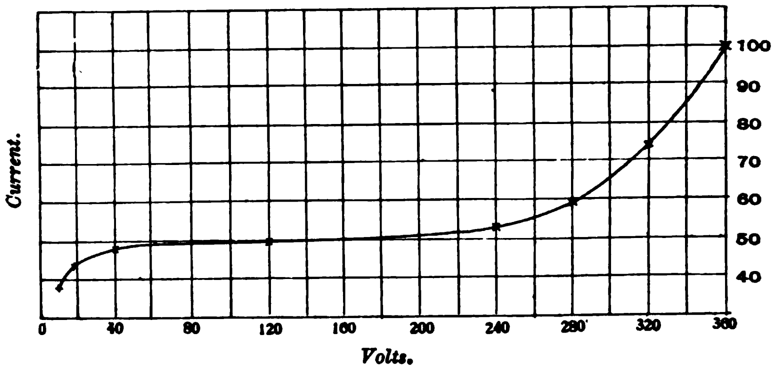


FIG. 4.

The pressure of the air during the above experiment was $\frac{2}{3}$ mm. and the temperature of the wire the same as in the experiment illustrated by the curve in Fig. 2. The conductivity when the wire is positive is very much less than when the wire is negative, as shall be referred to later in the paper. The above numbers are not to be compared as regards their absolute values with the numbers in the previous experiment when the wire is negative.

The rapid increase of current shown in the curve in Fig. 4 after the E.M.F. has reached a certain value would seem to indicate that the positive ions travelling out from the wire have produced secondary ionisation. The E.M.F. for which the secondary ionisation begins is not very different in the two cases, which would show that the masses in the two cases are not very different. All investigations on ionisation produced in air at pressures of 1 mm. and less, by Röntgen rays and ultra-violet light as well as in the

case of discharge in vacuum tubes, show that the negative ion is very small compared with the molecule, while the positive carrier is of molecular magnitude.

In Prof. Townsend's[1] work on the ionisation produced by Röntgen rays at low pressures the negative ion is active in producing ionisation by its collisions with molecules, while the positive ion is apparently inactive. In the case of the ionisation produced by the incandescent wire the positive ion is also apparently active in producing ionisation at pressures of 1 mm. and less. The experiments described above are at a pressure of $\frac{3}{8}$ mm.; we give another curve obtained at the same pressure with the wire again charged positively. The secondary ionisation is more marked in this case, due to the fact that the wire is at a lower temperature and consequently the velocity of the carrier greater for the same E.M.F.

Volts	Currents
20	17
80	24
160	32
240	39
320	55
360	71
400	100

This curve is plotted in Fig. 5.

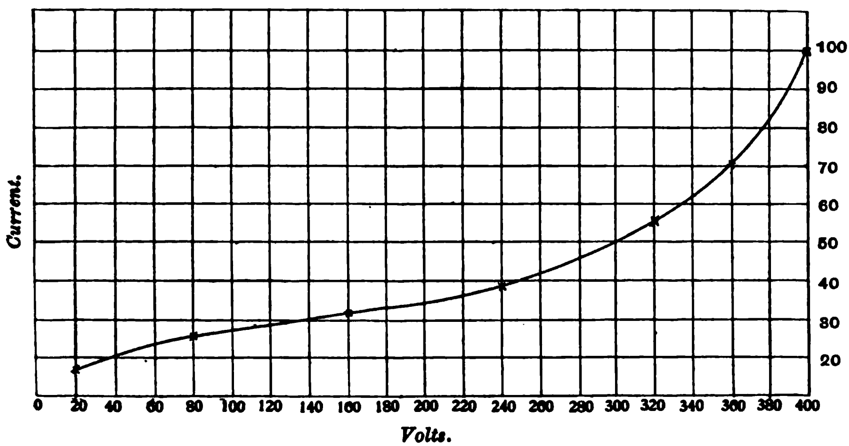


Fig. 5.

[1] *Phil. Mag.* Feb. 1901.

6. The numbers given below refer to experiments when the pressure of the air is $\frac{1}{10}$ mm.

Wire negative volts	Current in arbitrary units
40	57
80	68
160	80
240	86
320	94
400	100

Wire positive volts	Current in arbitrary units
40	7
80	13
160	20
240	32
320	54
400	100

As mentioned previously the current is always very much greater when the wire is charged negatively; the above numbers are not to scale in the two cases. The curves referring to these numbers are given in Fig. 6. The current when the wire is

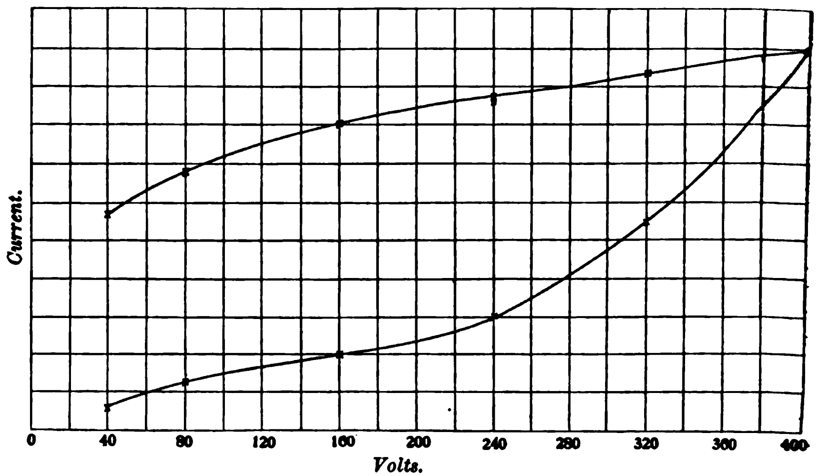


Fig. 6.

charged negatively does not increase rapidly with the higher potential, which means that at this pressure the velocity of the negative ions even with small voltage is such that ionisation is

produced at most of the collisions. The current when the wire is positive increases rapidly at the higher voltages.

The experiments at this pressure and at lower pressures would indicate that a smaller voltage is sufficient to produce ionisation by the collisions of the negative ion than what is required to give the necessary velocity to the positive. But they show that the positive ion we are dealing with can be given the velocity necessary to produce ionisation.

7. In experimenting with the ionisation produced by an incandescent wire in a gas at low pressure, there are many indications that we are dealing not only with the ions produced from the molecules of the gas but that we are actually getting ions off from the hot wire itself.

When the gas is at atmospheric pressure the number of positive ions is in excess of the number of negative except when the temperature of the wire is very high, when the amount of positive and negative is approximately the same. When the pressure is reduced the negative ions are greatly in excess, even at moderately high temperatures. Using the apparatus described, the current when the wire is negative may be 50 times what it is when the wire is positive, the pressure being 1 mm. or less. Such a difference from what is observed at atmospheric pressure suggests that at low pressure we get a copious supply of negative ions from the wire itself.

Again, the current when the wire is negative varies little with the pressure while the pressure is very small; for example, the current was practically constant when the pressure was varied from $\frac{1}{25}$ mm. to $\frac{1}{240}$ mm. This suggests that at such pressures the ions resulting from the ionisation of the gas molecules were small in number compared with those coming from the wire.

Possibly the positive ions which are active in producing secondary ionisation also come from the wire, which may account for the apparent difference between them and the positive ions investigated in other cases of ionisation.

However, more experiments are required before entering into any further discussion of this nature. The wire may produce ionisation at considerable distance from itself, in which case even when the wire is positive, the negative ions would travel for some distance through the gas and in this way produce secondary ionisation, but this action could scarcely explain the effects obtained.

It is intended to investigate more completely some of the effects described in the paper. I desire to thank Prof. Thomson for kind advice during the course of these experiments, which were carried out at the Cavendish Laboratory some time ago.

PROCEEDINGS

OF THE

Cambridge Philosophical Society.

On the Question of "Predisposition" and "Immunity" in Plants. By Professor H. Marshall Ward, D.Sc., F.R.S.

[*Read* 20 January 1902.]

Twenty years ago, in 1881–82, I expressed the conviction that *Hemileia vastatrix*, the fungus of the Coffee-leaf-disease in Ceylon, was an extended or adapted form of a native species of this Uredine found on another host-plant, allied to *Coffea*, and belonging to the genus *Canthium*; but although so convinced of the essential morphological identity of *H. Canthii* and *H. vastatrix*, all attempts to infect Coffee with *H. Canthii* and, reciprocally, to infect *Canthium* with *H. vastatrix*, failed, and the germ of the idea of adaptive parasitism fell to the ground.

On the other hand, I was able to show that *Hemileia vastatrix*, commonly supposed to be confined to *Coffea Arabica*, was capable of infecting *Coffea Liberica*, thus proving that the latter species was not immune to the parasite.

The questions which arose—but which were not then ripe for such clear enunciation as they have since become—were, Have we here a case, or cases, of differences of predisposition (susceptibility) or immunity (resistance) on the part of the host-plants; or is it a matter of adaptability, or of failure to adapt itself to the new host, on the part of the fungus?

Clearly both possibilities are theoretically to be expected, but very little advance was to be made by speculation.

The opinion has gained ground among planters, horticulturists and others, however, that different races of cultivated plants differ much in their susceptibility or predisposition to various fungi which induce diseases, and in view of the facts brought forward by those who grow such plants as *Chrysanthemums*, for instance, it is impossible to overlook the significance of the case as one for scientific enquiry.

The chief difficulty in connection with the whole matter turns on the vagueness of the statements frequently made, and on the lack of recognition even by plant-pathologists of the extremely complex nature of the phenomena. On several occasions I have attempted to narrow the issues involved, and to bring the subject into the region of practical investigation[1], but hitherto with but small measure of success, for it is evident that two conflicting sets of factors are concerned in every epidemic fungus disease—viz. (1) the external conditions, which may favour the one organism and place the other at a disadvantage in very many different ways and degrees; and (2) the reactions of the two organisms one to another, the host presenting obstacles to the entry of the fungus or offering it attractions in all conceivable degrees, while the fungus developes weapons of attack also in very various and graduated forms[2].

Within the last few years much has been done, especially under the stimulus of investigators into the problem of Wheat-Rust, towards clearing the way for further progress. Dating from De Bary's proof that the Rust of Wheat is a Uredine which developes its uredospores and teleutospores on the cereals, and is heteroecious—i.e. forms its æcidium and æcidiospores on another plant, viz. *Berberis*—much work has been accomplished, and many questions raised of which some are still controversial.

The results of this work have been to establish many other cases of Heteroecism—e.g. between *Gymnosporangium* on Junipers and *Roestelia* (the æcidium-form) on Pomaceæ; and to show that the old *Puccinia graminis*, the Rust of Wheat, included a number of forms or species of *Puccinia* which were not sufficiently distinguished by the earlier observers, but which have been shown to be quite different when investigated by modern methods.

For instance, it turned out that in addition to the true *Puccinia graminis* (Pers.) which is heteroecious on *Berberis*, there occur on our cereals at least two other species morphologically distinct and now easily separated by those who examine their microscopic characters. These species are *P. coronata* (Corda) with apical processes on its teleutospores, and which is heteroecious not on the Barberry but on species of *Rhamnus*, and *P. rubigo-vera* (D.C.) with brighter yellow aspect and heteroecious on neither *Berberis* nor *Rhamnus*, but on plants of quite a different Natural Order, viz. Boraginaceæ.

Closer and more prolonged investigations, and especially the industrious labours of Barclay in India, Eriksson in Sweden, Plow-

[1] See for instance "On the Structure and Life-history of Entyloma Ranunculi," *Phil. Trans.* 1886; and "On some relations between Host and Parasite" (Croonian Lecture), *Proc. R. S.*, vol. XLVII. 1890, p. 393.

[2] See also *Disease in Plants*, London, Macmillan & Co., 1901, where a general treatment of the whole subject is attempted.

right in England, Klebahn in Germany, and others, have led to the recognition of a state of affairs even more complex than this, however, for they have disclosed the facts that not only are there several species of rust on our cereals capable of distinction by visible characters, and, therefore, *morphological* species, but that each of these actual species may behave differently towards different species of cereals or other grasses, thus giving rise to *physiological* races or forms, each with its own distinctive biological characters, but indistinguishable by means of the microscope.

Eriksson especially has done so much, and such able work in connection with these points, that the reader may be referred to his papers for further details. Here it will suffice to point out that in his latest summary of his work on this species question[1], he accepts no less than thirteen "species" of *Puccinia* on the cereals, in place of the three hitherto mentioned, breaking up *P. graminis* into two, *P. rubigo-vera* into eight, and *P. coronata* into three distinct species, in addition to more than twenty specialised varieties or races. With these matters we are not now concerned: I mention them here merely to show that *P. graminis* of the text-books is not a single or simple species.

Of these "species" I selected some time ago one which forms the characteristic Brown Rust of the Bromes, for an exhaustive investigation. I may remark that at the time this work was begun, eighteen months ago, the species was known as *Puccinia dispersa* (Erikss.), a form or variety of the old *P. rubigo-vera* of De Candolle[2], but during the progress of the investigation two discoveries have been made, both of importance in other connections, but not bearing on my work, which is concerned only with the behaviour of the Uredo stage. It has been shown (1) by Eriksson[3] that this Brome-rust does not form its æcidium on *Anchusa*, as does the typical *P. dispersa* (the old *P. rubigo-vera*), and (2) by Müller[4] that the æcidium-form is developed not on *Anchusa* but on *Pulmonaria* and *Symphytum*.

The plan of my work was to test (1) how far and in what manner this particular Uredine is specialised to the Bromes, and the nature of this specialisation; (2) what are the conditions of infection and the relations between host and parasite during infection and incubation.

In September last[5] I gave a brief account of the principal results of a long series of infections, the details of which have only been withheld because I was anxious to solve some questions depending

[1] *Ann. des Sc. Nat.* 1901, vol. xiv. p. 101.

[2] Eriksson, *Die Getreideroste*, 18.

[3] Eriksson, *Ann. des Sc. Nat.* 1901, vol. xiv. p. 101.

[4] *Bot. Centr. Beihefte*, B. x. 1901, p. 182.

[5] See *British Association Report*, Glasgow, 1901, and *Ann. of Bot.* vol. xv. 1901, p. 560.

Table I.

Tabulated results of infections of pot-plants of species of *Bromus* with uredospores of *Puccinia dispersa* derived from pustules on *B. sterilis*, *B. mollis*, and *B. secalinus*. Spores tested and known to be vigorous in germination.

No. of Series	Date of inocu-lation	Host	Origin of Uredospores used	Number of plants in pot	Number inoculated	Number of plants which showed pustules (successful infection) on July 5	July 6	July 7	July 8	July 9	July 10	July 11	July 12	July 13	July 14	July 17	July 21	July 25	July 31	Remarks
735 d	June 27	B. sterilis	B. sterilis	17	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	
736 d	,,	,,	B. mollis	7	5	...	...	...	...	...	...	...	...	...	...	...	...	...	...	Many tips black
738 d	,,	,,	,,	17	6	...	...	...	...	...	...	...	...	...	...	...	...	...	...	,, ,, ,,
741 d	June 30	,,	B. secalinus	9	4	...	...	...	...	...	...	...	...	...	...	...	..	...	...	,, ,, ,,
744 d	July 2	,,	,,	7	5	...	...	...	...	...	...	...	...	...	...	...	...	...	...	,, ,, ,,
735 e	June 27	B. madritensis	B. sterilis	9	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	
736 e	,,	,,	B. mollis	12	7	...	...	...	...	...	...	...	...	...	...	...	...	...	...	Many corroded
738 e	,,	,,	,,	14	6	...	...	...	...	...	...	...	...	...	...	...	...	...	...	,, ,,
741 e	June 30	,,	B. secalinus	7	4	...	...	...	...	...	1	1	1	1	1	1	1	1	1	Doubtful if due to infection
745 e	July 2	,,	,,	8	5	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
735 c	June 27	B. maximus	B. sterilis	11	6	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
736 c	,,	,,	B. mollis	6	4	...	...	...	...	...	...	...	...	...	...	...	...	...	...	Some flecks
738 c	,,	,,	,,	11	6	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
741 c	June 30	,,	B. secalinus	11	4	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
744 c	July 2	,,	,,	8	5	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
735 i	June 27	B. secalinus	B. sterilis	12	6	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
736 i	,,	,,	B. mollis	11	6	2	6	6	6	6	6	6	6	6	6	6	6	6	6	
738 i	,,	,,	,,	5	3	2	3	4	4	4	4	4	4	4	4	4	4	4	4	Probably one leaf touched another
741 i	June 30	,,	B. secalinus	10	4	...	...	...	...	..	2	4	4	4	4	4	4	4	4	
744 i	July 2	,,	,,	7	5	...	...	...	...	...	4	5	5	5	6	6	6	6	6	Probably an extra leaf touched

735 b	June 27	B. velutinus	B. sterilis	7	5	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
736 b	,,	,,	B. mollis	9	5	...	4	5	5	5	5	5	5	5	5	5	5	5	5	
738 b	,,	,,	,,	12	7	4	6	7	7	7	7	7	7	7	7	7	7	7	7	
741 b	June 30	,,	B. secalinus	9	4	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
744 b	July 2	,,	,,	9	7	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
735 h	June 27	B. arvensis	B. sterilis	10	6	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
736 h	,,	,,	B. mollis	13	5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
738 h	,,	,,	,,	13	5	...	3	3	3	3	4	4	4	4	4	4	4	4	4	
741 h	June 30	,,	B. secalinus	12	4	...	...	...	...	...	2	4	4	4	4	4	4	4	4	
744 h	July 2	,,	,,	10	4	...	...	...	...	...	...	...	2	2	2	2	2	2	2	
735 a	June 27	B. mollis	B. sterilis	11	8	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
736 a	,,	,,	B. mollis	12	7	1	3	5	6	7	7	7	7	7	7	7	7	7	7	
738 a	,,	,,	,,	17	7	1	2	5	6	6	6	6	6	6	6	6	6	6	6	
741 a	June 30	,,	B. secalinus	10	4	...	...	...	...	...	...	...	...	...	...	...	...	...	...	Distinct flecks
744 a	July 2	,,	,,	7	4	...	...	...	...	...	...	...	2	2	3	3	3	3	3	
735 l	June 27	B. racemosus	B. sterilis	12	6	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
736 l	,,	,,	B. mollis	10	6	2	4	5	5	5	5	5	5	5	5	5	5	5	5	
738 l	,,	,,	,,	4	2	...	...	...	...	1	1	1	1	1	1	1	1	1	1	
741 l	June 30	,,	B. secalinus	11	4	...	...	...	...	...	...	1	1	1	1	1	1	1	1	
744 l	July 2	,,	,,	10	4	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
735 g	June 27	B. commutatus	B. sterilis	10	6	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
736 g	,,	,,	B. mollis	6	4	2	4	4	4	4	4	4	4	4	4	4	4	4	4	
738 g	,,	,,	,,	9	6	...	3	5	5	5	5	5	5	5	5	5	5	5	5	
741 g	June 30	,,	B. secalinus	12	4	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
744 g	July 2	,,	,,	10	5	...	...	...	...	...	...	...	...	...	...	...	1	1	1	
735 k	June 27	B. interruptus	B. sterilis	12	6	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
736 k	,,	,,	B. mollis	10	7	1	3	4	6	6	6	6	6	6	6	6	6	6	6	
738 k	,,	,,	,,	9	5	1	2	3	3	3	4	4	4	4	4	4	4	4	4	
741 k	June 30	,,	B. secalinus	12	4	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
744 k	July 2	,,	,,	16	5	...	...	...	...	...	...	1	1	1	1	1	1	1	1	

on further growth of the species of Bromes investigated, and to examine certain anatomical points. These results may be summarised as follows:

More than 2000 infections have been made on 22 species and varieties of *Bromus*, principally seedlings, carefully selected and cleaned and grown in pots under conditions in most cases so satisfactorily controlled that the rust-pustules only appeared (with very rare exceptions) on the spots where the sowings of spores on the leaves were made. These 22 species and varieties belonged to four out of the five sub-genera of the genus, viz. *Festucoides*, *Stenobromus*, *Serrafalcus* and *Ceratochloa*: I have not yet been able to experiment with *B. arduennensis* representing the sub-genus *Libertia* of Hackel.

In order to illustrate the nature of these infection experiments, let us take the following series from among the latest of those made during the past summer.

Series Numbered 735, 736 *and* 738. (Table I.)

The pots of seedlings were all under like conditions and infected similarly, by means of uredospores gathered separately and kept separate, and known to be vigorous on germination.

The annexed Table I. summarises the facts.

On comparing these results we find no room for doubt as to their meaning. The spores employed were in all cases germinating well: the infections were made at the same time, and the conditions were in every respect—including the age and treatment of the seedlings before infection—alike.

Nevertheless, we find as I had frequently found before[1], that spores from *B. sterilis*, while successfully infecting *B. sterilis* and its ally *B. madritensis*, failed on all the other plants of the series. On the other hand, the spores from *B. mollis* infallibly picked out their own species and its allies *B. velutinus*, *B. commutatus*, *B. arvensis*, *B. secalinus*, *B. interruptus* and *B. racemosus*, but failed on *B. sterilis* and its allies *B. madritensis* and *B. maximus*.

Series Numbered 741 *and* 744. (Table I.)

In the experiments where spores from *B. secalinus* were employed for inoculation we meet with exactly similar facts, only here the fungus attacks *B. secalinus* by preference and *B. arvensis* next in order. *B. sterilis*, *B. maximus*, *B. madritensis*, on the other hand, as also *B. velutinus*, *B. interruptus* and *B. racemosus*, were either entirely immune—to all appearance—or were only attacked very slightly.

[1] Details of the conditions and results of other experiments are in course of publication.

It should be noted that in this series, the seedlings of No. 741—the first named of each pair—were twenty days old, and, after infection, were more exposed to the air and sun. Moreover, they were infected two days earlier, with spores gathered the same morning: whereas No. 744 were infected with the same crop of spores, kept in a moist box for two days.

Here, again, it seems impossible to explain the results as due to anything but differences of susceptibility on the part of the host. For even if we lay stress on the irregularities of infection of *B. mollis*, *B. interruptus* and *B. racemosus*, we are still in face of the fact that in these two series, differing slightly in treatment, the spores growing on *B. secalinus* pick out that species first and especially, and also its near ally *B. arvensis*, and that they do not successfully infect any of the allies of *B. sterilis*. We cannot put this down to any lack of germinating power on the part of the spores: they were well tested and proved to be of excellent germinal capacity in every case.

Series Numbered 774 *and* 775. (Table II.)

In the following series I only used few plants as before so as to allow plenty of room in each pot, and infected them on the face of the leaf below the tip, in water only, and at a moderate temperature carefully watched. The infecting material (uredospores) was also carefully selected from rich green leaves early in the morning of a hot day, and allowed to develope further in glass dishes on damp filter paper kept near the optimum.

Moreover, the germinating capacity of *every infection was separately tested*, at the same temperature as that prevailing during infection, thus disposing of the criticism that failure to infect might be due to a failure of the spores to germinate.

During the first 24 hours the inoculated plants were kept outside at near 20° C., and the bell-jars lifted after another 24 hours, the plants thenceforth being on a table freely exposed to the mid-day sun. The weather was close and dull on the 21st, with hot sunny intervals: the shade temperature reached 26° C., and in the sun 36° to 39° were registered, but the infected seedlings were kept in the shaded area at 20—22° C.

On the 22nd a sharp fall of temperature occurred, and 20—21° C. in the shade was the highest recorded. The weather was dull and a cool wind prevailed.

On the 23rd it was still colder, and the shade temperature outside fell to 15·5—17·5° C.

On the 24th it rained heavily, and the shade temperature was 17—18° C., but a little sun appeared after 5 p.m.

The results are summarised in the following table:

TABLE II.

Tabulated results of infections of pot-plants of species of *Bromus*, with uredospores of *Puccinia dispersa* grown on *B. mollis* and *B. secalinus*. All spores vigorous.

No. of series	Date of inoculation	Host	Origin of uredospores used	No. of plants in pot	No. inoculated	July 30	July 31	Aug. 1	Aug. 2	Aug. 3	Aug. 4	
774 q	July 22	B. erectus	B. mollis	4	2	...	...	...	...	...	...	
774 g	,,	B. inermis	,,	10	5	...	...	...	...	...	...	
775 g	,,	,,	B. secalinus	8	5	...	..	...	...	...	...	
774 c	,,	B. ciliaris*	B. mollis	8	5	...	...	...	...	...	...	* Species needs further determination
775 c	,,	,,	B. secalinus	9	5	...	...	...	...	...	...	
774 b	July 21	B. tectorum	B. mollis	8	4	...	...	...	...	...	...	
775 b	July 22	,,	B. secalinus	6	4	...	...	...	...	...	...	
774 k	July 21	B. sterilis	B. mollis	6	4	...	...	...	...	...	...	
775 k	July 22	,,	B. secalinus	6	4	...	...	...	...	...	...	
774 m	July 21	B. madritensis	B. mollis	9	4	...	...	...	...	...	...	
775 m	July 22	,,	B. secalinus	7	4	...	...	...	...	...	...	
774 l	July 21	B. maximus	B. mollis	7	4	...	...	...	...	...	...	
775 l	July 22	,,	B. secalinus	6	4	...	...	...	...	...	...	
774 e	July 21	B. secalinus	B. mollis	10	5	3	3	4	5	5	5	
775 e	July 22	,,	B. secalinus	9	6	...	4	6	6	6	6	
774 i	July 21	B. velutinus	B. mollis	7	4	...	2	3	3	3	3	
775 i	July 22	,,	B. secalinus	10	5	...	3	4	4	4	4	
774 a	July 21	B. mollis	B. mollis	10	5	3	4	5	5	5	5	
775 a	July 22	,,	B. secalinus	7	4	...	...	...	...	...	...	
774 d	July 21	B. brizæformis	B. mollis	11	5	2	2	2	2	2	2	
775 d	July 22	,,	B. secalinus	8	5	...	...	3	3	3	3	
774 f	July 21	B. macrostachys	B. mollis	9	5	...	4	4	4	4	4	
775 f	July 22	,,	B. secalinus	9	5	5	5	5	5	5	5	
774 o	,,	B. unioloides	B. mollis	8	2	...	...	...	...	..	..	
775 o	,,	,,	B. secalinus	2	2	...	...	...	...	...	...	
774 n	,,	B. Schraderi	B. mollis	11	5	...	...	...	...	...	...	
775 n	,,	,,	B. secalinus	8	4	...	...	...	...	...	...	
774 p	,,	B. canadensis	B. mollis	2	2	...	...	...	...	...	...	
775 p	,,	,,	B. secalinus	2	2	...	...	...	...	...	...	

TABLE III.

Summary of results of infections of pot-plants, Summer of 1901, with uredospores of *Puccinia dispersa* f. sp. *bromina* (Erikss.).

Species of *Bromus* infected	Origin of the Uredospores used for the inoculation									
		B. mollis			B. sterilis			B. secalinus		
	Number of plants inoculated	Results +	Approx. per cent.	Number of plants inoculated	Results +	Approx. per cent.	Number of plants inoculated	Results +	Approx. per cent.	
B. erectus	37	1	2·7	60	...	...	...	...	...	
B. inermis	32	...	...	...	...	...	5	...	...	
B. ciliaris*	20	...	...	6	...	...	5	...	...	* Species to be revised later on
B. tectorum	21	...	...	5	...	...	4	...	...	
B. sterilis	90	4	4·4	84	68	81	18	...	...	
B. madritensis	77	...	..	61	38	62·3	13	1	7·7	
B. maximus	74	1	1·3	82	2	2·4	13	...	...	
B. secalinus	61	31	50·7	77	...	...	16	16	100	
B. velutinus	62	35	56·4	71	...	...	16	4	25	
B. arvensis	76	28	36·8	72	...	...	8	6	75	
B. mollis	85	60	70·6	84	...	...	8	3	37·5	
B. racemosus	56	15	26·8	72	...	...	12	1	8·3	
B. commutatus	45	14	31·1	72	...	...	9	...	...	
B. interruptus	49	19	38·8	72	...	...	9	1	11	
B. brizæformis	18	5	27	6	...	...	5	3	60	
B. macrostachys	12	4	33	...	...	...	5	5	100	
B. unioloides	2	...	...	...	...	...	2	..	...	
B. Schraderi	9	...	...	9	...	...	9	...	...	
B. canadensis	2	...	...	...	...	...	2	...	...	
B. giganteus	1	...	...	...	...	...	...	...	...	
B. pratensis†	14	...	...	3	...	...	8	...	...	† Species needs critical revision
	843			836			167			

In the annexed table (Table III.) I have put together the results of 1846 infection-experiments with these pot seedlings, arranged in such form as to show at a glance their significance in the present connection.

The results point without doubt to the conclusion that both the source of the spores employed and the specific peculiarities of the Brome inoculated are important factors in infection. For instance, uredospores derived from *B. sterilis* successfully infected *B. sterilis* in 68 cases out of 84 (81 °/₀), *B. madritensis* in 38 cases out of 61 (62·3 °/₀), and *B. maximus* in two cases out of 82 (2·4 °/₀), whereas they failed entirely to infect *B. erectus* [60[1]], *B. secalinus* [77], *B. velutinus* [71], *B. arvensis* [72], *B. mollis* [84], *B. racemosus* [72], *B. commutatus* [72], *B. interruptus* [72], *B. Schraderi* [9], and a number of others.

Now *B. madritensis* is closely, and *B. maximus* distantly related to *B. sterilis*, and all three come into the group *Stenobromus*. The evidence goes to show that the spores produced under the influence of the species of host *B. sterilis* (*Stenobromus* group) have been so modified by the circumstances of their nutrition and rearing, &c. that they can successfully attack other host-species of the same (*Stenobromus*) group, but are unable to overcome the obstacles to infection presented by the Bromes of the *Festucoides* (e.g. *B. erectus*), *Serrafalcus* (e.g. *B. mollis*, &c.), or *Ceratochloa* (e.g. *B. Schraderi*) groups.

Similarly with spores cultivated on *B. mollis* (*Serrafalcus*), these attacked *B. mollis* successfully in 60 cases out of 85 (70·6 °/₀) and other members of the *same cycle of relationship* (*Serrafalcus* group) in proportions diminishing more or less with the closeness of their relationship; but only in one case out of 31 did a pustule arise on *B. erectus* (*Festucoides*), and only in four cases out of 90 did infection succeed in *B. sterilis* (*Stenobromus*) as a consequence of sowing spores from *B. mollis* on these species.

Here arises the question, Is this a case of *spores raised on B. mollis adapting themselves to B. sterilis and B. erectus; or of the latter proving individually less resistant than their species generally to the infection?*

Even more striking were the results with spores which had been reared on *B. secalinus* (*Serrafalcus*). They infected *B. secalinus* 16 times out of 16 trials (100 °/₀) and *B. macrostachys* five times out of five trials, but were without result on *B. inermis* (*Festucoides*) [5], *B. sterilis* (*Stenobromus*) [18], *B. Schraderi* (*Ceratochloa*) [9], and so on; and here again the only exception to the generalisation that the spores from *B. secalinus cannot infect any Brome out of its own group Serrafalcus* was a single

[1] These numbers in square brackets denote the number of attempts made.

case (out of 13 tried) where *B. madritensis* (*Stenobromus*) showed pustules. Moreover, there was only one exception, viz. *B. commutatus* [9] to the generalisation that the spores from *B. secalinus* infect every species tried, nine in all, of its own group—*Serrafalcus*. However, in view of the fewer series of infections (see Table III.) with these spores I refrain from speculating on these results.

In the note quoted, I also referred to successful pure-cultures of this Uredo in glass tubes, aerated or not, and using sterilised "seeds" of the Bromes employed; to certain points not hitherto known, apparently, regarding the effects of temperature; to the fact that no explanation of the differences in infection could be derived from my examinations of the anatomy of the plant; and to the fact that the uredospores will germinate in extracts of leaves which they are unable to attack successfully in the living state.

The conclusions arrived at were, substantially, that the mutual relations between host and parasite depend not only on the influence—nutritive and otherwise—of the previous host on the spores themselves, making the latter "virulent" or "weak" towards any prospective host, but also are to be referred to positive reactions on the part of this prospective host, so that the latter is "predisposed" or "immune" to the attacks of the spores; and some remarks were added on the nature of the evidence supporting such conclusions.

The more immediate object of the present paper is to give some account of an attempt to bring this question of predisposition and immunity to a test of another kind.

It has been asserted and denied that certain structural peculiarities in the host, such as the amount of wax on the leaves, the number of stomata, the thickness of the cell-walls, and so forth are the determining factors as to whether infection can or cannot take place in such cases as we are considering here; but nothing has been done, so far as I can discover, in the way of a thorough comparative examination of these points in this connection.

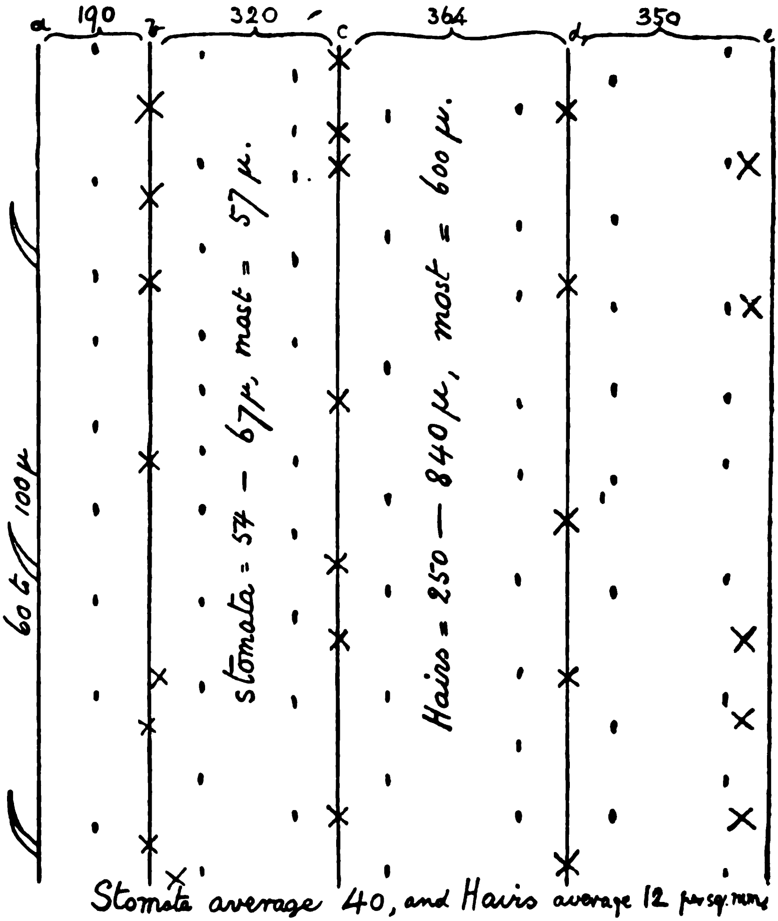
The plan decided upon was as follows. The leaves—first green leaf of the seedling—of two series of pot-plants employed for comparative infection experiments, were cut off and preserved at the end of the experiments, and subjected to rigorous examination during the autumn and winter to determine the following points. The numbers and sizes of the stomata and the hairs per square millimetre of surface, both above and below; the breadth and thickness of the leaf, and the proportions of vascular tissue to chlorophyll-tissue; the character of the motor cells, and so forth.

Care was taken to employ the same part of the same organ—

Table IV.

Facsimile of a plotting and direct counting of stomata, hairs, ribs, &c., on an area of squared paper representing a little more than one square mm. of leaf surface.

B. canadensis. Upper surface.



Explanation of diagram. The vertical lines are *a*, the margin; *b* to *e*, ribs with distances between. The dots represent the stomata and the crosses the hairs, each in its proper relative position. A square of glass representing exactly 1 sq. mm. was then placed in 5 positions on this area, and the average count taken.

i.e. the first green leaf,—grown under the same conditions in every case, and several methods were adopted for obtaining the measurements, so that one should check the other. For instance, in counting the stomata and hairs it will be noticed that I not only obtained averages as described below, but in the case of every species I also plotted a chart of a square millimetre and so obtained a direct count (see Table IV.), and other checks are described in the sequel.

It is of course impossible to suppose that the numbers obtained can be absolutely correct, but the tabulated results show such agreements amongst themselves, for the most part, that I am disposed to regard them as more accurate than is usually the case in such approximate determinations. It is unnecessary to dwell on the laborious nature of the determinations, but of course this limits the number of countings.

I append an account of that part of the analysis which concerns the stomata, and in Table V. the summary of results so far as the stomata are concerned.

Very little work appears to have been done on the dimensions, numbers and distribution of the stomata of grasses, and none, so far as I have been able to discover, on those of *Bromus* or on the seedlings of any genus.

It has been known since the time of Weiss[1] and even before him, that the stomata of grasses are arranged in longitudinal rows flanking the ridges, and that in some genera or species they occur only on one side, usually the upper, and in others on both sides of the leaf. In all the Bromes I have examined[2] they occur on both sides, and are more numerous on the upper than on the lower face of the lamina. Owing to their peculiar distribution on the flanks of the ridges it is not always easy to observe and count the stomata by direct observation of the faces of the leaf, since many of the stomata are seen obliquely from the side or are even more or less withdrawn from observation (unless great care is exercised) owing to the contractile action of the motor cells, which causes them to sink into the flanks of the narrow furrows as the leaf rolls inwards.

It seemed that, since infection occurs by means of the stomata, an accurate knowledge of their sizes and numbers was, then, an imperative necessity, in order to test whether any relation existed between these factors and the susceptibility to infection of the species.

Consequently, I undertook the very laborious task of counting the stomata per square unit of surface of the leaf, both above

[1] *Pringsh. Jahrb.* B. IV.

[2] With one exception, a species which requires further investigation in other respects.

and below, by several methods; arguing that—since each method was consistently applied to every species examined—the results obtained by one would serve to check those obtained by another, and so lead to results of considerable approximate accuracy.

The methods employed for obtaining the numbers of stomata per one square millimetre of leaf surface were as follows:

Direct counting of the numbers found in the field of view, the area of which was known[1]. Both surfaces were examined, and in each case an average of ten observations was recorded. Preliminary countings were made on leaves decolourised in alcohol, and rendered more translucent by warming in Eau de javelle, and transferred through the alcohols to glycerine and glycerine jelly; on leaves hardened in weak Flemming's solution and transferred to balsam; and on strips of the epidermis. The latter are difficult to obtain and yielded very unsatisfactory results.

In the end I decided that the best results were obtained by transferring the pieces of leaf, hardened as before in absolute alcohol, through alcohol and glycerine to glycerine jelly.

Having obtained the average numbers in the circular field of 900 μ diameter (column 5) it was of course a mere sum in proportion to calculate the number per one square millimetre of leaf surface (column 6)—(see Table V.). It will be noted that throughout the table A = Above; B = Below.

I also tried the following method:

Since the stomata are always disposed in longitudinal rows (column 2), with their long axes coinciding with that of their row, measurements of the average intervals (longitudinally) between the stomata (column 3) and of the distances (laterally) between the rows, and the measurements of the total breadth of the leaf (collected in a separate table) gave data which could be used to calculate the number per square millimetre, and these results (given in column 7) also afforded useful check-results, though in almost all cases the figures—derived from averages of twenty countings for each surface—are by this method a trifle too low.

Another method was to compare the numbers of stomata on the upper and lower surfaces of transverse sections of the leaves (column 4). Averages of ten counts each gave useful results, and possessed the advantage that each surface was compared on the same section. The drawbacks to this method are two, at least. In the first place it was found impracticable to obtain the uniformity of thickness and serial sequence of section[2] necessary to superpose them and so get the numbers per square millimetre of leaf surface direct. And, secondly, I frequently found that closely-

[1] With Zeiss Occ. 2 × Obj. C the diameter of the circular field was 900 μ.

[2] Ribbon sections were not used because I was obtaining averages from different parts of the area.

TABLE V. Stomata.

Species of *Bromus*	1 Length of stoma				2 Number of longitudinal series of stomata		3 Average distances between stomata		4 Average No. of stomata on the transverse section		5 Average No. of stomata counted in field-circle 900 μ diameter		6 Average No. of stomata per sq. mm. calculated from col. 5		7 Average No. of stomata per sq. mm. calculated from cols. 2 and 8 and a table of dimensions		8 Actual No. of stomata on a sq. mm. counted and plotted	
	A	Commonest	*B*	Commonest	*A*	*B*	*A*	*B*	*A*	*B*	*A*	*B*	*A*	*B*	*A*	*B*	*A*	*B*
B. mollis	51-70	60	64–70	67	6–8	6	165	230	6·2	2·7	17	13	27	20	36	25	35	28
B. tectorum	45–58	51	51–58	54	10–12	10	145	250	5·7	3·6	23	17	36	26	39	22	45	25
B. ciliaris*	51-64	57	51-64	57	10–12	10	200	220	8·1	4·7	19	15	30	23	28	25	39	30
B. brizæformis	45–51	48	32–45	38	10	10	130	184	6·8	3·4	31	15	48	23	38	27	52	40
B. secalinus	57–70	60	64–77	70	10–12	10–12	188	220	6·7	3·7	27	14	42	27	22	15	41	25
B. macrostachys	50-65	57	65–77	71	10	10	140	220	6·9	4·5	22	12	34	19	39	25	43	27
B. inermis	57–64	60	60–65	62	10–12	10–12	160	240	5·25	3	22	15	34	23	50	33	43	30
B. pratensis*	33–60	46	0		6–8	0	140	0	5·2	0	38	0	56	0	60	0	64	0
B. velutinus	64–73	68	64–100	82	10	10–12	170	240	7·7	6·3	14	8	22	11	29	21	29	24
B. sterilis	45–60	52	51–58	54	10–12	10	102	138	8·8	5·2	33	18	51	28	49	36	53	33
B. maximus	64–97	80	58–83	70	14	14	220	290	9·9	6·2	14	9	22	14	21	16	23	18
B. madritensis	58–68	63	58–70	64	10	10	170	300	8·2	3·7	24	9	38	14	35	19	37	21
B. Schraderi	38–58	48	45–58	52	14	14	150	220	11·2	6·4	25	14	39	22	44	26	45	29
B. unioloides	45–64	51	48–64	56	12–14	12	140	210	11·8	5·7	29	18	45	28	34	20	51	28
B. racemosus	58–75	70	45–70	65	10–12	10	130	160	8·9	6·6	34	17	53	26	43	35	47	39
B. commutatus	65–77	70	64–70	65	6	6	140	190	6·3	3·7	26	17	40	26	38	28	42	32
B. interruptus	58–70	64	51–64	57	6–8	6	140	174	5·5	3·6	21	10	33	16	42	34	49	32
B. arvensis	45–58	52	45–58	56	6–8	6	125	200	4·9	2·5	39	20	61	31	48	25	53	30
B. canadensis	54–67	57	57–65	58	10–12	10	139	330	7·3	2·8					40	19	40	18

* This species requires further examination.

packed stomata were cut through twice, or two stomata on the same row occurred in the thickness of a section, or a stoma was merely shaved by the razor, and such were apt to be missed in the counting, and so on. The numbers thus obtained were only of use in establishing generally the fact that those above are more numerous than those below.

On the whole, useful though the above methods were for checking results, there were considerable divergencies, and the best way was found to be that of carefully counting the stomata on a given area somewhat larger than one square millimetre, and plotting them out on squared paper. On then placing over the chart a glass plate, covered all over except a square representing, on the same scale as the paper, one square millimetre, it was easy to count the stomata in five positions of the square and obtain more correct results (see Table IV.).

I may add that I checked this last method, so far as *B. mollis* is concerned, by repeating it on a seedling germinated in December, and at a much lower temperature and under very different conditions as regards illumination, &c. On the upper surface the number of stomata, 35 per square millimetre, was the same as before, though that of the hairs was fewer than the average.

It only remains to add—so far as Table V. is concerned—that the measurements of the lengths of the stomata (column 1) were obtained from 10 measurements on each surface, and that I have given the extremes as well as the commonest lengths. The variations in breadth were too small to measure accurately with medium powers, and I concluded that as all the stomata appeared shut (except in the transverse sections) no data of value could be obtained from them.

The next part of the plan adopted was as follows:

Assuming that the number of experimental infections—1846 were employed—is large enough to warrant the procedure, I construct a curve of percentages of successful infections for each species with each kind of spore (see Table III.). Thus, with the spores obtained from *B. mollis*, *B. mollis* gave 70·6 °/₀, *B. velutinus* 56·4 °/₀, *B. secalinus* 50·7 °/₀ of successful infections, and so on. By joining the upper ends of ordinates proportional to these percentages, a diagram-curve is obtained which we may call the infection-curve. See Table VI.

I then take the measurements of—say the sizes of the stomata —and construct a diagram-curve in similar manner of each of the measurements—extremes, commonest sizes, above and below—by joining the upper ends of ordinates proportional to the lengths, and taking the species in like order.

It appeared that in this way it ought to be possible to express graphically any correspondences or discrepancies between the

general average trend of corresponding parts of these curves, and so test the question, Does the predisposition or immunity of a given species bear any relation to the size of the stomata? And, similarly, to the numbers of stomata, the sizes or numbers of hairs, the chlorophyll area, and so forth?

So far as the work has gone I find such glaring and irreconcilable dissimilarities between the curve of infection and any of the curves of measurements, together with such obvious general agreements in the latter curves among themselves, that it is impossible to regard the predisposition or immunity of any particular species as a function of the sizes or numbers of stomata or hairs, or, at any rate at present, of the area of the leaf and volume of chlorophyll-tissue exposed to the attacks of the fungus, and unless further research shows some much greater variations in the factors mentioned than have appeared up to the present, must conclude that we have here a method which enables us to say at once that the infecting tube of the fungus does not enter the leaf of, e.g., *B. mollis* sixty times out of 85 (70·6 °/$_{\circ}$) cases tried, and the leaf of *B. maximus* only once out of 74 (1·3 °/$_{\circ}$) times, simply because the stomata of the former are more numerous than those of the latter—which is the case—because if that were so it seems impossible to understand why *B. sterilis* with more stomata than either is not also more susceptible. Nor can the predisposition of *B. mollis* be due to the fact that its stomata are larger than other species: those of *B. maximus* and *B. racemosus* are larger still.

Appended are diagrams of the curves for stomata (Tables VI. and VII.), which illustrate the principles involved.

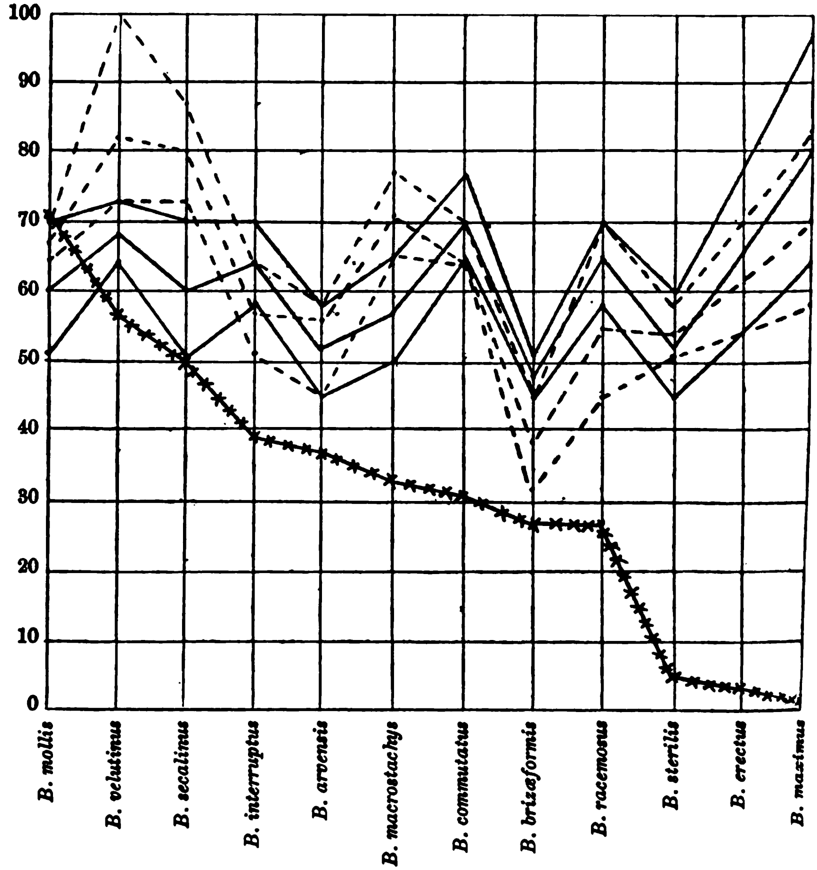
It seems clear from the foregoing that we are face to face with one issue only, viz. that infection does not depend on the ease of access afforded to the germ-tubes by the number and sizes of the stomata, and since similar negative results have been met with in trying the curves of other structural features, the temptation to the following generalisation is irresistible:—

The capacity for infection, or for resistance to infection, is independent of the anatomical structure of the leaf, and must depend on some other internal factor or factors in the plant.

If this is accepted, however, we are driven back on to those mysterious factors, the properties of the cell or the constitution of the plant, for an explanation of the relative immunity from or predisposition to the disease. The failure to find any structural or mechanical explanation of the phenomenon, in the sense here implied, does not necessarily involve the assumption that there is no mechanism in the living plant which is answerable for the obstruction, or aid, to infection exhibited by the species. It only points to the conclusion that the mechanism is of that more

refined and subtle nature which determines such fundamental properties as specific relationship, variation, heredity and other biological phenomena.

TABLE VI.



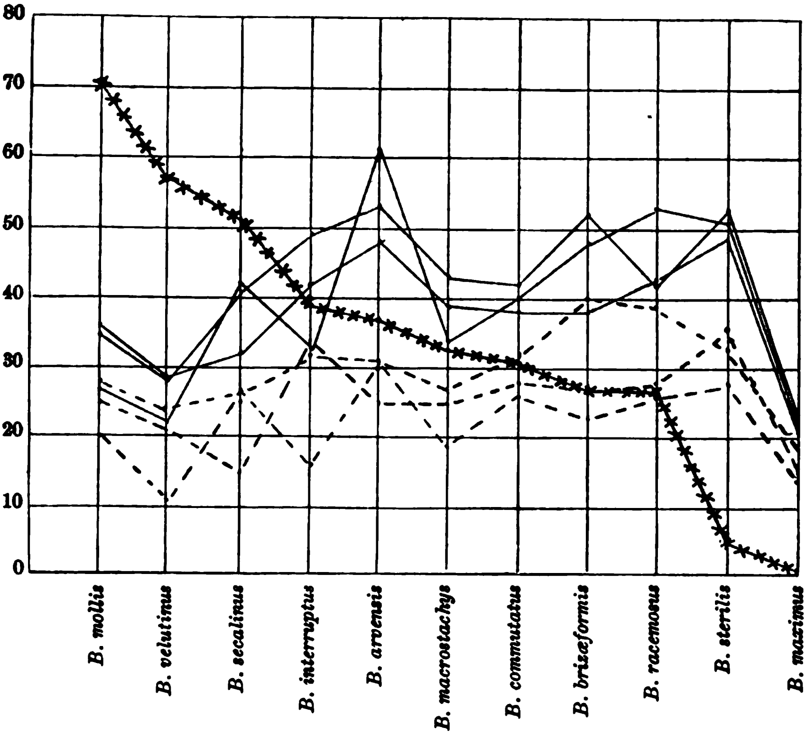
Size of stomata (extremes and commonest sizes). First green leaf.

The vertical heights are in multiples of 1μ. The measurements of stomata are extremes and commonest lengths for both upper and lower surfaces.

We are, in fact, driven to assumptions such as that the spores of the fungus derived from a given species *A* of the host-plant are able to attack the species or variety *B*, but are unable to successfully infect the species or variety *C*, because some resisting substance or condition exists in the living cells of *C* which is absent

from those of *B*. Or, to put the matter in another way, a given species of host-plant *x* is susceptible to infection from the fungus derived from a given species or variety *y*, but immune to the same fungus derived from a species or variety *z*, because the cells of *x* contain a substance, or offer conditions, favourable to the fungus derived from *y*, but not to that derived from *z*.

TABLE VII.



Above = ——— Below = - - - - Infection = ×–×–×–×

Number of stomata per 1 sq. mm. on first green leaf.

Vertical heights = multiples of 1 μ. The measurements of numbers of stomata given are obtained by three distinct methods (see Table V.) for each surface.

But there are yet other possibilities. The influence of its former host *A* on the fungus may have been such—in virtue of the substances afforded it in nutrition, or of conditions of other kinds—that they have imparted to it properties which enable it to overcome the obstructive substances or conditions offered by the living cells of the prospective host *B*, but not such as render

it meet to cope with those offered by C, and we term B susceptible and C immune.

Or, finally, it is conceivable that the conditions—substantial or otherwise—of its original substratum, a living host x, have so affected the fungus that it is unable to avail itself of the favourable conditions offered by a species or variety y, and so does not successfully infect it, but is quite capable of infecting z, and benefiting by the substances, or conditions, offered by the cells of the latter.

In discussing these questions, it seems to me that plant-pathologists have not sufficiently clearly borne in mind the probability that two distinct classes of phenomena are here concerned—(1) the activities of the host, on the one hand, towards the fungus may be attractive or repulsive: in health they should be the latter. (2) The relations of the fungus towards the host may be aggressive or passive: they are usually aggressive.

Theoretically any or all of these possibilities may exist, and the resultant may be the outcome of a very complex condition of affairs, viz. attractions and repulsions of the fungus towards and from the host, and attractions and repulsions of the host towards and from the fungus.

Moreover, as we know from the recent advances in the study of enzymes and of chemotactic substances, of toxins and antitoxins, such substances and conditions as I have postulated above do actually occur in the living cells, and there is not only no absurdity but, on the contrary, every show of probability that—since the structural features elucidated by the microscope are not responsible for the phenomena of immunity and susceptibility on the part of the host, or of capacity or incapacity to infect on the part of the fungus—it is in the domain of the invisible biological properties of the living cell that we must expect the phenomena to reside.

This brings me to the consideration of some remarkable resemblances, or coincidences, between the behaviour of these fungus-spores towards their host, and, reciprocally of the host to the parasite, and that of the pollen and stigma the one towards the other.

It is obviously not straining the facts to compare the physiological behaviour of a uredospore and its germ-tube towards the tissues of a leaf, with that of a pollen-grain (which is also a spore) and pollen-tube towards the tissues of the stigma and style on which it germinates and into which it penetrates.

Just as the hyphæ of parasitic fungi are attracted by chemotactic substances[1] so are the pollen-tubes[2], and even if we did not

[1] Miyoshi, *Jahrb. f. wiss. Bot.* 1895, B. XXVIII. p. 269.
[2] Miyoshi, *Bot. Zeitg.* 1894, p. 23.

know that the latter contain enzymes[1], it would have to be inferred, because some pollen-tubes (e.g. *Alopecurus*) pierce and travel in the middle lamella, and others (e.g. *Agrostemma*) penetrate the cell-walls of the stigmatic cells, and pass through from cell to cell of the style[2] in a manner exactly analogous to the passage of a *Botrytis* hypha in the substance of the cell-wall[3]. Moreover, no one who has traced the corrosive action of the pollen-tube of *Pinus* in the tissues of the nucellus[4], or that of a pollen-tube of a chalazogamic plant in the nucellus of the ovule (e.g. *Casuarina*[5]) can hesitate to compare the action to that of a parasitic fungus.

Even more striking in some respects is the behaviour of the pollen-grains of *Vaccinium* and the spores of *Sclerotinia*, a fungus parasitic on the *Vaccinium*. As Woronin showed[6], insects convey both the fungus-spores and the legitimate pollen to the stigma of the *Vaccinium*. Here they both germinate, side by side, and the germ-tubes of the fungus race the pollen-tubes down the style to the ovules.

Now we may clearly compare this case to that of two different pollens placed side by side on the same stigma, and it will be remembered that Charles Darwin[7] proved that several orders of events may follow according to the origin of the two pollens employed in such an experiment.

The case which chiefly concerns us here is that in which the prepotent pollen is that which has originated from a plant of the same species or variety as the one to be pollinated. Darwin cited several cases where it was found that if the pollen from a given stamen was placed on the stigma of its own flower, together with pollen from another flower of the same species or variety, the former pollen was prepotent. This case affects the present question less directly than the following, however, because it turns on the origin of the pollen from the individual plant or flower.

Many cases are known where the stigma of a given plant *A* may be successfully pollinated by pollen from its own flower, or from a flower on another plant of the same species or variety, but will be either only partially or not at all successfully pollinated by pollen from a flower of another variety or species *B* of the same genus.

It is a sort of rule that the best results in cross-breeding are

[1] Green, *Phil. Trans.* 185, 1894 B. p. 385.

[2] Strasburger, *Befruchtungsvorgang bei den Phanerogamen*, 1884, Taf. 1, Figs. 55, 56.

[3] Marshall Ward, *Ann. Bot.* 1888, Vol. II., Plate XXIV.

[4] Strasburger, *Angiosp. u. d. Gymnosp.* 1879.

[5] Treub, *Ann. du Jard. Bot. de Buitenzorg.* X. p. 145; Benson, *Linn. Trans.* 1893, p. 409.

[6] Woronin, *Mém. de l'Acad. Imp. de St Pét.* T. XXXVI. 1888, p. 25.

[7] Darwin, *Cross and Self-fertilization in Plants*, 1876.

obtained between different individuals of the same variety or species, and that the more the variety or species which yields the pollen differs from the variety or species to which the pollinated stigma belongs, the less likely is the pollen to "take"[1].

But if for "pollen" we read "uredospore," for "pollen-tube," "germ-tube," for "pollination," "infection," and for "stigma" we read "leaf," we have an exactly similar order of events in the capacity for infection of the plants I am discussing, and it would be quite in accordance with scientific accuracy to say that the best results in infection are obtained by dusting the leaves with spores from different individuals of the same varieties or species, and that the more the variety or species which yields the spores differs from the variety or species to which the infected leaf belongs, the less likely is the infection to "take."

The parallelism of the two cases up to this point is clear, and some important results seem to me to follow. It is conceded by all who have had to do with hybridisation that the obstacles to crossing are not merely differences in observable structure in the flowers concerned—though such may occur in particular cases—but reside in obscure inter-relations between the cells of the stigma, ovules, &c. of the one plant, and the pollen and the pollen-grains, &c. of the other. And that is exactly what occurs in the case of the Bromes and their Rust.

But, such being the case, we may expect that just as cultural variations affect the possibilities of crossing, so too they will affect the possibilities of infection. Just as variation in the properties of stigma and pollen are brought about by changes of environment, so, too, variations in the properties of the leaf-cells and of the uredospores, and in consequent predisposition or immunity, will be brought about by such changes.

[1] See also Bailey, *Plant Breeding*, 1896.

The Genito-urinary Organs of Dipnoan Fishes. By J. GRAHAM KERR, M.A., Christ's College.

[*Read* 20 January 1902.]

LEPIDOSIREN.

Pronephros.

The pronephros forms the functional kidney of *Lepidosiren* up to about stage 35.

The archinephric duct arises as a solid flattened rod which splits off from the mesoblast just at the junction of myotome and lateral plate. The splitting-off process gradually spreads backwards, and the archinephric rudiment is separated completely from the underlying mesoblast except anteriorly, where through a region extending through two myotomes it remains continuous with the main mass of mesoblast along its inner edge. This attached region is the rudiment of the pronephros. The archinephric rudiment lies close under the epiblast from which however it is sharply marked off by its coarsely-yolked character. At its posterior end it passes directly into mesoblast. At no time is it continuous with the epiblast.

The mesoblastic structures are at first completely solid. About stage 23 the first traces of coelomic cavity appear as a split traversing the region of the nephrotome, extending up slightly into the myotome, and outwards slightly into the region of the perivisceral cavity. The coelomic cavity so arising rapidly spreads up into the pronephric rudiment as the cavity of the tubule. There are on each side two nephrostomata as in Urodelan Amphibians. Round the cavity of each nephrostome the mass of mesoderm becomes as it were carved out to form the wall of the funnel. From the nephrostome the cavity gradually extends backwards along the rudiment of the archinephric duct. The coelomic split opposite the nephrostomes widens out to form a considerable pronephric chamber, and in stage 24 the splanchnic wall of this forms a dome-shaped projection into the cavity opposite each nephrostome. In the interior of this sinus-like cavities are formed, rudimentary blood-vessels, containing round corpuscles. The two dome-shaped structures in question upon each side become continuous and give rise to the glomus. At first lying on

the ventral wall of the pronephric chamber, the glomus becomes later on by differential growth carried inwards and upwards so as to be attached to the median wall of the chamber close to the rudiment of the dorsal aorta.

The myocoele never becomes more than a narrow split, but the perivisceral cavity extends with further development ventralwards of the pronephros. About stage 31 the perivisceral cavity forms a considerable chamber on each side, the glomus hanging freely into it from its medio-dorsal angle, and the pronephros forming a bulging inwards of its dorsal wall. In stage 32 the pronephric swelling comes into contact with the oesophageal rudiment and fuses with it so as to separate off a chamber lying above it, into which the nephrostomes open and which contains the glomus, from the perivisceral cavity. This fusion never extends round the posterior end of the pronephros, so that here the pronephric chamber remains freely open to the general coelom.

During the period in which the cloacal aperture is temporarily closed the archinephric duct becomes greatly dilated, apparently distended by the continued forcing into it of coelomic fluid by the pronephrostomes. A similar dilatation of the cavities of the pronephros was observed in the case of the frog by Marshall and Bles, the obstruction to the outflow of fluid being in this case, however, caused by the occlusion of the archinephric duct itself. In *Lepidosiren* the prevention of the outflow of excretory products to the exterior is associated with the fact that during this period the larvae are lying crowded together practically motionless in the still water at the bottom of the nest. It is probably to be looked upon as an adaptation to diminish pollution of the surrounding water[1].

The substitution of the mesonephros for the pronephros as the functional renal organ is a gradual process which takes place in the stages before and during metamorphosis.

Mesonephros.

The mesonephros forms the functional kidney of the adult. The rudiments of its tubules are first noticeable about stage 30 as

[1] The enormous dilatation of the homologue of the Amphibian bladder seen in the allantois of the Amniota is probably an analogous phenomenon, the embryo being here for the first time completely shut up in a shell through which diffusion of fluid hardly takes place at all. The urinary secretion therefore, instead of being allowed to pass into the space round the embryo—thence to pass away by diffusion, is retained within the tissues of the embryo (both yolk sac and allantois being morphologically parts of the embryo, in spite of ordinary Text-book usage) in the homologue of the urinary reservoir of the anamniotic ancestral form. That the cloacal bladder of Amphibia—the homologue of the allantois of Amniota—is physiologically a urinary bladder is indicated by an abnormal specimen of *Rana temporaria* examined by me, in which the cloacal bladder was absent. In this specimen the whole of the rectum was greatly distended by accumulated urine.

slight aggregations of nuclei and protoplasm in the mesoblast internal and slightly dorsal to the archinephric duct. It is remarkable that the tubules, at first strictly metameric, become at a very early stage increased in number so as to be about twice as numerous as the myomeres in the same region. Each tubule rudiment assumes the form of a U, the longer limb coming into contact with the wall of the archinephric duct. About stage 31 a lumen appears in the better developed rudiments, which thus become tubular. The outer end of the tubule opens into the archinephric duct. The inner end, the end of the shorter limb of the U, expands, becomes thin walled and soon becomes dilated to form a chamber which in a transverse section appears nearly as large as the section of the archinephric duct. The cavity in question is the rudiment of the Malpighian capsule, and thus the capsular coelom in *Lepidosiren* develops secondarily as a cavity in the mesoblast, quite distinct from the perivisceral cavity. In *Lepidosiren* peritoneal funnels connected with the mesonephros do not exist at any period. For some time there is no glomerulus projecting into the capsule, but by stage 35 the glomerular projection is quite distinct.

Testicular network of Dipnoi.

The existence of a connexion between testis and mesonephros was first established by Ehlers for *Lepidosiren*. Semon has recently chronicled the discovery of spermatozoa in the Malpighian capsules of *Ceratodus*—pointing to a connexion in that Dipnoan. I have myself described the testicular kidney connexion in *Protopterus*, and I have also worked out in detail and figured for the first time the precise relations in the adult male *Lepidosiren*.

These relations may be shortly summarised as follows:

In *Lepidosiren* the very much elongated testis is divided into two regions—a larger formative region somewhat cylindrical in shape and lying immediately ventral to the mesonephros, and a shorter, much narrower, vesicular region which, imbedded in a sheath of dense connective tissue, similarly lies ventral to and closely apposed to the ventral edge of the kidney. The connexions of testis with kidney are limited to the posterior portion of the vesicular region, whence about half-a-dozen vasa efferentia pass into the substance of the kidney. Here they break up into branches which open into the cavities of Malpighian capsules. From these the spermatozoa reach the cloaca by the kidney tubules and Wolffian duct.

In *Protopterus* essentially similar conditions hold, modified slightly by the fact that in the fully-developed Protopterus the kidneys and also the vesicular portions of the testes have become

fused posteriorly across the middle line. In *Protopterus*, however, there passes off to each side only a single vas efferens which arises from the extreme posterior end of the fused testes.

The presence of a testicular network in Dipnoi is of great morphological interest, adding as it does another to the great groups of gnathostomatous vertebrates in which such an arrangement occurs. These groups are the Selachians, the Dipnoans, the Amphibians, the Reptiles, the Birds, and the Mammals. In fact, the only groups in which it does not characteristically occur are those of the Crossopterygii, and the Teleostomi. But in the latter group we have only to look at those forms which the common consensus of Zoologists regards as most primitive—the Ganoids—and we here find a testicular network of the most typical kind (*Acipenser*, *Lepidosteus*, *Amia*). There only remain then the higher Teleostomes, viz. the Teleostean fishes, and the Crossopterygians, in which the network is absent. We should, I think, require very strong evidence before we could believe that an arrangement characteristic of a single one of the groups of Gnathostomata, and of the admittedly more highly specialized members of a second group, was the primitive one amongst gnathostomatous vertebrates, rather than another arrangement which is characteristic of all the remaining groups. Evidence of the required weight is, I believe, completely wanting. Against such a view we have, in the first place, the balance of morphological probability. We know that it is one of the most fundamental characteristics of the coelomata that two main functions—that of nitrogenous excretion and that of reproduction—are carried on by the cells lining the coelom, their products passing into that cavity. Surely it is probable that these two products should primitively find their way to the exterior, by the same ancient channels of communication of that cavity with the exterior,—the nephridial tubes.

Secondly, we have the fact that the conditions in *Lepidosiren* and *Protopterus* when taken with certain facts in regard to other groups, furnish a very simple explanation of how the arrangements in Crossopterygians and Teleosts may have been derived from those common to the other groups of Gnathostomes.

If we regard the different forms of anurous Amphibians[1] we find that typically there is an extensive testicular network, connecting testis and kidney over a considerable part of their total length. In the male *Bombinator* the anterior transverse canals of the network have become enlarged, and their course to the kidney duct has become direct. In the male *Alytes* the same happens, but here the (usually two) enlarged canals are the sole representatives of the network.

[1] *v.* Gadow, *Cambridge Natural History*, vol. VIII. p. 51.

Finally, in *Discoglossus* the transverse canals are reduced to a single one lying anteriorly and connecting testis with kidney duct directly. I am not aware that any morphologist would derive the complex testicular network so characteristic of most frogs from a condition such as that of *Alytes*: on the contrary it would be probably universally admitted that in *Alytes* we have a condition derived from that which is more usual by a reduction of the testicular network to a vestige at its anterior end.

Passing then to a consideration of the conditions described in the two-lunged Dipnoans we find that they fall in perfectly with the belief that they may have come about by a similar process to that which has taken place amongst the Anura—that the single connexion between testis and kidney apparatus in *Protopterus* represents the posterior one of the metamerically repeated connexions of *Lepidosiren*, and that the little series of such connexions in *Lepidosiren* represents in turn the posterior members of a once much more extensive series, stretching through the greater part of the length of testis and kidney.

I have pointed out in my paper in *Proc. Zool. Soc. London*, 1901, vol. II., that it is but a step from the condition found in *Protopterus* to that found by Budgett and Jungersen in *Polypterus*: that the step is a very short one is indicated by what has taken place in the Anura. Jungersen's work on Teleostean development also shows that he is justified in saying that the relations of testis and duct in *Polypterus* lead up directly to the relations of those structures in Teleosts.

I believe then that the conditions shown to be present in *Protopterus* and *Lepidosiren* afford the strongest evidence for accepting in the meantime as our working hypothesis the view that the common ancestral forms of the various groups of gnathostomatous vertebrates possessed a well-developed testicular network, and that the cases where this is now absent have arisen through the enlargement of the posterior element of the network and the atrophy of the remainder.

Further Observations upon the Biological Test for Blood. By GEORGE H. F. NUTTALL, M.A., M.D., Ph.D., Christ's College; University Lecturer in Bacteriology and Preventive Medicine.

[*Read* 20 January 1902.]

In various publications which have appeared since May 1901, I have described the results of investigations upon the blood, which indicate that we are able by means of this test to study what I have termed the "blood-relationship" amongst animals. I will refer those interested in the subject to my other papers for particulars regarding the mode of preparation of the test-sera[1].

By means of *anti-human* serum, I have been able to establish the fact that all the Primates, excepting the Lemuridae, possess some common quality in their blood which brings about a precipitation upon the addition of anti-human serum. I have tested 42 samples of blood from apes and monkeys with uniform results, and in addition I found that there were quantitative differences in the reaction obtained, the Simiidae giving, as Dr Grünbaum of Liverpool has also found, a reaction practically equivalent to that obtained with human blood, whilst I have found in addition that the Cercopithecidae bloods give less reaction, the least reaction being however obtained with the bloods of the Hapalidae and Cebidae. In other words the amount of reaction would appear to correspond with the degree of relationship existing amongst the Anthropoidea.

Tests made with *anti-dog* serum have only yielded positive results with bloods of other Canidae, eight different species of which have been examined.

Anti-horse serum only produced a reaction with the blood of the horse and donkey. The anti-sera for ox and sheep blood have given reactions which indicate that a bond of consanguinity exists between various true ruminants (Ox, Sheep, Goat, Antelope, Deer). It is interesting to note that the bloods of Tragulidae and Camelidae have given no such indication of relationship with the true ruminants.

Since I published these results I have continued to collect

[1] *Brit. Med. Journ.* 1901 (11 May), vol. I. p. 1141: (1 Sept.), vol. II. p. 669. *Journ. of Hygiene,* 1901 (1 July), vol. I. pp. 367—387. *Proceedings of the Royal Society,* vol. LXIX. pp. 150—153 (Paper read 21 Nov.). *Journ. of Tropical Med.* 1901 (16 Dec.), pp. 405—408. *American Naturalist,* vol. XXXV. pp. 927—932.

bloods and to test them with the anti-sera named. I have now examined about 440 blood-samples and found my previous observations confirmed.

To-day, I wish to briefly report upon the results I have obtained on testing about 250 bloods of all classes of animals, vertebrates chiefly, by means of other anti-sera which I have since succeeded in producing, namely anti-sera for the blood of the pig, of the fowl, for fowl's egg albumen, as well as anti-alligator, anti-turtle, anti-frog, and anti-lobster serum. Whereas the first three have already been obtained from rabbits by French and German observers, the others are new. I am at present engaged in preparing other anti-sera upon which I hope to report upon a future occasion.

Anti-pig serum has been found to produce a reaction only in dilutions of pig's blood, no other bloods of Suidae having as yet been examined. On the other hand, this anti-serum was observed to produce a marked clouding in a number of mammalian bloods: that of man, several species of monkey, bear, dog, opossum, racoon, cat, coati, genet, stoat, rat, mouse. Only once did it produce a very slight clouding in a non-mammalian blood. (Experimental error[1]?)

Anti-fowl serum on the other hand has been found to behave quite differently to the anti-sera for mammalian bloods, whose action is limited—within time limits—to definite, and at times, small groups of animals. Anti-fowl serum was found to produce a reaction not only in solutions of fowl's blood, and that of the closely related pheasant, turkey, etc., but also with the bloods of widely divergent species, such as the parrot, various species of duck, the woodcock, sheathbill, heron, eagle, owl, condor, pigeon, a number of small passerines, as also that of the American rhea. A marked clouding was moreover produced in the blood of the swallow, rook, landrail, stork, swan, and African ostrich. What I have termed a "marked clouding" is probably to be regarded as an indication of a more remote relationship. In some cases a slight reaction may be classified as but a clouding, this being due to experimental errors which I hope with time to exclude, possibly by quantitative methods of determination. Anti-fowl serum produced clouding in but one mammalian blood, and this may be due to experimental error[1].

Anti-egg serum produced only a marked reaction in solutions of the egg-albumen of the fowl, but a *feeble* reaction with the blood of the fowl. It produced a marked clouding in dilutions of the bloods of the parrot, swan, heron, stork, grebe, conure, crow, emu, in dilutions of the white-of-egg of the emu, and in blood dilu-

[1] Blood samples, collected on filter-paper, sent me from other places may have been in contact with other bloods.

tions of *Alligator sinensis* and *Testudo ibera*, a faint clouding with *A. mississippiensis* and *Chelone midas*. This anti-serum produced no clouding in mammalian or other bloods. Should further investigations show similar reactions with other reptilian bloods, then we may perhaps be able to assume that the egg still possesses a vestige of reptilian character. It is however too early to draw any conclusions.

Anti-alligator serum, prepared by injecting rabbits with the serum of *A. mississippiensis*, produced a reaction on being added to serum dilutions from *A. mississippiensis* and *A. sinensis*, a faint clouding in the blood of *Chelone midas*.

Anti-turtle serum (from *Chelone midas*) acted upon turtle blood, but in addition produced a marked clouding in the blood of *Testudo ibera*, a very faint clouding with *A. mississippiensis*, no other bloods being affected.

Anti-lobster serum (from *Homarus vulgaris*) reacted with lobster serum dilutions, produced marked clouding with blood of *Astacus fluviatilis*, but exerted no effect whatever on any of the 250 bloods examined, as in the preceding cases[1].

I come finally to the consideration of the reaction which takes place in non-homologous or distantly related bloods when these are allowed to stand. From the first, I found it necessary, in making these tests, to put a time limit upon them. This may appear to be a rather arbitrary proceeding. My time limit has usually been five minutes at average temperatures in the laboratory. A powerful anti-serum will certainly have acted within that time upon its homologous blood-dilution; with powerful fresh anti-sera the reaction takes place almost instantaneously. On the other hand, if we allow the mixtures of anti-sera and bloods to stand, a reaction takes place slowly with non-homologous bloods. The results I have hitherto obtained tend however to prove that anti-mammalian sera only produce these later reactions in mammalian bloods, and anti-avian sera act similarly on avian sera alone. I am at present engaged in carefully observing these slower reactions. The results above recorded are based upon observations made within the arbitrary time limit.

[1] Note whilst going through the press:—I have since found that anti-lobster serum produced slight reactions or marked clouding with blood dilutions of five species of crab, viz. *Portunus depurator*, *P. puber*, *Carcinus maenas*, *Cancer pagurus*, *Eupagurus bernhardus*.

Note on a Method for determining the Concentration of Hydrogen Ions in Solution. By H. O. JONES, B.A., Clare College, and O. W. RICHARDSON, B.A., Coutts Trotter Student, Trinity College.

[*Read* 3 February 1902.]

The following investigation was suggested by a series of observations by Mr Fenton and one of the authors[1]. They showed that oxalacetic hydrazone decomposed in presence of water at 100°C. into carbon dioxide and pyruvic hydrazone. In the presence of strong acids, however, the reaction proceeded differently and pyrazolone carboxylic acid and water were formed. With weak acids or very dilute strong acids the two reactions occurred simultaneously and it was shown that the amounts of carbon dioxide given off, using acids of the same normality, were in the inverse order of their "affinity constants."

It was suggested that these reactions might be explained by supposing that the formation of pyruvic hydrazone was due to the decomposition of the negative ion according to the following equation:

$$\begin{array}{llll} \mathrm{COOH\,.\,C\,.\,CH_2\,.\,\overset{-}{CO_2} + \overset{+}{H} = CO_2 +} & \mathrm{COOH\,.\,C - CH_3} & & (1) \\ \quad\ \ \Vert & \quad\ \ \Vert & & \\ \quad\ \mathrm{N\,.\,NH\,.\,C_6H_5} & \quad\ \mathrm{N\,.\,NH\,.\,C_6H_5}\,; & & \end{array}$$

whereas the undissociated molecule decomposed as follows:

$$\begin{array}{lllr} \mathrm{CH_2 - CO\,.\,OH} & \mathrm{CH_2 - CO} & & \\ \vert \qquad\qquad \mathrm{.\,H} & \vert \qquad\ \ \vert & & \\ \mathrm{C : N\,.\,N\,.\,C_6H_5} = & \mathrm{C : N\,.\,N\,.\,C_6H_5 + H_2O.} & & (2) \\ \vert & \vert & & \\ \mathrm{CO_2H} & \mathrm{CO_2H} & & \end{array}$$

The effect of acids would thus be to diminish the concentration of the negative ion and hence to reduce the amount of carbon dioxide produced.

The experiments here described were undertaken with the view of further developing this hypothesis, and of testing its quantitative validity. The theoretical problem is one of some complexity, since it is necessary to take into account the simultaneous occurrence of two reactions, the products of each of which exert an effect on the rate at which the other proceeds. In the case where the concentration of the hydrogen ions due to the hydrazone is negligible compared with that due to the acid used,

[1] Fenton and Jones, *Jour. Chem. Soc.* 1901, LXXIX. p. 91; *Proc. Chem. Soc.* 1901, p. 24; *Proc. Camb. Phil. Soc.* XI. 2, p. 108.

the following expression has been deduced for the amount of pyruvic hydrazone formed:

$$\frac{c_1}{c_2} = 1 + ax, \qquad (3)$$

where c_1 = original concentration of oxalacetic hydrazone,

c_2 = final concentration of pyruvic hydrazone,

x = concentration of hydrogen ions,

and a is a constant which is equal to the velocity constant of the reaction (2) divided by the product of the velocity constant of the reaction (1) into the dissociation constant of oxalacetic hydrazone. The amount of carbon dioxide produced can be readily expressed in terms of the final concentration of the pyruvic hydrazone to which it is proportional. On theoretical grounds it was to be expected that with a solution of ·1 gram of the hydrazone in 100 c.c. the ionisation of the substance could be neglected compared with that due to $\frac{N}{20}$ sulphuric acid, and as a matter of fact the experiments show that this holds with solutions stronger than one-fiftieth normal.

In the experiments the amount of carbon dioxide produced when ·1 gram of the hydrazone was heated with 100 c.c. of acid of various strengths was estimated by absorption in baryta and titration of the excess. The results obtained, as shown in the following table, fully confirm the theoretical conclusions.

The values in column (4) were calculated from Kohlrausch's[1] measurements at 18°, whereas the values for ax are proportional to the concentration at 100°. This may explain the slight variation in the values of the constant a near the top of column (5): the gradual increase towards the bottom being due to the increasing effect of the ionisation of the hydrazone itself in the more dilute solutions.

This reaction affords a simple, quick and easy method of measuring the concentration of hydrogen ions in solution and therefore of determining the dissociation constants of acids. To determine the concentration of the hydrogen ions in a given acid solution, ·1 gram of oxalacetic hydrazone is heated with 100 c.c. of the solution to 100°C. and the amount of carbon dioxide evolved is estimated. Let this be b c.c. at 0° C. and 760 mm. Then the concentration of the hydrogen ions in the solution is given by

$$x = \frac{1}{a}\left(\frac{c_1}{b} - 1\right) = \frac{1}{50}\left(\frac{10}{b} - 1\right).$$

[1] Kohlrausch, *Wied. Ann.* xxvi. 161.

To test the method, experiments were made with an $\frac{N}{20}$ HCl solution. Four experiments gave 2·95, 2·75, 2·95 and 2·95 c.c. of carbon dioxide respectively. Taking the mean we get for the concentration of hydrogen ions $x = \cdot 049$. The value obtained from Krannhals' measurements of the conductivity at 100° C. is $x = \cdot 0475$. The two methods therefore give concordant results.

Strength of H_2SO_4 used	Volume of CO_2 from ·1 gm.	$ax = \frac{c_1}{c_2} - 1$	x from conductivity	a
$\frac{N}{10}$	2·42 c.c.	3·13	·06	52
$\frac{N}{15}$	3·17 "	2·15	·043	50
$\frac{N}{20}$	3·87 "	1·58	·0335	47
$\frac{N}{30}$	4·88 "	1·05	·023	46
$\frac{N}{60}$	5·95 "	·68	·013	52
$\frac{N}{120}$	6·85 "	·46	·007	65
$\frac{N}{\infty}$	8·5 "	·18	0	∞

The authors are making similar experiments with other derivatives of oxalacetic acid and are also investigating the velocity of the reaction. In conclusion they wish to thank Mr Fenton for his kindness in allowing them to proceed with this work, and Professors Liveing and Dewar for kindly placing the resources of their laboratories at their disposal.

The oxidation of Glucosone to Trioxybutyric Acid. By R. S. MORRELL, M.A., Gonville and Caius College.

[*Read* 3 February 1902.]

An aqueous solution of glucosone obtained by the action of hydrogen peroxide 20 vols. on glucose in the presence of ferrous sulphate (*C. S. J.* 1899, 345; 1900, 1219), was oxidized by bromine at 40° C. The excess of bromine was removed by a current of air, and the yellow liquid treated with excess of lead carbonate. After standing for 24 hours the liquid was filtered from lead bromide and excess of lead carbonate; sulphuretted hydrogen was passed into the solution, and after filtration, the filtrate concentrated in a vacuum on the water bath to a small bulk. After boiling with calcium carbonate and decolorising with animal charcoal the solution was concentrated in a vacuum on the water bath to a small bulk, and then poured into absolute alcohol. The calcium salt is obtained as a granular precipitate, very soluble in water. It was purified by treatment with oxalic acid and calcium carbonate and reprecipitating with alcohol. This was done several times, and the purified salt gave numbers agreeing with those required for calcium trioxybutyrate.

·417 gram calcium salt dried at 110° gave ·1780 gram $CaSO_4$; indicating 12·6 % Ca *12·9 % Ca

·2155 gram calcium salt dried at 110° gave

·239 gram CO_2	30·2 % C	*30·9 % C
·0895 gram H_2O	4·6 % H	*4·5 % H

·1995 gram calcium salt dried over sulphuric acid in a desiccator gave

·2072 gram CO_2	28·3 % C	†27·7 % C
·0872 gram H_2O	4·8 % H	†5·2 % H

* calculated for $(C_4H_7O_5)_2Ca$.

† calculated for $(C_4H_7O_5)_2Ca, 2H_2O$.

The calcium salt gave with normal lead acetate a precipitate which is insoluble in dilute acetic acid (characteristic test for trioxybutyric acid). Calcium trioxybutyrate obtained from erythrite by the action of nitric acid was found to have the same properties as this calcium salt prepared from glucosone.

Reduction of Calcium Trioxybutyrate by Hydriodic Acid and Phosphorus.

Fifteen grams of calcium trioxybutyrate were heated for 8 hours on a sand bath with 130 c.c. of hydriodic acid, B.P. 127°, and 5 grams of amorphous phosphorus. The brown liquid was diluted with an equal volume of water and shaken with ether. On evaporation of the ether a brown oil was left, which was treated with dilute sulphuric acid and zinc dust, and then distilled in a current of steam. The distillate was neutralised with calcium carbonate and the filtrate, after concentration on the water bath in a vacuum, was treated with silver nitrate. The silver salt precipitated was silver butyrate, which was confirmed by its peculiar odour and by a silver determination.

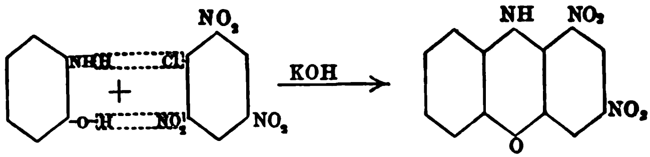
·2435 gram silver salt gave ·1345 gram silver; °/ₒ Ag = 55·4;
·1708 „ „ ·0948 „ °/ₒ Ag = 55·5;

calculated for $(C_4H_7O_2)$ Ag = 55·3.

The formation of Di-nitro-phenoxazines. By J. C. CROCKER, B.A., St John's College.

[*Read* 3 February 1902.]

In a paper by G. S. Turpin (*C. S. J.* 1891, 714), the use of picric chloride as a reagent for the isolation and characterisation of amines was recommended. It was there shewn that in the case of ortho-amido-phenol, the reaction did not take the normal course. The *o*-hydroxy-picramido derivative first formed, condensed in the presence of alkali, to a di-nitro-oxazine, with the elimination of nitrous acid from the hydroxyl and picryl radicles. Thus:—



At Dr Turpin's suggestion I have investigated the generality of the above reaction and the following results have been obtained by the reaction of picric chloride with various orthoxy-amido-compounds, in the presence of alkali.

Eikonogen (amido-β-naphthol sodium sulphonate).

The substance was dissolved in dilute alcohol and treated in hot solution with the calculated molecular proportions of picric chloride in dilute alcohol and with alcoholic soda. Some nitrogen was evolved in the reaction and from the purple-brown solution formed, there separated a muddy-looking precipitate, which was filtered off after cooling, and washed with water. Recrystallised from hot water, in which it is only moderately soluble, the substance formed minute bronze plates. The sodium determination shewed the body to be the expected substance, naphtho-di-nitro-phenoxazine-sodium sulphonate.

The bronze plates dried at 130° :—

I.	·2862 gm.	gave	·0465 Na_2SO_4
II.	·3048	„	·0473 „
III.	·3920	„	·0606 „

	I.	II.	III.	Calc.
Na	5·26	5·03	5·01	5·38

Amido-phenol-sulphonic acid ($NH_2 : OH = 1 : 2$).

The amido-phenol-sulphonic acid was dissolved in dilute alcoholic soda and added to the hot alcoholic solution of picric chloride, stirring meanwhile. The liquid became almost solid from the deposition of minute bronze plates. This was filtered after cooling, and washed with dilute alcohol, then recrystallised from hot water, in which it is only moderately soluble. 7·6 gm. of the amido-phenol-sulphonic acid gave 11·3 of the product: about 70 °/₀ yield. After drying at 150° for some time, the substance was obtained as a rouge-coloured powder and the percentage of sodium agreed with that required for di-nitro-phenoxazine-sodium-sulphonate.

I.	·4120 gm.	gave	·0760 Na_2SO_4
II.	·4157	„	·0763 „

	I.	II.	Calc.
Na	5·97	5·95	6·13

Its aqueous solution was dark-red and gave the characteristic violet-blue of the oxazines with hot potash solution. It is almost insoluble in alcohol and acetic acid.

Amido-β-naphthol ($NH_2 : OH = 1 : 2$).

To 1·95 gm. of the hydrochloride of amido-β-naphthol, dissolved in dilute alcohol, was added a hot solution of 1·2 gm. NaOH in dilute alcohol, then the mixture was stirred into a hot solution of 2·5 gm. picric chloride in dilute alcohol. On cooling and filtering a red-brown powder was left, which when recrystallised from aniline formed silky hair-like light-brown needles. The aniline was washed out with alcohol and the crystals were dried at 180°. Analysis agreed with the expected body, naphtho-di-nitro-phenoxazine.

I.	·2285 gm.	gave	25·2 c.c. N at 15° and 771 bar. pr.
II.	·1734	„	20·6 c.c. „ 25°·2 and 752 bar. pr.
III.	·1989	„	·4305 CO_2 and ·0550 H_2O

	I.	II.	III.	Calc.
N	13·11	13·30	—	13·01
C	—	—	59·02	59·44
H	—	—	3·07	2·79

The molecular weight of this body was determined.

The solution of ·1208 gm. substance in 20 gm. naphthalene lowered the freezing-point of the solvent ·135° C.

	Experiment	Theory
Molecular weight	313	323

When heated in a melting-tube the substance decomposed slowly at about 274° with the formation of a brown film.

Amidol (di-amido-phenol $OH : NH_2 : NH_2 = 1 : 2 : 4$).

2·5 gm. of picric chloride were dissolved in hot dilute alcohol, and the solution was mixed with a dilute alcoholic solution of ·98 gm. amidol hydrochloride, the temperature being kept at 55°—65°. ·7 gm. potash in alcohol (2½ molecules KOH) was run in carefully, drop by drop, from a burette, the liquid being stirred well. Minute dark-red plates settled out during the process and on cooling. The yield was 2·4 gm. It melted at 180—5°. The latter melting-point was obtained after twice crystallising from glacial acetic acid.

Analysis shewed the substance to be di-picramidol.

I. ·2190 gm. dried 100° gave 39·4 c.c. N at 20° and 759 bar. pr.

II. ·2370 gave 42·0 c.c. N at 22° and 762 bar. pr.

	I.	II.	Calc. $C_6H_3\begin{cases}OH\\NH.Pi^*\\NH.Pi\end{cases}$
Nitrogen	20·69	20·30	20·55

* $Pi = C_6H_2(NO_2)_3$.

The substance consisted of minute dark-red plates, difficultly soluble in alcohol, fairly easily soluble in glacial acetic acid, easily soluble in cold potash to a dark-brown solution, reprecipitated by HCl. This shewed the phenolic character of the body. Heated in a dry tube it first melted and then decomposed explosively.

The phenolic hydrogen is replaced by the acetyl group when the substance is acted on by a mixture of acetyl chloride and sodium acetate.

2·6 gm. of di-picramidol were heated for one hour in a reflux apparatus, with excess of acetyl chloride and sodium acetate. A yellow precipitate formed. The whole was poured into water, filtered and washed. The yield was 2·5 gm. of a substance, which melted, after recrystallisation from dilute acetic acid, at 223°. The substance crystallised in minute reddish-yellow plates.

·2670 gm. gave 45·3 c.c. N at 19° and 757 bar. pr.

		Calc.
Nitrogen	19·56	19·05

Hence the substance was di-picramidylic acetate

$$C_6H_3\begin{cases}O.Ac\\NH.Pi\\NH.Pi\end{cases}$$

When di-picramidol was dissolved in aniline, in which it is

extremely soluble, and alcohol added to the solution, a chocolate-brown crystalline precipitate was formed. These crystals contained aniline, and when heated evolved aniline and then exploded. The high melting-point (269°) suggested that condensation of the phenol-body to the oxazine had taken place and this was proved by the abundant presence of nitrite in the aniline liquors. Moreover, alcoholic potash gave no coloration in the cold, but when heated the blue-violet of the oxazines was immediately apparent. Owing to their extreme solubility in aniline, and the necessity for using alcohol as precipitant, these crystals were not obtained quite pure. The aniline found approached most nearly to two molecules of aniline of crystallisation. After evaporation of the aniline, the residual brown powder was crystallised from dilute acetic acid. It was then obtained as a dark-bronze powder, M.P. 277°. Analysis gave results in accordance with picramido-di-nitro-phenoxazine.

·2115 gm. gave 37·0 c.c. N at 18·5° and 751 bar. pr.

$$C_6H_3\begin{cases} NH.Pi \\ NH \\ O \end{cases}\!\!\!>C_6H_2(NO_2)_2$$

Nitrogen	20·04	19·68

Attempts to get the oxazine direct from amidol only resulted in a small yield of oxazine, which deposited first, and a good yield of the phenol-body. Excess of KOH should be carefully avoided. The use of alkaline sulphite in place of alkali makes the process simpler and gives an excellent product, but the yield of the phenol-body is not so good. It was not found possible to condense the phenol-body by the use of strong alkalis like potash. Feebly alkaline bodies, e.g. aniline or sodium phosphate, on the other hand, usually cause the oxazine condensation. The following method gave satisfactory results.

4 gm. of the phenol-body were added to 350 c.c. alcohol and heated on water-bath. 46 gm. of pulverised sodium phosphate were gradually added while stirring. The substance dissolved and minute bronze sheeny plates immediately settled out. The substance, recrystallised from glacial acetic, consisted of minute bronze-coloured plates, melting at 277°, which give the characteristic oxazine reaction with potash. It was difficultly soluble in alcohol, and fairly easily soluble in glacial acetic acid. The yield of crude product was 3·5 gm. Analysis gave results in accordance with picramido-di-nitro-phenoxazine

$$C_6H_3\begin{cases} NH.Pi \\ NH \\ O \end{cases}\!\!\!>C_6H_2(NO_2)_2.$$

I. ·1266 gm. gave 21·8 c.c. N at 20° and 757 bar. pr.
II. ·2127 „ ·3370 CO_2 and ·0420 H_2O.

	I.	II.	Calc.
Carbon	—	43·20	43·30
Hydrogen	—	2·16	1·80
Nitrogen	19·80	—	19·68

Amidol represents a limiting case, where the influence of the second amido-group prevents the usual condensation taking place, except to a slight extent; the condensation however being subsequently effected by weakly alkaline reagents.

Incidentally the reaction of picric chloride and alkali on amido-thymol was examined. Only one picryl group entered, the phenol OH, as might be expected, remaining intact.

Amido-thymol ($CH_3 : NH_2 : C_3H_7 : OH = 1 : 2 : 4 : 5$).

One molecular proportion each of picric chloride and amido-thymol were separately dissolved in hot dilute alcohol. Two molecular proportions of soda in dilute alcohol were now added rapidly, stirring meanwhile. The solution was now diluted with an equal volume of water and allowed to cool, when minute bronze-coloured plates separated out together with a little "tar." These were filtered and washed with dilute alcohol. Crystallised from a mixture of acetic and alcohol, a reddish bronze crystalline powder separated out, melting at 209°. On drying at 150° a purple-brown powder was obtained and the M.P. was raised to 212°. The substance was easily soluble in alcohol or acetic acid. Soda easily dissolved it to a red solution which darkened to reddish-brown. Hence the hydroxyl group was probably intact. Crystallised from strong alcohol the substance was obtained as dark-red needles. A nitrogen determination shewed it to be the expected substance picramido-thymol.

I. ·1786 substance gave 23·2 c.c. N at 18° C. and 752 bar. pr.
II. ·2770 „ „ 35·9 „ 17° „ 751 bar. pr.

	I.	II.	Calc.
Nitrogen	14·94	14·94	14·89

Calculated for $C_6H_2(CH_3)(C_3H_7)(OH)NH.C_6H_2(NO_2)_3$.

The interaction of Thiocyanates, Picric Chloride, and Alcohols. By J. C. CROCKER, B.A., St John's College.

[*Read* 3 February 1902.]

The action of picric chloride on lead and mercury thiocyanates in benzene and toluene solutions, was investigated by Dixon (*C. S. J.* 1896, 868). He could observe no reaction under these conditions. When, however, picric chloride is allowed to act on ammonium thiocyanate in absolute alcoholic solution a peculiar reaction readily takes place. A yellow solid substance is formed and hydrochloric acid is found in solution.

10 gm. of picric chloride were dissolved in absolute alcohol and the hot solution was mixed with a hot solution of 3·2 gm. ammonium thiocyanate in absolute alcohol. The liquid almost immediately became turbid and an oil separated out which quickly solidified to a crumbling mass. On cooling, this was filtered off, washed with water, alcohol and ether. The yield was 10 gm. After recrystallisation from glacial acetic acid diluted with alcohol (1:1) the product appeared as golden-yellow, compact, heavy crystals, insoluble in water or ether, difficultly soluble in alcohol, and easily soluble in glacial acetic acid. The crystals melted at 138°. Analysis gives the empirical formula $C_{15}H_9N_7SO_{13}$.

The substance must thus contain two picryl groups, one of which, as will be shewn later, is attached to a nitrogen atom. The substance is very stable and undergoes changes with difficulty, its stability towards reagents being probably due in some measure to its insolubility in most solvents. When boiled with very concentrated potash, ammonia is evolved and ethyl alcohol can be detected, by the iodoform reaction, in the distillate. The substance is perfectly unchanged when heated to 100° for several hours, but if the temperature is raised to 130° for a short time, the whole mass is completely charred—owing to the high percentage of NO_2 groups contained in the body.

The production of ethyl alcohol by potash distillation suggested the presence of an ethoxy-group and the amount of this was subsequently estimated by the Zeisel method, an additional bulb of dilute $CuSO_4$ solution being interposed to absorb the sulphuretted hydrogen produced, the whole apparatus being kept at 85°—90°.

The view that has been taken with regard to the formation and constitution of this body is as follows. The picryl chloride and thiocyanate probably first react to form picryl thiocyanate,

$$C_6H_2(NO_2)_3 - \boxed{Cl} + \boxed{NH_4} - S - C \equiv N$$
$$\rightarrow C_6H_2(NO_2)_3 - S - C \equiv N.$$

Another picryl chloride now adds on across the triple "bond" in the $-C\equiv N$ group, the chlorine going to the carbon and the picryl radicle to the nitrogen,

$$C_6H_2(NO_2)_3-S-C\equiv N+C_6H_2(NO_2)_3Cl \rightarrow C_6H_2(NO_2)_3-S-\underset{\textstyle Cl}{\underset{|}{C}}=\underset{\textstyle C_6H_2(NO_2)_3}{\underset{|}{N}}$$

This would be a strongly acidic chloride, and would act on the alcohol, giving hydrochloric acid and an ethoxy-body,

$$C_6H_2(NO_2)_3-S-\underset{\textstyle Cl}{\underset{|}{C}}=\underset{\textstyle C_6H_2(NO_2)_3}{\underset{|}{N}}+EtOH \rightarrow C_6H_2(NO_2)_3-S-\underset{\textstyle EtO}{\underset{|}{C}}=\underset{\textstyle C_6H_2(NO_2)_3}{\underset{|}{N}}+HCl$$

This body is a picrimido-picryl-thio-carbonic ester. Putting Pi for $C_6H_2(NO_2)_3$, the relation to carbonic ester is easily seen:

$$Pi-S-\underset{\textstyle N-Pi}{\underset{\|}{C}}-O-Et$$

If this view of the constitution is correct, the body should hydrolyse somewhat in this way:

$$Pi-S-\underset{\textstyle N-Pi}{\underset{\|}{C}}-O-Et+\begin{array}{c}O\\H_2\end{array} \rightarrow PiNH_2+Pi-S-\underset{\textstyle O}{\underset{\|}{C}}-O-Et$$

And indeed one of the products of hydrolysis is picramide. The above explanation suggested that different alcohols should give new bodies of similar structure. Both methyl and normal propyl alcohols gave corresponding derivatives. It was somewhat unexpected, however, to discover that there were two isomeric methoxy-derivatives, easily transformable one to the other. Mere crystallisation from different solvents effected the transition, the form produced in some cases being dependent on the concentration of the solutions. The more stable form melted at 169° and the less stable at 157—8°. The former occurred in woolly masses of a pale yellow colour, while the latter appeared as compact, heavy, brilliant crystals of a golden yellow colour, so alike the ethoxy-body as to be indistinguishable from it, by mere inspection. The analysis in both cases gives the empirical formula $C_{14}H_7N_7SO_{13}$.

The number of methoxy-groups was determined in both cases and was found to correspond to that required by theory.

The β-isomer (169°) is prepared in the same way as the ethoxy-body, using methyl alcohol instead of ethyl alcohol.

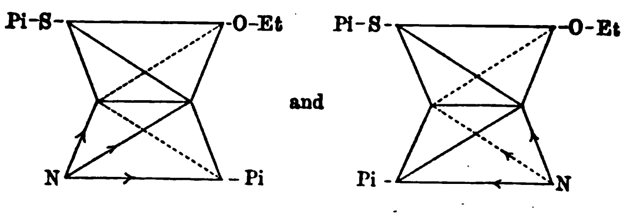
2·5 gm. picric chloride and ·8 gm. NH_4SCN were dissolved separately in methyl alcohol, and then mixed while hot. Separation of an oil and a crumbling precipitate almost immediately took place. This was filtered and washed with water, alcohol and ether. The mother-liquor was found to be acid in this case also. The product on crystallisation from glacial acetic acid or acetic acid diluted with methyl alcohol, appeared as a light-yellow mass, M.P. 169°. It was difficultly soluble in alcohol or ether, easily soluble in glacial acetic acid.

The α-isomer which was investigated first, is obtained from the β-isomer.

5 gm. of the β-isomer (169°) were crystallised from 140 c.c. of ethyl alcohol and acetic acid (1 : 1), and the crystallisation carefully watched.

After six hours the product was filtered off rapidly and washed with alcohol and ether. The yield was 1·2 gm. of the α-isomer (157—8°). The mother-liquor deposited the β-isomer. The α-isomer could not be further purified by crystallisation since it was partially retransformed to the β-isomer. It could be entirely converted to the β-isomer by simple crystallisation from methyl alcohol and glacial acetic (1 : 1).

The ease with which these bodies can be transformed one to the other, suggests stereo-isomerism, and this can be easily explained on the Hartzsch-Werner theory of the trivalent nitrogen atom. The forms would then correspond to the "syn" and "anti" configurations.



Which structure to assign to each isomer has not yet been determined, and further experiments are in progress to elucidate these points.

These compounds are also of interest, since they afford means of obtaining well-crystallised derivatives of the alcohols, which may serve for their characterisation and identification.

Note on the Reduction of a Ternary Quantic to a Symmetrical Determinant. By Dr A. C. DIXON.

[*Received* 25 January 1902.]

The reduction of a ternary quartic to the form of a symmetrical determinant of the fourth order with linear constituents is important in the theory of the bitangents. The object of this note is to discuss the corresponding problem for a plane curve of any degree.

Let U be an n-ic in x, y, z. The problem is to express U as a symmetrical determinant of order n whose constituents shall be linear in x, y, z.

Consider the system of curves of degree $n-1$ touching $U=0$ in $\frac{1}{2}n(n-1)$ points. The system will be $(n-1)^{\text{ply}}$ infinite, since there are $\frac{1}{2}(n+2)(n-1)$ parameters subjected to $\frac{1}{2}n(n-1)$ conditions. Let $v_{11}=0$ be one of the curves, and denote its points of contact with $U=0$ collectively by α_1. Let the most general $(n-1)^{\text{ic}}$ through α_1 be

$$\lambda_1 v_{11} + \lambda_2 v_{12} + \lambda_3 v_{13} + \ldots + \lambda_n v_{1n} = 0,$$

where $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the arbitrary parameters.

Then the curve $v_{12}^2 = 0$ passes through all the intersections of $v_{11}=0$, $U=0$. Hence quantics v_{22}, w_{1122} must exist of degrees $n-1$, $n-2$ such that

$$v_{12}^2 = v_{11}v_{22} - Uw_{1122}.$$

In the same way quantics v_{rs}, w_{11rs} must exist such that

$$v_{1r}v_{1s} = v_{11}v_{rs} - Uw_{11rs}.$$

The $(n-1)^{\text{ics}}$ $v_{22}, v_{33}, \ldots, v_{nn}$ will each touch U in $\frac{1}{2}n(n-1)$ points and v_{rs} will meet U in the points of contact of v_{rr}, v_{ss}. The products $v_{rs}v_{ij}$, $v_{ri}v_{sj}$, $v_{rj}v_{si}$ differ by expressions in which U is a factor. The $(n-1)^{\text{ic}}$ $V=0$, where $V \equiv \lambda_1^2 v_{11} + \lambda_2^2 v_{22} + \ldots + 2\lambda_1\lambda_2 v_{12} + 2\lambda_1\lambda_3 v_{13} + \ldots$, touches $U=0$ at $\frac{1}{2}n(n-1)$ points and these points lie on each of the curves $\dfrac{\partial V}{\partial \lambda_1}=0$, $\dfrac{\partial V}{\partial \lambda_2}=0, \ldots$.

The determinant of the expressions v, say Δ, is the discriminant of V as a quadratic in $\lambda_1, \lambda_2, \ldots$, and we have seen that each of its minors of the second order contains U as a factor. Hence each minor of order r contains U^{r-1} as a factor. The determinant Δ itself is then a constant multiple of U^{n-1} and its first minors contain the factor U^{n-2}, whose degree is $n(n-2)$. But each first minor is of the degree $(n-1)^2$ and is therefore equal to U^{n-2} multiplied by a linear expression. Let $U^{n-2}\beta_{rs}$ be the minor of v_{rs}. The determinant formed by these minors is the $(n-1)^{\text{th}}$

power of the original determinant Δ and is therefore a constant multiple of $U^{(n-1)^2}$. Taking the factor U^{n-2} out of each row we have the result that the determinant of the linear expressions β is a constant multiple of U, so that the desired reduction has been accomplished, unless the determinant Δ of the expressions v is identically zero.

Now this will be the case if $\lambda_1, \lambda_2, \ldots \lambda_n$ can be so chosen that V contains a squared linear factor. Suppose for instance that $v_{11} = x^2\phi$, ϕ being of the degree $n-3$. Then we may take $v_{12} = xy\phi$, $v_{13} = xz\phi$, $v_{22} = y^2\phi$, $v_{23} = yz\phi$, $v_{33} = z^2\phi$ and also

$$v_{1r} = x\chi_r, \quad v_{2r} = y\chi_r, \quad v_{3r} = z\chi_r \quad (r = 4, 5, \ldots n).$$

For v_{1r} is an $(n-1)^{\text{ic}}$ through the n points in which $x=0$ meets U and hence v_{1r} must contain x as a factor; similarly v_{2r} contains y and v_{3r} contains z. The other intersections with U are the same for v_{1r}, v_{2r}, v_{3r}. Hence the second $(n-2)^{\text{ic}}$ factor is the same for each.

The first three rows of Δ are now the same but for the factors x, y, z; hence Δ vanishes with all its first minors and the method fails.

Suppose on the other hand that no curve of the system V breaks up into an $(n-3)^{\text{ic}}$ and a double straight line. Take any line, say $z=0$, cutting $U=0$ in n distinct ordinary points, $c_1, c_2, \ldots c_n$. Then we may take v_{11} to touch U at $c_2, c_3, \ldots c_n$, v_{22} at $c_1, c_3 \ldots c_n$, v_{rr} at $c_1, \ldots c_{r-1}, c_{r+1} \ldots c_n$. The curves of the system V thus determined are all different, since otherwise a curve of the system would consist in part of the line $z=0$, which is against our supposition. It may be noticed also that the curves $v_{11}, v_{22} \ldots$ are uniquely determined, for the condition that V should pass through a given point is generally a quadratic in $\lambda_1, \lambda_2 \ldots$, but when the point lies on $U=0$ the quadratic expression becomes a perfect square and the condition is linear.

Since now $$v_{11}v_{22} - v_{12}^2 = Uw_{1122}$$

$v_{12} = 0$ passes through $c_1, c_2, \ldots c_n$ and must contain z as a factor; the same is true for $v_{13}, v_{14} \ldots$. Hence the only terms in Δ that do not contain z are those in the leading term $v_{11}v_{22} \ldots v_{nn}$ which certainly exist, and therefore Δ does not vanish identically.

There is therefore one reduction of the ternary quantic to the form of a symmetrical determinant with linear constituents for every theta-function of even characteristic which does not vanish for zero values of the arguments. (Compare Baker, *Abelian Functions*, pp. 268—270.)

When U has been thus reduced the functions $v_{11}, v_{12} \ldots$ are the first minors of the determinant and V may be derived by bordering U with a row and column each consisting of the quantities $\lambda_1, \lambda_2, \ldots \lambda_n, 0$.

The Zeros of a Polynomial. By J. H. GRACE, M.A., Peterhouse.

[*Received* 11 November 1901.]

(1) Between two real zeros of a polynomial with real coefficients there is at least one real zero of the derived function. I regard this theorem as giving a limitation for the roots of $f'(z)=0$ when two roots of $f(z)=0$ are given, and I propose to consider the more general question, viz., When two roots real or imaginary of the equation $f(z)=0$ whose coefficients are possibly imaginary are given, do any corresponding limitations exist for the roots of $f'(z)=0$? It will be found that if A, B represent the given roots in the Argand diagram, then there is at least one root of the equation $f'(z)=0$ within a circle whose centre is the middle point of AB and whose radius is $\frac{1}{2}AB\cot\frac{\pi}{n}$, where n is the degree of the polynomial $f(z)$.

(2) *If all the roots of $f(z)=0$ lie inside a given oval curve in the Argand diagram, then so also do all the roots of $f'(z)=0$.*

This well-known theorem can be easily established by elementary mechanical considerations. In fact if the roots of $f(z)=0$ be the positions of equal centres of force attracting according to the law of the inverse distance, then the roots of $f'(z)=0$ represent the equilibrium points. Now if all the poles are on one side of a straight line, all the equilibrium points are on that side, because for points on the line or on the opposite side of the line the components of force perpendicular to the line are all in the same direction. Hence allowing the line to envelope an oval curve enclosing all the centres of force, we see that all the equilibrium points are within the oval. The same reasoning applies to any convex polygon enclosing the centres of force, and it applies even in the extreme case in which all the centres lie on the boundary, except that if they all lie on the same straight line then all the equilibrium points lie on that line.

(3) Suppose now that the oval enclosing the roots of $f(z)=0$ is a circle, and consider the effect of inverting with respect to an external point O. Bearing in mind that this is equivalent to a homographic transformation of z such that the point at infinity becomes O and also that the expression $f'(z)$ is the first polar of the point at infinity, we infer from the theorem of (2) that if all

the roots of $f(z)=0$ lie within a circle, then all the roots of the first polar of any external point lie within that circle. The result is still true when all the zeros of $f(z)$ lie on the circle, but if O be also on the circle, then all the zeros of the first polar are also.

Similarly if all the zeros lie outside the circle, then all the zeros of the first polar of an internal point lie outside the circle.

(4) *If the form $f(z)$ be apolar to a given form ϕ, then it has a zero lying within any circle enclosing all the roots of $\phi=0$.*

For a moment let the forms be homogeneous, i.e. replace z by x_1 and make the expression homogeneous by means of x_2.

Then let

$$f(x_1x_2)=a_x{}^n,$$

and let $\phi(x_1x_2)=b_x{}^n$ be the apolar form so that we have $(ab)^n=0$, or if the zeros of f be $(y_1, y_2)(z_1, z_2)\dots(w_1, w_2)$ we have

$$b_yb_z\dots b_w=0.$$

Thus of the n ratios $(y_1, y_2)\dots(w_1, w_2)$ all but one may be chosen arbitrarily and the nth is then given by the mixed polar of the preceding values with respect to

$$b_x{}^n=0.$$

Now returning to the Argand diagram suppose that the roots of the fixed apolar form

$$\phi(z)=0$$

are all inside a given circle S and suppose that $P_1, P_2, \dots P_n$ are the roots of

$$f(z)=0,$$

so that P_n is determined by equating to zero the mixed polar of $P_1, P_2, \dots P_{n-1}$ with respect to $f(z)$.

If P_1 be outside S then all the roots of its first polar $f_1(z)=0$ are inside S, if P_2 be outside S then all the roots of its first polar with respect to $f_1(z)$, say $f_2(z)=0$, are inside S, and so on.

But by continued polarization we come to a linear form

$$f_{n-1}(z)=0,$$

which determines P_n, hence if $P_1, P_2, \dots P_{n-1}$ are all outside the circle S then P_n is inside the circle, i.e. the equation $f(z)=0$ has at least one root within a circle enclosing all the roots of the fixed equation

$$\phi(z)=0.$$

If the circle S contains all the roots of $\phi(z)$, then when $P_1, P_2, \dots P_{n-1}$ are all on the circle so also is P_n.

(5) *If the coefficients of the form $f(z)$ satisfy any linear homogeneous relation, then the equation $f'(z)=0$ always has a root within a certain circle.*

Let $$f(z) = a_0 z^n + a_1 z^{n-1} + a_2 z^{n-2} \dots + a_n,$$
and suppose the relation in question is
$$a_0 b_0 + a_1 b_1 + a_2 b_2 \dots + a_n b_n = 0.$$

Now $f(z)$ is apolar to the form
$$\phi(z) \equiv c_0 z^n + c_1 z^{n-1} \dots + c_n,$$
when
$$a_0 c_n - \frac{1}{n} a_1 c_{n-1} + \frac{2}{n(n-1)} a_2 c_{n-2} - \frac{3!}{n(n-1)(n-2)} a_3 c_{n-3} \dots = 0,$$
so that in our case $f(z)$ is apolar to the form given by
$$c_n = b_0,$$
$$c_{n-1} = -nb_1,$$
$$c_{n-2} = +\frac{n(n-1)}{2} b_2,$$
etc.,

i.e. to the form
$$\psi(z) \equiv b_0 - nb_1 z + \frac{n(n-1)}{2} b_2 z^2 \dots (-1)^n b_n z^n,$$

and hence it always has a root within any circle enclosing all the roots of $\psi(z)=0$. If we choose the least of these circles we have a definite circle within which there is always a root of the equation
$$f(z) = 0.$$

(6) Now let $f'(z)$ be given, namely
$$f'(z) = p_0 z^{n-1} + p_1 z^{n-2} \dots p_{n-1},$$
so that
$$f(z) = \frac{p_0}{n} z^n + \frac{p_1}{n-1} z^{n-1} \dots + p_{n-1} z + p_n,$$
where p_n is an arbitrary constant.

Hence if $f(z)$ has two given roots α and β, we have
$$\frac{p_0}{n}(\alpha^n - \beta^n) + \frac{p_1}{n-1}(\alpha^{n-1} - \beta^{n-1}) + \dots p_{n-1}(\alpha - \beta) = 0,$$
a homogeneous linear relation between the coefficients of $f'(z)$.

Consequently by the result just proved $f'(z)$ is apolar to the form

$$\frac{\alpha^n - \beta^n}{n} - z\frac{n-1}{n-1}(\alpha^{n-1} - \beta^{n-1}) + z^2\frac{(n-1)(n-2)}{2}\frac{1}{n-2}(\alpha^{n-2} - \beta^{n-2})$$
$$+ z^3\frac{(n-1)(n-2)(n-3)}{3!}\frac{1}{n-3}(\alpha^{n-3} - \beta^{n-3}) \ldots = 0,$$

or to

$$(\alpha^n - \beta^n) - nz(\alpha^{n-1} - \beta^{n-1}) + \frac{n(n-1)}{2}z^2(\alpha^{n-2} - \beta^{n-2}) \ldots = 0,$$

that is to the form of degree $n-1$,

$$(\alpha - z)^n - (\beta - z)^n.$$

Thus if $f(z) = 0$ has two given roots α and β, then $f'(z)$ is apolar to the form $(\alpha - z)^n - (\beta - z)^n$.

(7) Now the roots of the equation

$$(\alpha - z)^n - (\beta - z)^n = 0$$

are easily constructed, for they are given by

$$(\alpha - z) = \omega(\beta - z),$$

where ω is any nth root of unity which is not unity itself.

To construct the points representing these roots take A, B for the points α, β and let O be the middle point of A, B, then we have

$$\frac{z-\alpha}{z-\beta} = e^{\frac{2r\pi}{n}\iota}, \quad r = 1, 2, \ldots n-1;$$

$$\therefore \frac{z - \dfrac{\alpha+\beta}{2}}{\dfrac{\alpha-\beta}{2}} = \iota\cot\frac{r\pi}{n};$$

$$\therefore z = \frac{\alpha+\beta}{2} + \iota\cot\frac{r\pi}{n}\frac{\alpha-\beta}{2},$$

which shews that to arrive at z we have to travel a distance

$$OA\cot\frac{r\pi}{n}$$

from O along a line at right angles to AB.

The greatest distance of one of these $(n-1)$ points from O is

$$OA\cot\frac{\pi}{n},$$

and hence all the roots[1] of

$$(\alpha - z)^n - (\beta - z)^n = 0$$

lie within a circle whose centre is O and whose radius is

$$OA \cot \frac{\pi}{n}.$$

But $f'(z)$ is apolar to the form

$$(\alpha - z)^n - (\beta - z)^n,$$

and hence $f'(z)$ must have at least one zero within this circle.

Cor. The zeros of $f'(z)$ cannot all lie on the same side of the line bisecting AB at right angles.

This follows from the above reasoning by regarding the line in question as a circle of infinite radius which for the purpose of our argument practically encloses all the roots of

$$(\alpha - z)^n - (\beta - z)^n = 0.$$

(8) The results just arrived at can be illustrated geometrically when the equation $f(z) = 0$ is a cubic.

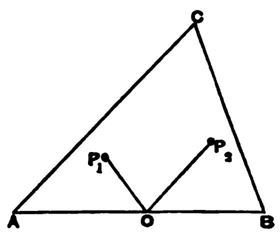
In fact suppose that A, B, C represent the roots of

$$f(z) = 0,$$

and that P_1 and P_2 represent those of

$$f'(z) = 0,$$

then P_1, P_2 are the foci of the maximum ellipse inscribed in the triangle ABC.



Hence if O be the middle point of AB we have

$$OP_1 \,.\, OP_2 = \text{square of semi-diameter parallel to } AB$$

$$= AB^2 \,.\, \frac{3}{4},$$

so that if A and B are given we have

$$OP_1 \,.\, OP_2 = 3OA^2;$$

[1] Except two which lie on the circle. If $n > 3$ some roots certainly lie inside the circle, but if $n = 3$ all the roots lie on the circle and we can construct an apolar form having its two roots on this circle.

hence either P_1 or P_2 lies inside a circle whose centre is O and whose radius is $OA\sqrt{3} = OA \cot\frac{\pi}{3}$, unless both points lie on this circle.

Further since OP_1, OP_2 are equally inclined to AB it follows that they are on different sides of the line bisecting AB at right angles.

Oxidation in Presence of Iron. By HENRY J. HORSTMAN FENTON, M.A., F.R.S.

[*Received* 1 March 1902.]

The remarkable influence which is exerted by traces of iron salts in determining and regulating the oxidation of various organic substances was first observed by the author in 1876, and the observation has since opened up a very wide and fruitful field for investigation. Since the researches which have been published in this direction extend over a considerable period and have appeared in various journals, it has been considered advisable to communicate to the Society a brief summary of the work together with an account of recent results not hitherto published.

The function of the iron in these reactions is twofold: in the first place it increases the *activity* of the oxidizing agent to a remarkable extent, and secondly it has a *selective* or *regulating* influence so that certain groups are attacked and others left. This second influence may be regarded as the essential and novel feature of the actions referred to, oxidations being effected in this way which are not possible by any other means; the mere increase of activity may of course be induced by other agents and is not peculiar to iron.

It has been considered necessary to emphasize this point and also to draw attention to the dates of publication, since considerable misconceptions with regard to these matters appear to exist. [Compare G. Ollendorff, *Inaugural Dissertation*, Berlin, July 1900, 11.]

The first substance investigated in this direction was tartaric acid; it was observed that this acid, or its salts, when treated with certain oxidizing agents in presence of ferrous iron gave, on addition of caustic alkali, a beautiful violet colour [*Chemical News*, 1876, 33. 190; 1881, 43. 110]. The explanation of this effect and the isolation and identification of the active product was at first a matter of very considerable difficulty owing to the unstable character of the substance in aqueous solution. In 1894 this product was isolated and proved to be the hitherto missing dioxymaleic acid. From this acid a large number of interesting derivatives have been obtained, and its decompositions and transformations afford direct and simple means of preparing various other compounds which can, by other methods, be obtained only with great difficulty or not at all.

DIOXYMALEIC ACID.

$$\begin{array}{c} OH.C.COOH \\ \| \\ OH.C.COOH. \end{array}$$

[*Trans. Chem. Soc.* 1894. 899, 1896. 546, *Proc. Chem. Soc.* 1898, etc.]

This acid is prepared by slowly adding strong hydrogen dioxide to a nearly saturated solution of tartaric acid containing a little ferrous tartrate, or other ferrous salt, and the separation of the product is effected by the cautious addition of fuming sulphuric acid or other dehydrating agent. The operations must be conducted at a low temperature and the mixture kept in motion by means of a current of air or other device. After standing for a short time the acid begins to separate in lustrous, transparent, diamond-shaped plates. These, the dihydrate $C_4H_4O_6 . 2H_2O$, when isolated, are remarkably stable at the ordinary temperature and lose their water of crystallization at 80—90°. They are very sparingly soluble in cold water and the aqueous solution is unstable, decomposing slowly at the ordinary temperature and very rapidly at 50—60°. The aqueous solution acts as a powerful reducing agent, it gives with ferric salts a blackish colour which is changed to an intense and beautiful violet by addition of caustic alkalis; and with phenylhydrazine acetate it gives the *salt* $C_4H_4O_6 . 2N_2H_3Ph$ which separates in brilliant silver-like plates.

The di-methyl and di-ethyl esters are crystalline solids and both exist in two well-marked distinct crystalline forms. The diethyl ester has the remarkable property of becoming liquid when kept in dry air although it is permanent in moist air or in a vacuum desiccator; this property has recently been explained in a short communication to the Society [Fenton and Ryffel, *Proc. Camb. Phil. Soc.* 1901. 109].

The molecular weight of the acid was arrived at by the vapour pressure method of Will and Bredig and was confirmed by the freezing-point method in case of the methyl ester. The constitution of the acid follows from various considerations, among which may be mentioned the formation of di-acetyl or di-benzoyl derivatives both of the acid and esters, the di-basicity of the acid, the formation of salts only by the actions of phenylhydrazine and hydroxylamine, and the colour reactions with ferric chloride.

Dioxyfumaric Acid.

$$\begin{array}{c} OH.C.COOH \\ \| \\ COOH.C.OH. \end{array}$$

When dioxymaleic acid is treated with a concentrated solution of hydrobromic acid it undergoes a remarkable change in crystalline form without change of composition, the resulting product consisting of long hair-like needles. From the mode of formation of this modification and from the fact that its acid aniline salt appears to be more stable than that of the previously described modification, there seems little doubt that it is the fumaroid form.

Both dioxymaleic and dioxyfumaric acids are fairly strong acids, their affinity-constants approaching that of oxalic acid. [Skinner, *Trans. Chem. Soc.* 1898. 483.]

Dioxytartaric Acid.

$$\begin{array}{l} C(OH)_2.COOH \\ | \\ C(OH)_2.COOH. \end{array}$$

[*Trans. Chem. Soc.* 1895. 48; 1898. 71, 167 and 472.]

When dioxymaleic acid is covered with glacial acetic acid and treated with rather more than the calculated quantity of bromine, at the ordinary temperature, it is oxidized almost quantitatively to dioxytartaric acid

$$C_4H_4O_6.2H_2O + Br_2 = C_4H_6O_8 + 2HBr.$$

In this reaction the presence of water is essential, and it is shewn that the change is a reversible one, since dioxytartaric acid when heated with excess of hydrogen bromide yields dioxymaleic acid and *free bromine.*

In this manner large quantities of pure dioxytartaric acid may be prepared in a very short time as a white crystalline powder melting at 114—115°. Previously this acid had only been obtained in relatively minute quantities from the sodium salt by the action of hydrogen chloride in ether, the product so prepared melting at 98°.

Dioxytartaric acid is very easily soluble in water and is easily recognised by its sparingly soluble sodium salt and its characteristic osazone. It may be reduced to dioxymaleic acid either by hydrogen bromide as above mentioned, or by zinc and sulphuric acid.

The stability of this acid is much greater than would be expected from its remarkable constitution, and the dry substance may be heated to 90° without loss of weight.

Preparation of Tartronic Acid.

An aqueous solution of dioxytartaric acid when heated on a water-bath to about 60—70° decomposes quantitatively into tartronic acid and carbon dioxide

$$C_4H_6O_8 = C_3H_4O_5 + H_2O + CO_2,$$

and after concentration of the solution by evaporation the acid is obtained in a pure state in the form of long transparent prisms. This reaction affords probably the simplest and most direct method for the preparation of tartronic acid.

The affinity-constant of tartronic acid as previously determined was abnormal, being smaller than that of malonic acid; but Skinner [*Trans. Chem. Soc.* 1898. 489] taking advantage of this new method of preparation, has redetermined the value and obtains the number 0·5, that of malonic acid being 0·158.

Salts of dioxytartaric acid and separation of the alkali metals.

Hitherto the only metallic salt of this acid which was known with certainty was that of sodium, the barium salt of Barth [*Sitz. Acad. Wien*, 82. ii. 1024] being of doubtful composition. Having now command of the free acid in quantity a further study of the salts was undertaken. Especially interesting is a comparison of the solubilities of the alkali-metal dioxytartrates, all of which are easily obtained by neutralization of the free acid with the respective carbonates. The ratio of the solubilities of these at 0° is as follows—

$$Li : Na : K : Rb : Cs = 2 : 1 : 66 : 166 : 576,$$

and it is evident therefore that the differences might be made use of in separating the alkali metals.

Calcium dioxytartrate, obtained by adding calcium chloride to a soluble dioxytartrate, is a remarkable substance, being obtained in the form of a perfectly colourless transparent jelly.

Quantitative Estimation of Sodium.

Sodium dioxytartrate is probably the least soluble sodium salt known, its solubility at 0° being 0·039. Since no direct method for the quantitative estimation of sodium had hitherto been known, experiments were made with a view of devising such a method by the use of dioxytartaric acid. The free acid gives a partial

precipitation even with the sodium salts of strong acids, such as sodium chloride or sulphate, and the precipitation is completed by neutralization with ammonia or potassium carbonate; but since the dioxytartrates of ammonium and potassium are also sparingly soluble, the best method was found in using a solution of the potassium salt itself as precipitant. The latter is added in excess to a neutral solution of the sodium salt to be estimated and the mixture kept at 0° for about half an hour with occasional stirring; under these conditions a complete precipitation of sodium occurs. Instead of weighing the sodium dioxytartrate the simplest method is to dissolve it in excess of dilute sulphuric acid and titrate with potassium permanganate, the reaction being sharp and definite at the ordinary temperature and taking place according to the relation—

$$C_4H_6O_8 + 3O = 4CO_2 + 3H_2O.$$

This volumetric method gives excellent results if carefully performed. [Compare Sutton, *Volumetric Analysis*, 8th Edition, p. 66.]

Potassium dioxytartrate forms a most convenient and delicate reagent for the qualitative detection of sodium and easily indicates the presence of one part of sodium in about 2000 parts of water.

Glycollic Aldehyde or Diose.

$$\begin{array}{l} CH_2OH \\ | \\ CHO. \end{array}$$

[*Trans. Chem. Soc.* 1895. 774.]

Dioxymaleic acid is, as above stated, sparingly soluble in cold water; it dissolves easily in warm water but the solution is very unstable; decomposition begins even at the ordinary temperature and at 50—60° carbon dioxide is rapidly evolved. When the decomposition is complete it is found that approximately two molecules of carbon dioxide are evolved from one molecule of the acid, and the resulting liquid has very powerful aldehydic properties. It instantly restores the colour to Schiff's reagent, gives a silver 'mirror' with ammoniacal silver nitrate, reacts with phenylhydrazine and hydroxylamine and is easily oxidized by bromine. The product of the action of phenylhydrazine is found to be glyoxal osazone $\begin{array}{l} CH : N_2HPh \\ | \\ CH : N_2HPh \end{array}$ and the product of oxidation

is glycollic acid. From these and other considerations it is shewn that the substance produced is glycollic aldehyde. This aldehyde which is of special interest as being the first representative of the (aldose) sugars had hitherto only been obtained in aqueous solution and mixed with other substances, but by the present method it is easily isolated in the pure state and in the crystalline form. In the decomposition mentioned above a small quantity of an acid (apparently glyoxylic) is formed at the same time and, in order to obtain the pure aldehyde, the mixture is neutralized with chalk and the calcium salt removed by treatment with alcohol. On evaporation of the alcoholic solution in a vacuum desiccator and heating to 40°, the aldehyde is left in a pure state in form of a wax-like solid.

The molecular weight of this product as ascertained by the freezing-point method corresponds to the double formula $C_4H_8O_4$, but its aqueous solution still gives the same osazone as before with phenylhydrazine.

If the amorphous aldehyde be heated to about 100° in a vacuum the greater part undergoes polymerization in the manner to be described below, but during this operation crystals usually appear in the upper part of the apparatus. These crystals were subsequently further examined [Fenton and Jackson, *Trans. Chem. Soc.* 1899. 575] and found to be the pure aldehyde. In this case also the molecular weight determined by the freezing-point method initially indicates the doubled formula but, after the aqueous solution has remained standing for several hours, the number obtained exactly corresponds to the single formula. This point is of much interest in view of the fact that crystalline glyceraldehyde which had just previously been isolated by Wohl, and is the second representative of the aldose sugars, shews an exactly similar behaviour [Wohl, *Ber.* 1898, 31. 2394].

Glycollic aldehyde crystallizes in colourless transparent oblique plates melting at 95—97°. It has a sweet taste, is fairly easily soluble in water and may to a considerable extent be vaporized unchanged.

In order to prepare the crystalline aldehyde the initial purification above described is not necessary; dioxymaleic acid is decomposed by warming with water, and the resulting solution fractionally distilled under reduced pressure; the aldehyde passes over at 90—100° under a pressure of about 20 mm. as a syrupy liquid which crystallizes on standing.

Synthesis of Hexoses.

[*Trans. Chem. Soc.* 1897. 375.]

When glycollic aldehyde is heated in a vacuum to about 100° it partly vaporizes unchanged, as previously mentioned; the remainder is found on cooling as a solid transparent gum which is somewhat brittle and has a sweet taste. Analyses and molecular weight determinations by the freezing-point method shew that this product has the formula $C_6H_{12}O_6$. Its aqueous solution reduces Fehling's solution and gives various colour reactions characteristic of sugars, and when heated with water to 140° it yields furfural. After purification by treatment with alcohol it gives with phenylhydrazine acetate a normal hexosazone

$$C_{18}H_{22}N_4O_4.$$

If the product obtained from condensation of glycollic aldehyde by heat, and purification with alcohol as above described, be further heated for several hours in a vacuum to 100—106°, it becomes more brittle and its composition then approximates to the formula $C_6H_{10}O_5$.

The synthetical formation of sugars in the manner here described is of especial interest as tending to throw light on the natural formation of carbohydrates, more particularly when the following facts are considered.

When tartaric acid or its salts are exposed to sunlight and air in presence of small quantities of ferrous salts, a notable quantity of dioxymaleic acid is produced. For this experiment to succeed the three conditions *presence of iron*, *oxygen* and *sunlight* are absolutely necessary; in absence of any one of these conditions the change does not occur. Since dioxymaleic acid spontaneously decomposes, even at ordinary temperatures, in aqueous solution giving glycollic aldehyde, and the latter readily undergoes condensation to hexose (and probably to hexose 'anhydrides' also) it is evident that a theory as to the natural formation of carbohydrates may be formulated which perhaps compares favourably in some respects with the popular 'formaldehyde hypothesis' of Baeyer. [Compare Fenton, *B. A. Report*, 1895.]

With regard to the chemical nature of the sugar obtained in the manner here described it may be remarked that it closely resembles the condensation products obtained by Löew and by Fischer from formaldehyde and by Fischer and Tafel from 'glycerose.' In order to throw further light upon the matter glycollic aldehyde was afterwards condensed in presence of dilute alkalis instead of by heat, and from this product a hexosazone was obtained which corresponds exactly in melting point and

other properties with *β-acrosazone* (Fenton and Jackson, *B. A. Report*, 1899). Later, Jackson (*Trans. Chem. Soc.* 1900. 129) by similar treatment found also an osazone closely resembling α-acrosazone.

It is not improbable that glycollic aldehyde may be produced as an intermediate stage in the formation of 'formose' from formaldehyde, and that the two condensation products may be identical; this suggestion is supported by the fact that Pechmann (*Ber.* 1897. 2459) has obtained glyoxalosazone from formaldehyde and accounts for its formation by the aldol condensation of formaldehyde to glycollic aldehyde.

Degradation of Sugars.

[*Trans. Chem. Soc.* 1900. 1294.]

The direct and systematic transformation of a higher to a lower member of the aldose sugars was first effected by Wohl [*Ber.* 1893. 740 *et seq.*]. The oxim of the sugar is first obtained by the action of free hydroxylamine and is then acted upon by acetic anhydride in presence of sodium acetate. In this way the nitrile of the corresponding acid results and all the hydroxyl groups become acetylated. Glucoseoxim for example yields the nitrile of pentacetylgluconic acid, and arabinoseoxim gives the nitrile of tetracetyl arabonic acid. These products readily lose the elements of hydrocyanic acid by the action of silver oxide or of alkalis yielding a derivative of the lower sugar.

This method has now been applied in the case of glycollic aldehyde. Alcoholic hydroxylamine gives the corresponding oxim in a syrupy and impure condition, but when the product is treated with acetic anhydride as in Wohl's experiments it gives the acetyl derivative of glycollic acid nitrile, or acetoxyacetonitrile,

$$CH_3CO . O . CH_2CN.$$

This when treated with ammoniacal silver oxide yields a product which when acted upon by dilute sulphuric acid gives *formaldehyde.* Wohl has since obtained glycollic aldehyde from glyceraldehyde, so that only the degradation of tetrose to triose is wanting to complete the entire series from hexose.

A second systematic method of bringing about the degradation of sugars is that based upon the oxidation of the corresponding acids by hydrogen dioxide in presence of iron. This was first effected in case of tartaric acid, a normal oxidation-product of tetrose, in the manner described above; the degradation here extends over two stages—tetrose to diose—and an intermediate product—dioxymaleic acid—can be isolated. The initial operation

is effected at a low temperature and the intermediate compound subsequently decomposed by heating.

Ruff [*Ber.* 1898. 1573 *et seq.*] has since applied this method with much success to several other processes of similar degradation in the case of monobasic acids and brings about the operation in a single stage. He modifies the method by using a ferric salt instead of a ferrous salt, and operates at a higher temperature at starting, without isolation of an intermediate product. In this way, for example, he obtains *d.* arabinose from gluconic acid, and *d.* tetrose from *d.* arabonic acid.

Oxidation of Polyhydric Alcohols.

[Fenton and Jackson, *Trans. Chem. Soc.* 1899. 1.]

The influence of ferrous iron in assisting and regulating oxidation is again illustrated to a marked degree in the case of polyhydric alcohols. These substances when treated with hydrogen dioxide alone are either unaffected or very slowly and incompletely oxidized, but in presence of a small quantity of ferrous iron a vigorous oxidation immediately ensues with considerable evolution of heat. In this manner glycollic aldehyde is formed from glycol, glyceraldehyde from glycerol, tetrose from erythritol and mannose from mannitol. Hexoses are also obtained from dulcitol and sorbitol. This method probably affords the simplest and most direct method of oxidizing the polyhydric alcohols. Monohydric alcohols of the $C_nH_{2n+1}OH$ series appear to be unaffected.

In the cases of glycol, glycerol, and erythritol the oxidations may, as in the case of tartaric acid, be effected by atmospheric oxygen in presence of iron when exposed to sunlight.

Oxidation of Acids.

[Fenton and Jones, *Trans. Chem. Soc.* 1900. 69. Fenton, *B. A. Report*, 1899.]

As in the case of tartaric acid and the polyhydric alcohols the presence of ferrous iron is found to exert a similar remarkable influence in the oxidation of all hydroxy-acids, in addition to many cyclic and unsaturated acids. Owing to the *selective* character of this oxidation-process it is possible in many cases to obtain products which cannot be directly prepared in any other way. Some of the more important results obtained may be briefly summarised as follows—

Saturated aliphatic acids which do not contain hydroxyl or

ketonic groups are generally unacted upon by hydrogen dioxide, either in presence or absence of iron, at the ordinary temperature, or at any rate the amount of change, if any, is altogether insignificant in comparison with the 'active' acids above mentioned. [But that such acids may to some extent be oxidized in this manner is shewn by the recent investigations of Hopkins and Cole (*Proc. Roy. Soc.* 1901. 21), who find that a notable quantity of glyoxylic acid may result from acetic acid by oxidation and that the change is accelerated by the presence of iron.]

All the hydroxy-acids examined are oxidized almost instantaneously in this manner, generally with evolution of heat, and the operations have, as a rule, to be conducted with special cooling arrangements to avoid rise of temperature.

In this manner *glyoxylic* acid is obtained from *glycollic* acid, *pyruvic* from *lactic*, and *mesoxalic* from *tartronic* acid. *Glyceric* acid yields a product which gives a violet coloration with ferric salts and with phenyl-hydrazine gives Nastvogel's osazone,

$$CHN_2HPh\,.\,CN_2HPh\,.\,COOH\,;$$

for these and other reasons it is shewn that the product must be either hydroxypyruvic acid, $CH_2(OH)\,.\,CO\,.\,COOH$, or dihydroxyacrylic acid, $CH(OH):C(OH)\,.\,COOH$.

Malic acid yields *oxalacetic* acid: owing to the importance of the latter acid the reaction has been fully investigated and the results will be described below.

Mucic and *saccharic* acids yield products which give a violet coloration with ferric chloride; these products are still under examination.

Oxalacetic Acid.

$$\begin{array}{l} CH_2\,.\,COOH \\ | \\ CO\,.\,COOH. \end{array}$$

[Fenton and Jones, *Trans. Chem. Soc.* 1900. 77, 1901. 91, and *Proc. Chem. Soc.* 1901. 24.]

When malic acid is treated with rather less than the calculated quantity of hydrogen dioxide in presence of ferrous iron, the liquid turns dark red, and if the temperature be allowed to rise a violent decomposition takes place with evolution of carbon dioxide. But if the mixture be very carefully cooled and the reagent slowly added no gas is evolved, and if strong sulphuric acid is cautiously added and the mixture extracted by ether a white crystalline substance is obtained which proves to be pure oxalacetic acid. The esters of this acid have of course been known

for a considerable time, but the free acid was hitherto practically unknown. By the hydrolysis of ethyl ethoxyfumarate, Nef (*Annalen*, 1893. 230) had prepared an acid apparently identical with this, but it was not obtained in a pure state.

Oxalacetic acid melts at 176—180°[1] and is fairly easily soluble in water. Its solutions give an intense blood-red colour with ferric chloride, and the aqueous solution on heating evolves carbon dioxide giving pyruvic acid.

It is a well-marked dibasic acid; with phenylhydrazine it yields a beautifully crystalline hydrazone,

$$COOH . CH_2 . CN_2HPh . COOH,$$

and with hydroxylamine the oxim

$$COOH . CH_2 . C : NOH . COOH$$

is produced. Hydrazine hydrate gives the hydrazine salt of the hydrazone together with Rothenburg's pyrazolon carboxylic acid.

When oxalacetic acid is oxidized with bromine a product is obtained which gives a violet coloration with ferric chloride, and with phenylhydrazine yields Nastvogel's osazone; very probably this product is identical with that obtained by oxidation of glyceric acid in presence of iron.

Comparison of the affinity-constants of acids.

The hydrazone of oxalacetic acid, described above, when heated with pure water, yields the hydrazone of *pyruvic acid* and *carbon dioxide*,

$$\begin{array}{l} CH_2COOH \\ | \\ C : N_2HPh \\ | \\ COOH \end{array} = \begin{array}{l} CH_3 \\ | \\ C : N_2HPh + CO_2 \\ | \\ COOH, \end{array}$$

but in presence of acids of sufficient concentration an entirely different decomposition occurs, *water* being lost and Wislicenus' *pyrazolon carboxylic acid* resulting

$$\begin{array}{l} CH_2 . COOH \\ | \\ C : N . N\begin{matrix} \diagup H \\ \diagdown Ph \end{matrix} \\ | \\ COOH \end{array} = \begin{array}{l} CH_2 . CO \\ | \qquad\ \ | \\ C : N . NPh + H_2O \\ | \\ COOH. \end{array}$$

With acids of insufficient concentration both changes occur simultaneously, the amount of carbon dioxide evolved being less

[1] Wohl and Oesterlin (*Ber.* 1901. 1139) have since prepared oxalacetic acid from diacetyl tartaric anhydride; they shew that the acid, whether prepared by this method or that of the present authors, is capable of existing in two forms which melt at 176° and 146° respectively.

as the concentration of the acid increases. Based upon this observation the authors have devised an extremely simple method by which the relative affinity-values of various acids at 100° may be compared. A given weight of the hydrazone is heated with a known amount of the acid in question, and the resulting carbon dioxide is measured.

The nature of the above decomposition is explained by assuming the instability of the negative ion

$$COOH . CN_2HPh . CH_2CO_2$$

and causes which prevent ionization, such as a sufficient concentration of hydrogen ions, should have the effect of diminishing the amount of carbon dioxide produced. This hypothesis was tested in various ways with very satisfactory results; a salt in presence of its own acid, for example, greatly diminished the influence of the latter, and solvents having different ionizing powers gave widely different results, the greatest yield of carbon dioxide being obtained when water was used as solvent, less with amyl alcohol, and least with toluene and nitrobenzene.

MESOXALIC SEMI-ALDEHYDE.

$$\begin{array}{l} CHO \\ | \\ CO \\ | \\ COOH. \end{array}$$

Hitherto all the principal observations upon the results of oxidation in presence of iron have been made with the use of hydrogen dioxide as oxidizing agent. In the earlier experiments on tartaric acid however it was observed that the same effect could be produced, to some extent at any rate, by electrolysis with use of an iron anode and also by chlorine, hypochlorites and other oxidizing agents.

The effect of oxidizing agents other than hydrogen dioxide is now being investigated by Mr Ryffel and the author, and the first experiment tried has given an important and interesting result.

Tartaric acid was again first selected for experiment, with chlorine as the oxidizing agent. When chlorine is passed into a solution of tartaric acid at the ordinary temperature the absorption is very slight, although it is shewn that a certain amount of change does occur; but in presence of ferrous iron (preferably ferrous tartrate) the action is greatly facilitated. Under the latter condition, after saturation with chlorine, the resulting yellow solution soon becomes colourless, and the treatment is then

repeated in a similar way several times, allowing the mixture to stand for some hours between each saturation. The resulting product now gives with phenylhydrazine a bright orange-yellow precipitate, which when recrystallized from hot chloroform melts at 222—224° and is shewn to be identical with Nastvogel's osazone $CHN_2HPh.CN_2HPh.COOH$ which has been mentioned above. [The investigation of this product was in the first instance considerably retarded owing to the fact that all previous observers have given a value to the melting-point of this osazone which is far too low. A complete re-examination of the osazone has now proved that the true melting-point is as here stated, 222—224°.]

By concentrating the liquid under reduced pressure and allowing the unaltered tartaric acid to crystallize out as far as possible, a syrupy liquid is obtained, which however still retains some tartaric acid, and which gives a very abundant yield of the above osazone. The oxidation-product is shewn to contain no chlorine, since it may be converted into a barium salt which is free from chlorine, and which when decomposed with dilute sulphuric acid gives the same osazone. Under these circumstances it is evident, from the established constitution of the osazone (Nastvogel, *Annalen*, 1888. 85; Will, *Ber.* 1891. 400, etc.), that the product must be either hydroxypyruvic acid (or dihydroxyacrylic acid), tartronic semi-aldehyde or mesoxalic semi-aldehyde.

It gives no violet coloration with ferric chloride, so that it is not identical with the glyceric acid oxidation-product above referred to.

In order further to decide the nature of the product the following experiments were tried, and the results of these, apart from other considerations, definitely prove its constitution.

Action of Hydroxylamine.

The tartaric acid-chlorine oxidation-product obtained as above described after removal of most of the unaltered tartaric acid was mixed with excess of hydroxylamine hydrochloride in aqueous solution and neutralized with sodium carbonate. After standing over night it was acidified with dilute sulphuric acid and extracted several times with ether. Most of the ether was distilled off and the remaining liquid allowed to remain in a vacuum desiccator.

The resulting solid reddish mass was dissolved in the least possible ammonia solution and acidified with strong hydrochloric acid, the operations being carried out with careful cooling. Long colourless needles separated, which were purified by again dissolving in ammonia and acidifying with dilute sulphuric acid. These melted at 178—180° and were found by analysis to have the formula $C_3H_4N_2O_4$. They are somewhat sparingly soluble in cold water, and the aqueous solution gives with ferrous sulphate

and a little caustic alkali a beautiful but transient violet colour. Ferric chloride gives a deep red colour, and cupric acetate an olive-green precipitate.

This compound is identical in every respect with the 'secondary' dioximido-propionic acid CH(NOH) . C(NOH) . COOH which Söderbaum obtained from dibromopyruvic acid.

Oxidation with cupric hydroxide.

The oxidation-product mentioned above when further oxidized with cupric hydroxide, in presence of alkalies, gives a large yield of *mesoxalic acid.*

Taking this result together with those above described conclusive evidence is afforded as to the constitution of the product—that it is the semi-aldehyde of mesoxalic acid.

The formation of this substance from tartaric acid under the conditions mentioned might be explained in several ways. At first it was thought probable that dioxytartaric acid was the initial product, and that this gave rise to mesoxalic semi-aldehyde with loss of carbon dioxide and water. But all attempts to prepare the substance from pure dioxytartaric acid, by heat, action of acids or of iron salts, were unsuccessful or unsatisfactory. It was observed however that the product of the action of chlorine on tartaric acid in presence of iron gives, in the first instance, a notable reaction of dioxymaleic acid with iron salts, and it seemed probable that the latter acid might be the initial product in formation of the semi-aldehyde. Starting upon this assumption attempts were made to prepare the semi-aldehyde directly from pure dioxymaleic acid, and these eventually were completely successful.

When a solution of dioxymaleic acid is treated with a ferric salt, such as ferric sulphate, it is almost quantitatively converted into mesoxalic semi-aldehyde and carbon dioxide,

$$C_4H_4O_6 + Fe_2(SO_4)_3 = C_3H_2O_4 + 2FeSO_4 + H_2SO_4 + CO_2.$$

After removal of the iron salt and free numeral acid the aldehyde remains as a thick syrup. It has not yet been induced to crystallize but experiments are being made in this direction.

The typical oxidation-products of glycerol which are theoretically possible are eleven in number, namely—glyceric, hydroxy-pyruvic, tartronic and mesoxalic acids, glyceraldehyde, dihydroxy-acetone, tartronic semi- and di-aldehydes, hydroxy-pyruvic aldehyde, and mesoxalic semi- and di-aldehydes. Of these the first six are well known; mesoxalic di-aldehyde is known only in form of oxim, and the remaining four were hitherto unknown. The isolation of the semi-aldehyde of mesoxalic acid therefore brings the series one stage nearer completion.

This semi-aldehyde is of especial interest in consequence of its relationship to uric acid, since its aldehyde-hydrate $CH(OH)_2 . CO . COOH$ may be regarded as tautomeric with trihydroxy-acrylic acid $C(OH)_2 = C(OH) . COOH$.

Oxidation of Carbohydrates.

[Cross and Bevan, *Trans. Chem. Soc.* 1898. 463, 1899. 747. Morrell and Crofts, *ibid.* 1899. 786, etc.]

This branch of the research has hitherto been undertaken, in communication with the author, especially by Messrs Cross and Bevan, and Morrell and Crofts. Amongst the most important results which have been obtained in this direction may be mentioned the observations of the latter chemists, who shew that both dextrose and lævulose when oxidized by hydrogen dioxide in presence of ferrous iron yield *hexosone* $CH_2OH(CHOH)_3 . CO . CHO$, and that similar results are obtained with arabinose and rhamnose, which give pentosone and methyl-pentosone respectively.

Galactose, presumably owing to its special configuration, gives a different result, which is being further investigated.

Messrs Cross and Bevan have also studied the action on acetylene and benzene, which yield acetic acid and phenol respectively. Furfural also yields a colour-giving substance, which is probably hydroxy-furfural.

Function of the Iron.

In the present state of these researches, which are still in active progress, it is somewhat premature to draw general conclusions as to the probable nature of the influence of iron in the oxidations mentioned, but the following statements may be made with considerable confidence:

(*a*) The influence of the iron belongs to the class of chemical changes usually described as catalytic, an almost infinitesimal proportion being in many cases sufficient to determine the oxidation.

(*b*) It is probably essential in all cases that the iron should be present in the *ferrous* state, and in the few instances where a ferric salt has been successfully employed it is probable that previous reduction to the ferrous state initially occurs.

(*c*) Judging from the colour changes and other effects it is highly probable that the iron undergoes alternate changes in state of oxidation or valency; the higher state may be the ferric condition or it may correspond to a still higher form, such as FeO_2. (Compare Manchot, *Zeit. Anorg. Chem.*, 1901, 420.)

(*d*) Ferric iron although probably inactive in inducing the initial oxidation has the effect of encouraging the breaking down of the oxidation-product with evolution of carbon dioxide. (Compare Ruff's experiments (*loc. cit.*) and the formation of mesoxalic semi-aldehyde.)

(*e*) The oxidizing influence in presence of ferrous iron is selective, certain groups only being prone to attack. Nearly all the results may be primarily expressed by the replacement of hydrogen by hydroxyl, and this action appears to be chiefly confined to hydrogen atoms which are directly associated with hydroxyl groups. Hydrogen directly associated with a cyclic nucleus however, as in benzene and pyromucic acid, and with certain unsaturated linkages in aliphatic compounds, is also liable to attack. The aldehydic HC : O group appears to be remarkably resistant to the oxidation-process in question.

(*f*) In some cases the changes could be most easily explained by the supposition that non-hydroxylic hydrogen atoms are simply withdrawn, as for example in the oxidation of tartaric acid to dioxymaleic acid. This explanation in the latter instance although satisfactory in other respects would, according to the hypothesis of van 't Hoff and Wislicenus, necessitate the formation of a fumaroid derivative, whereas a maleinoid form actually results. This question has recently been investigated by W. F. Cooper of Clare College and the author, and their experiments shew that the di-acyl derivatives of tartaric acid do not appear to lend themselves to the oxidation-process in question, so that the hydroxylic hydrogen atoms would appear to be essential.

Incidental Observations.

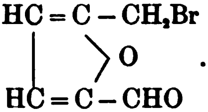
In the process of working out the problems above described a large number of interesting observations, not directly connected with the primary objects, have been made, and in following up some of these side issues results of considerable importance have been obtained. It has been considered advisable in the present communication to include a very brief account of these in order to give an idea of the extent of ground which has been covered.

Esterification by Ethyl Ether. [Fenton and Gostling, *Trans. Chem. Soc.* 1898. 554.] In studying the properties of dioxymaleic acid it was found that hydrogen bromide acted in an interesting manner, and various solvents were employed for the reaction. In using perfectly pure, dry ethyl ether an unexpected result was obtained, the product being ethyl dioxymaleate. An excellent yield is obtained in this way, and the observation was extended to other

acids with the result that the reaction proves to be a general one, that is to say, a large number of ethyl esters may be prepared by the use of ethyl ether in place of alcohol, using dry hydrogen bromide as dehydrating-agent.

Characteristic colour reaction for certain carbohydrates. [Fenton and Gostling, *Trans. Chem. Soc.* 1898. 556, and 1901. 361.] Continuing the observations upon the action of hydrogen bromide in ether on substances other than acids, it was found that certain carbohydrates give an intense purple colour with the reagent, and that the rapid production of this colour is characteristic of ketohexoses or those containing a ketohexose nucleus.

Derivatives of Methyl-furfural and the constitution of 'cellulose.' [Fenton and Gostling, *Trans. Chem. Soc.*, 1899. 423, and 1901. 807.] The cause of the remarkable purple colour mentioned in the last section was the next subject of enquiry, and after considerable initial difficulties a new substance was isolated in a crystalline state, which proves to be *bromo-methyl-furfural*,



This substance gives an intense purple colour with hydrogen bromide, and on oxidation yields the corresponding bromopyromucic acid. Later observations shew that the chloro-derivative may be produced in a similar manner by the action of hydrogen chloride on the carbohydrates mentioned. A large number of typical carbohydrates have been investigated, and it is proved that these methyl-furfural derivatives result only from *ketohexoses* such as lævulose and sorbose, or substances which give rise to ketohexoses on hydrolysis, such as cane-sugar and inulin. It may in fact be stated with confidence that the production of bromo- or chloro-methyl-furfural in the manner described is a specific test for the presence of a ketohexose nucleus. Later it has been shewn that all forms of *cellulose* give large yields of these methyl-furfural derivatives by the action of hydrogen bromide or chloride, and the presence in cellulose of a ketohexose nucleus is consequently proved. After extracting the methyl-furfural derivative from the action on cellulose, the residue is found to contain ordinary *d.* glucose (*Proc. Chem. Soc.* 1901. 166), a fact of much interest in view of the recent isolation of 'cellose,' $C_{12}H_{22}O_{11}$ from cellulose by Skraup and König.

A large number of other interesting derivatives of methyl-furfural have also been obtained by the authors.

On a method of increasing the sensitiveness of Michelson's Interferometer. By H. C. POCKLINGTON, M.A., St John's College.

[*Received* February 1902.]

1. The object of this paper is to suggest a modification of the interferometer, which should have a considerably greater sensitiveness. This is done by making the interfering beams consist of circularly polarised light, a retardation of one beam then causing a rotation of the plane of polarisation of the resultant. This enables us to make use of the great sensitiveness of the apparatus used for measuring the rotation of the plane of polarisation. The general method is described in § 2, and a particular method of arranging the apparatus is discussed in § 3. An objection to this method is mentioned in § 4, with an alternative method whereby it is avoided. In § 5 it is shown how to eliminate the noxious light reflected from the inclined plate of the interferometer.

2. Let a beam of plane polarised light be incident on the inclined plate. This beam can be resolved into two, polarised respectively in and perpendicular to the plane of incidence. Each of these beams is divided into two parts at the inclined plate. One pair of these parts is reflected down to a mirror and then back through the inclined plate to the eye. The ratio of these parts as they reach the eye is not in general unity, but can be made unity by suitably choosing the plane of polarisation of the incident light. Let now a plate of mica or other suitable substance, of such thickness that it produces a relative retardation of an eighth of a wave, be placed between the inclined plate and the mirror that we are considering, and let its principal lines be parallel to and perpendicular to the line of intersection of the plate and the mirror. Since the light passes twice through the mica, a relative retardation of a quarter wave-length is produced, and the light, after passing through the inclined plate, is circularly polarised. The same can be done[1] in the case of the light that passes through the inclined plate, is reflected at the other mirror, and is finally reflected by the inclined plate to the eye. If the beams are oppositely polarised they will combine to produce

[1] This cannot be done exactly unless the plane of polarisation of the incident light is slightly altered, and the resulting circularly polarised beam has not the same intensity as that previously mentioned. See below.

a plane polarised beam, the plane of polarisation depending on the amount of retardation that one of the interfering beams may have undergone owing to a displacement of one of the mirrors or to the interposition of some body in the path of one of the beams. As the intensity of the resultant beam is independent of the retardation, nothing will be observed when the emergent light is examined with the naked eye; but if a nicol is used, all light polarised in a certain plane will be quenched, and the interference phenomena will be seen as usual. For example, if one of the mirrors is slightly inclined to the image of the other in the inclined plate, a series of parallel bands is seen. If the analyser be turned through any angle, the effect produced on the phenomena is the same as if one of the two interfering beams had been retarded by a proportional amount (a retardation of a wave-length corresponding to a rotation of 180°), and the bands will move uniformly across the field. I have verified this experimentally with an interferometer having an unsilvered inclined plate, and find that when white light is used, the bands as they move change their colour in accordance with the position they occupy, thus *e.g.* each band as it comes to the place of the central band becomes black, and after passing it reverses the order of its colouring. In order to secure the maximum sensitiveness it would theoretically seem best to work with each mirror coincident with the image of the other in the inclined plate. The state of polarisation of the emergent beam is then uniform, and the plane of polarisation can be determined as in any form of polaristrobometer. The error, under favourable circumstances, should[1] not exceed 3′, which corresponds to an error of 1/3600 of the breadth of a fringe when the observations are made in the usual way. Even if we assume that the twentieth part of a fringe can be measured in the latter case, the accuracy can be increased nearly two hundred-fold by the use of circularly polarised light. For many of its uses the interferometer is already sufficiently sensitive, but for experiments on the drift of the ether, and perhaps also on the influence on transmitted light of the layer of air that a surface of glass condenses on itself, the increased sensitiveness would be useful.

3. In arranging the details of the apparatus we see that the inclined plate cannot be silvered, on account of the difference in the retardation it produces in the two perpendicularly polarised parts of either beam. Further, one of the interfering beams has undergone four refractions, and the other two if no compensating plate is used, or six if one is used. The beams are hence of different intensities. Since the two parts of a beam are

[1] H. Landolt, *Handbook of the Polariscope*.

diminished in different ratios by a refraction, the azimuth of polarisation of the incident light requisite to cause one of the emergent beams to be circularly polarised is different from that required for the other. Both these difficulties can be overcome by placing the compensating plate parallel and very close to the inclined plate, the distance apart being that which corresponds to the first bright ring in Newton's rings. This arrangement has the further advantage that the amount of light reflected at the thin film of air is four times as great as that reflected at a single air-glass surface.

If the amplitude of the light-waves incident on the air-film is A, the displacement in reflected wave is

$$Ab\,\frac{2(1+b^2)\sin^2\dfrac{\delta}{2}\sin\phi-(1-b^2)\sin\delta\cos\phi}{1-2b^2\cos\delta+b^4},$$

and that in the transmitted wave is

$$A(1-b^2)\frac{(1-b^2\cos\delta)\sin\phi-b^2\sin\delta\cos\phi}{1-2b^2\cos\delta+b^4},$$

where b is the ratio of the amplitude of the wave reflected at a single air-glass surface to that of the incident wave, δ is the relative retardation of beams reflected at the two surfaces of the film, and ϕ is an expression of the form $r(t-x/V)$. Since the value of b depends on the plane of polarisation of the incident light, the two parts of each beam in general undergo a different alteration of place at reflection or refraction. In our case however $\delta=\pi$ and no alteration of place occurs. We see however that the condition $\delta=\pi$ must be satisfied with considerable accuracy. The above expressions are then

$$A\,\frac{2b}{1+b^2} \text{ and } A\,\frac{1-b^2}{1+b^2},$$

respectively, where b is given by Fresnel's theory of reflection.

Each beam is reflected once at and transmitted once through the thin film, and undergoes four other refractions, two at an air-glass surface, and two at a glass-air surface. If therefore the incident light is polarised in a plane making an angle ϕ with the plane of incidence, the parts of either emergent beam that are polarised in and perpendicular to the plane of incidence are respectively

$$A\cos\phi\cdot\frac{2b_1}{1+b_1^2}\,\frac{1-b_1^2}{1+b_1^2}\,c_1^2f_1^2,$$

and

$$A\sin\phi\cdot\frac{2b_2}{1+b_2^2}\,\frac{1-b_2^2}{1+b_2^2}\,c_2^2f_2^2,$$

where c, f are the ratios in which the amplitude is changed by the refractions into and out of glass. Since $b^2 + cf = 1$, these can be written

$$A \cos \phi \,.\, \frac{2b_1 (1 - b_1^2)^2}{(1 + b_1^2)^2},$$

$$A \sin \phi \,.\, \frac{2b_2 (1 - b_2^2)^2}{(1 + b_2^2)^2},$$

where $$b_1 = \sin (i - r)/\sin (i + r),$$

and $$b_2 = \tan (i - r)/\tan (i + r).$$

If $i = 45°$ and $\mu = 1{\cdot}5$ these parts of either beam are $\cdot 38\, A \cos \phi$ and $\cdot 18\, A \sin \phi$, and to make these equal we must have $\phi = 65°$.

4. The main objection to be urged against this method of obtaining two circularly polarised beams is that it requires that the interval between the two inclined plates shall be very accurately adjusted. Whether it is better to use this method, or use the ordinary form of interferometer (with unsilvered inclined plate) and allow for the want of exactitude of circularity in the polarisation when calculating the result, can only be determined by experiment.

5. The phenomena in either case are rendered less brilliant by the light reflected from the other face of the plate. This light can be eliminated either by making the plates slightly wedge-shaped, or by using a circular hole in a screen as a source of light and forming a real image of it beyond the apparatus by means of a lens. The light to be eliminated now passes through an image distinct from that formed by the light which we want, and can be cut off by a screen.

[Note, *Received* 14 March 1902.]

One of the most sensitive Polaristrobometers is Laurent's form. In this the light in half the field has its plane of polarisation rotated through a certain small angle. On rotating the analyser first one half and then, after a further small rotation, the other half of the field becomes dark. The source of the great sensitiveness is partly that the ratio of the two intensities varies from 0 to ∞ for a small rotation of the analyser, and partly that this rotation can be accurately measured.

The same principle can be used with the interferometer without utilising circularly polarised light. The mirror should be

adjusted so as to give an even shade, and half of one mirror should be covered with a thin plate which causes a small retardation (or a plate may be cut in two and one half slightly tilted). If now the whole of the light be retarded by a further accurately measurable amount (say by inclining a moderately thin plate placed in the path of the light and provided with circles or with mirror and scale) the two halves of the field will become dark in close succession, and all the above-mentioned conditions of sensitiveness are satisfied. There is still the difficulty of the unequal intensities of the interfering beams, and the difficulty can be overcome as before, without the counterbalancing difficulty due to change of phase being met with, for we may here use plane polarised light.

The Influence on Light reflected from and transmitted through a Metal of a Current in the Metal. By P. V. BEVAN, B.A., Trinity College, Cambridge.

[*Received* 19 February 1902.]

The equations applicable to metallic media under the influence of light vibrations may be written

$$(a + \iota b)\frac{d\mathbf{E}}{dt} + (c + \iota d)[\mathbf{EH}] = V \text{ curl } \mathbf{H},$$

$$\frac{d\mathbf{H}}{dt} = - V \text{ curl } \mathbf{E},$$

taking $\mu = 1$ as we only deal with vibrations of such rapidity that the magnetic properties of the metals become lost.

The constants in the first equation are obtained by considering the effect of corpuscles in the metal which are of two types, those which are confined to a certain region and can perform oscillations about an equilibrium position, and others which can move more or less freely from one region to another in the metal. (Cf. Drude, Reports presented to the International Congress on Physics, Paris 1900.) I propose to consider the case of light reflected at the surface of a metal carrying a current.

The first of the above equations must be modified since we have a constant external force $\mathbf{E}_1$ which is steady, as we are only considering the case of a steady current. This produces in the plate a steady flow of ions

$$\frac{\mathbf{E}_1}{\sigma} + \frac{\lambda}{\sigma}[\mathbf{E}_1\mathbf{H}_1],$$

where the first term is the ordinary conduction term according to Ohm's Law and the second term represents the Hall effect, σ representing the specific resistance of the metal.

The state of the metal is now different from its state when no current passes, since the force $\mathbf{E}_1$ may be sufficient to dislodge some of the ions which before the current was flowing were performing oscillations and the freely moving corpuscles have now on the average a velocity in the direction of the force $\mathbf{E}_1$.

Leaving out of account the constants c, d since they involve the ratio of the velocity of a corpuscle to that of light, we confine ourselves to the consideration of the constants a and b.

Suppose a corpuscle has mass m, charge e, and that an elastic force $p\xi$ acts on it, where ξ is its displacement in the direction of the x axis, κ a frictional coefficient, then the equations of motion for the corpuscle are

$$m\frac{d^2\xi}{dt^2} = eX - \kappa\frac{d\xi}{dt} - p\xi \text{ etc.,}\quad \mathbf{E} \equiv (X, Y, Z).$$

For corpuscles which are not acted on by a harmonic force we have

$$m\frac{d^2\xi}{dt^2} = eX - \kappa\frac{d\xi}{dt}.$$

Writing $\qquad \dfrac{2\pi\kappa}{p} = \alpha,\quad \dfrac{4\pi^2 m}{p} = \beta,\quad \dfrac{4\pi^2 e^2 N}{p} = \gamma,$

where N is the number of charges e per unit volume, then we obtain

$$a + \iota b = \left[1 + \Sigma_1\left(\frac{\gamma}{1 + \iota\dfrac{\alpha}{\tau} - \dfrac{\beta}{\tau^2}}\right) + \Sigma_2\left(\frac{4\pi e^2 N}{\iota\dfrac{2\pi\kappa}{\tau} - \dfrac{4\pi^2}{\tau^2}m}\right)\right],$$

where τ is the period of the luminous waves and Σ_1 refers to corpuscles of the first type, Σ_2 to corpuscles of the second type. (Drude, *loc. cit.*)

For the modification to be introduced when a current is flowing in the metal we first consider Σ_2. This sum will be unaltered by the constant external force since the equations are linear both for the motion of the corpuscle due to the light field and the electric force producing the current. There will then be no change in Σ_2.

We have mentioned the possibility of some of the corpuscles of the first type changing character owing to the electric force driving the current, and becoming corpuscles of the second type. But as Ohm's Law holds very accurately we see that the number of corpuscles passing over from one type to the other is small compared to the whole numbers in the two types provided the temperature of the metal does not change, so that there will not be much change in the values of a and b and any variation in these constants will be of the order of variations from Ohm's Law.

If now $\mathbf{E}'$, $\mathbf{H}'$ represent the electric and magnetic forces due to the light and $\mathbf{E}_1\mathbf{H}_1$ the part due to the battery producing the current, the total electric flux is

$$(a + \iota b)\frac{d\mathbf{E}'}{dt} + (c + \iota d)[\mathbf{E}', \mathbf{H}_1 + \mathbf{H}'] + \frac{\mathbf{E}_1}{\sigma} + \frac{\lambda}{\sigma}[\mathbf{E}_1, \mathbf{H}_1 + \mathbf{H}'],$$

$\mathbf{E}_1$ being constant.

Now in the second term $\mathbf{H}_1$ is not very large and $\mathbf{E}'$ is small compared with $\mathbf{E}_1$ so that the second term is to the fourth in the ratio $\mathbf{E}' : \mathbf{E}_1$ and can be neglected, both terms being small compared to the other terms in this expression.

The equation is therefore

$$(a + \iota b)\frac{d\mathbf{E}'}{dt} + \frac{\mathbf{E}_1}{\sigma} + \frac{\lambda}{\sigma}[\mathbf{E}_1, \mathbf{H}_1 + \mathbf{H}'] = V \operatorname{curl} (\mathbf{H}_1 + \mathbf{H}'),$$

and as

$$\frac{\mathbf{E}_1}{\sigma} + \frac{\lambda}{\sigma}[\mathbf{E}_1\mathbf{H}_1] = V \operatorname{curl} \mathbf{H}_1,$$

$$(a + \iota b)\frac{d\mathbf{E}'}{dt} + \frac{\lambda}{\sigma}[\mathbf{E}_1\mathbf{H}'] = V \operatorname{curl} \mathbf{H}',$$

as the first equation connecting the electric and magnetic forces due to the light vibrations.

We have also

$$\frac{d\mathbf{H}'}{dt} = - V \operatorname{curl} \mathbf{E}'.$$

For convenience we write the equations

$$a\frac{d\mathbf{E}}{dt} + \iota\gamma[\mathbf{PH}] = V \operatorname{curl} \mathbf{H},$$

$$\frac{d\mathbf{H}}{dt} = - V \operatorname{curl} \mathbf{E},$$

and we suppose the current parallel to Ox so that $\mathbf{P}$ representing the current E. M. F. is $(P, 0, 0)$.

If then the forces in a disturbance propagated in the metal vary as $e^{\iota(lx+my+nz+pt)}$, we have

$$\left.\begin{aligned} apX \qquad &= V(mN - nM) \\ apY - \gamma PN &= V(nL - lN) \\ apZ + \gamma PM &= V(lM - mL) \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots (1),$$

and $\quad p(L, M, N) = - V(mZ - nY, nX - lZ, lY - mX) \ldots\ldots (2).$

We have therefore

$$ap\Sigma lX + \gamma P(Mn - Nm) = 0,$$

$$\Sigma Ll = 0,$$

$$\Sigma LX = 0.$$

And therefore the magnetic force is in the wave front, the electric force is perpendicular to the magnetic force but not accurately in the wave front.

If P were $= 0$, $\Sigma lX = 0$ and

$$ap^2X = V^2\{\Sigma l^2 . X - l\Sigma lX\} = V^2X\Sigma l^2,$$

so that

$$ap^2 = V^2\Sigma l^2.$$

Writing

$$\frac{p}{V} = f,$$

we have when P is not zero,

$$af^2 - \Sigma l^2 = c,$$

where c is small.

And then

$$cX = - l\Sigma lX,$$

$$cY = - m\Sigma lX - \frac{\gamma P}{V}(lY - mX),$$

$$cZ = - n\Sigma lX + \frac{\gamma P}{V}(nX - lZ),$$

and

$$cL = \frac{\gamma P}{V}(Mm + Nn) - l\Sigma Ll$$

$$= - \frac{\gamma P}{V}Ll,$$

and

$$c + \frac{\gamma P}{V}l = 0 \text{ or } L = 0:$$

if $L = 0$ equations (2) give

$$M\left(c + \frac{\gamma P}{V}l\right) = 0,$$

$$N\left(c + \frac{\gamma P}{V}l\right) = 0.$$

Hence if $c + \frac{\gamma Pl}{V}$ is not zero, $M = 0$ and $N = 0$, and there is no wave.

So if $c + \frac{\gamma Pl}{V} = 0$, a wave can be propagated in the metal; the velocity and coefficient of absorption being determined by the equation

$$af^2 - l^2 - m^2 - n^2 + \frac{\gamma Pl}{V} = 0 \quad\text{.................(3).}$$

This equation gives for any direction of wave motion only one velocity, so that in whatever plane the light is polarised the velocity is the same and there is no difference of phase of the components of any vibration introduced on progression through the metal.

For light incident normally $l=0$ and the effects on reflected and transmitted light are the same as when no current flows.

Consider light, polarised in the plane of incidence, incident on the metal, the forces varying as $e^{\iota(lx+my+nz+pt)}$. The values of l, m, n in equation (3) will be l, m and say ν.

The ratio of the amplitude of the reflected light to that of the incident is

$$\frac{n-\nu}{n+\nu}.$$

The metal we suppose thick enough for the first surface only to be taken into account.

Putting $$\nu = \nu_1 - \iota\nu_2,$$

so that ν_1 and ν_2 are positive, the effect becoming zero for z large and negative, we have,

$$\text{the ratio of the two amplitudes is } = \frac{n-\nu_1+\iota\nu_2}{n+\nu_1-\iota\nu_2},$$

and putting this in the form $Re^{\iota\phi}$, we have

$$\phi = \tan^{-1}\frac{2n\nu_2}{n^2-\nu_1^2-\nu_2^2},$$

so that the change of phase of the reflected light is, when represented as a time,

$$\frac{1}{p}\tan^{-1}\frac{2n\nu_2}{n^2-\nu_1^2-\nu_2^2}.$$

Now from equation (3), since $\alpha = a + \iota b$, and $\iota\gamma = \frac{\lambda}{\sigma}$,

$$\left.\begin{aligned} a\gamma^2 - l^2 - m^2 - \nu_1^2 + \nu_2^2 &= 0 \\ b\gamma^2 + 2\nu_1\nu_2 - \frac{\lambda Pl}{\sigma V} &= 0 \end{aligned}\right\}.$$

Suppose the plane of incidence is that of zx, containing the direction of the current.

Then
$$\left.\begin{aligned} a\gamma^2 - l^2 - \nu_1^2 + \nu_2^2 = 0 \\ b\gamma^2 + 2\nu_1\nu_2 - \frac{\lambda Pl}{\sigma V} = 0 \end{aligned}\right\} \quad\text{..................(4)},$$

m being equal to 0.

If we put $P = 0$ in equation (4) we obtain the change of phase T due to the metal without the current flowing into it. The additional change of phase due to the current is

$$\mathfrak{T} = \frac{\partial T}{\partial \nu_1}\delta\nu_1 + \frac{\partial T}{\partial \nu_2}\delta\nu_2,$$

where
$$T = \frac{1}{p}\tan^{-1}\frac{2n\nu_2}{n^2 - \nu_1^2 - \nu_2^2},$$

and $\delta\nu_1$, $\delta\nu_2$ are the changes in ν_1, ν_2 due to the retention in equations (4) of the term

$$\frac{\lambda Pl}{\sigma V}.$$

We have then from (4)

$$\nu_1\delta\nu_1 = \nu_2\delta\nu_2,$$
$$\nu_1\delta\nu_2 + \nu_2\delta\nu_1 = \frac{\lambda Pl}{2\sigma V} = \phi \text{ say}.$$

And therefore
$$\delta\nu_1 = \phi\frac{\nu_2}{\nu_1^2 + \nu_2^2},$$
$$\delta\nu_2 = \phi\frac{\nu_1}{\nu_1^2 + \nu_2^2}.$$

We obtain then

$$\mathfrak{T} = \frac{2n\nu_1\phi(n^2 - \nu_1^2 + 3\nu_2^2)}{p(\nu_1^2 + \nu_2^2)\{(n^2 - \nu_1^2 - \nu_2^2)^2 + 4n^2\nu_2^2\}}.$$

From this expression, using (4) with $P = 0$, we obtain

$$\mathfrak{T} = \frac{2n\nu_1\phi\{f^2(1-a) + 2\nu_2^2\}}{p\gamma^4\{(1-a)^2 + b^2\}(\nu_1^2 + \nu_2^2)}.$$

From equations (4)

$$\nu_1^2 = \tfrac{1}{2}[(a\gamma^2 - l^2) + \sqrt{\{(a\gamma^2 - l^2)^2 + b^2\gamma^4\}}],$$
$$\nu_2^2 = \tfrac{1}{2}[-(a\gamma^2 - l^2) + \sqrt{\{(a\gamma^2 - l^2)^2 + b^2\gamma^4\}}].$$

Now
$$a = n'^2(1 - k^2),$$
$$b = -2n'^2k,$$

where n' and $n'k$ are the quantities corresponding to the refractive index of the metal and the absorption coefficient, and from Drude's

values (*loc. cit.* p. 43) both a and $2b$ for several metals are greater than 10. For a rough approximation we can, in the expressions for ν_1 and ν_2, neglect l^2 compared with af^2 and bf^2, and write

$$\nu_1^2 = \frac{f^2}{2}(a + \sqrt{a^2 + b^2}),$$

$$\nu_2^2 = \frac{f^2}{2}(-a + \sqrt{a^2 + b^2}),$$

or

$$\nu_1 = fn',$$

$$\nu_2 = fn'k.$$

That is, ν_1, ν_2 have approximately the same values as for light incident perpendicularly on the surface.

We thus obtain

$$\mathfrak{T} = \frac{2n\phi\,\{1 - n'^2(1 - k^2) + 2n'^2k^2\}}{n'pf^3(1 + k^2)[\{1 - n'^2(1 - k^2)\}^2 + 4n'^4k^2]}$$

$$= \frac{\lambda Pln}{pf^3\sigma V}\,\frac{1 - n'^2(1 - k^2) + 2n'^2k^2}{n'(1 + k^2)\,[\{1 - n'^2(1 - k^2)\}^2 + 4n'^4k^2]}.$$

If now ϑ is the angle of incidence, $l = f\sin\vartheta$, $n = f\cos\vartheta$, $f = \frac{p}{V}$, and $p = \frac{2\pi}{\tau}$, where τ is the period of the vibration.

We obtain therefore

$$\mathfrak{T} = \frac{A\lambda\tau^2}{4\pi^2\sigma} P \sin\vartheta \cos\vartheta,$$

where
$$A = \frac{1 - n'^2(1 - k^2) + 2n'^2k^2}{n'(1 + k^2)\,[\{1 - n'^2(1 - k^2)\}^2 + 4n'^4k^2]}.$$

The fraction of the time of vibration or the fraction of the wave-length, if we regard the change of phase as a length, is owing to the current

$$\frac{A\lambda\tau}{4\pi^3\sigma} P \sin\vartheta \cos\vartheta.$$

The units adopted so far are Gaussian; to express this result in electromagnetic units we observe that σ is in these units the same as in electrostatic units, and the unit of resistance in the electrostatic system is v^2 the electromagnetic unit; if then σ' is the number expressing the specific resistance in electromagnetic units $\frac{\sigma'}{v^2}$ is the number expressing the same quantity in Gaussian

units, v being 3.10^{10}, so $\sigma = \dfrac{\sigma'}{9.10^{20}}$; λ is in these units the same as in the electromagnetic system. If P' is in volts $P' = 3.10^2 P$, P being in electrostatic units; if σ'' is in ohms $\sigma' = 10^9 \sigma''$, so that the change of phase is, when the quantities are in the ordinary practical units, dropping the dashes,

$$3.10^9 \frac{A\lambda\tau}{4\pi^2\sigma} P \sin\vartheta \cos\vartheta.$$

The Hall coefficient C is related to λ by the equation

$$\frac{C}{\sigma'} = \lambda.$$

The change of phase is therefore

$$\frac{3}{4\pi^2} \frac{AC\tau}{\sigma^2} P \sin\vartheta \cos\vartheta;$$

P being in volts, σ in ohms, and C in electromagnetic units as given by Hall.

Now $\dfrac{P}{\sigma}$ is the current per unit area of the metal in ampères. If we have a metallic mirror 1 centimetre broad and t centimetres thick with a current G flowing through it, then

$$\frac{P}{\sigma} = \frac{G}{t},$$

G being in ampères.

The change of phase is therefore

$$\frac{3}{4\pi^2} \frac{AC\tau}{t\sigma} G \sin\vartheta \cos\vartheta.$$

If we take the case of Bismuth which has a large Hall coefficient, viz. $8{\cdot}58.10^{-6}$, and for which $\sigma = 1{\cdot}4.10^{-4}$, we get the change of phase

$$\frac{12}{\pi^2} A \frac{\tau}{t} G 10^{-2} \sin\vartheta \cos\vartheta,$$

which as A is less than 1, and τ is 2.10^{-15} for yellow light, is a quantity too small for any measurement since G cannot be made very large for a thin film of metal.

For light polarised perpendicularly to the plane of incidence, this plane containing the direction of the current, we have the ratio of the reflected wave's amplitude to that of the incident wave

$$\frac{an - v}{an + v}.$$

And again with $\nu = \nu_1 - \iota\nu_2$, α being $\equiv a + \iota b$, this ratio

$$= \frac{na - \nu_1 + \iota(nb + \nu_2)}{na + \nu_1 + \iota(nb - \nu_2)},$$

so that in this case the change of phase on reflection is

$$T = \frac{1}{p}\tan^{-1}\frac{2(a\nu_2 + b\nu_1)n}{n^2(a^2 + b^2) - \nu_1^2 - \nu_2^2}.$$

And for the additional change of phase due to the current

$$\mathfrak{T} = \frac{\partial T}{\partial \nu_1}\delta\nu_1 + \frac{\partial T}{\partial \nu_2}\delta\nu_2,$$

where $\delta\nu_1$, $\delta\nu_2$ as before, are equal to

$$\phi\frac{\nu_2}{\nu_1^2 + \nu_2^2}, \quad \phi\frac{\nu_1}{\nu_1^2 + \nu_2^2}.$$

We get the change of phase in this case as a fraction of the wave-length

$$\frac{\lambda P \tan\vartheta \,.\, \tau}{4\pi^2\sigma}\,\frac{1 - 3k^2}{n'^2(1 + k^2)^2},$$

for incidences not very far removed from the normal, so that again the change is too small to be measured.

Consider now the effect of the current on the velocity of the light transmitted through the metal. In the metal the velocity of the light is

$$V' = \frac{p}{\sqrt{l^2 + \nu_1^2}},$$

and this

$$= \frac{p}{\sqrt{af^2 + \nu_2^2}}.$$

The velocity V' will therefore be different when the current flows in the metal from its value in the metal without any current. The change in the velocity is

$$\frac{\partial V'}{\partial \nu_2}\delta\nu_2,$$

and is therefore

$$-\frac{p\nu_2\delta\nu_2}{(af^2 + \nu_2^2)^{\frac{3}{2}}},$$

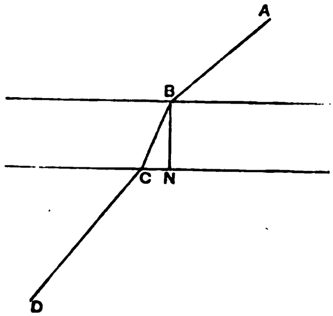
which is

$$-\frac{p\nu_1\nu_2\phi}{(af^2 + \nu_2^2)^{\frac{3}{2}}(\nu_1^2 + \nu_2^2)}.$$

Let us now consider the case of light passing through a thin metal plate, and then find the effect on the emergent light of

a current made to pass through the plate in the direction of the intersection of the plate and the plane of incidence. We have seen that the surface effects are of an order too small to be measured, so that we shall have to consider a change of phase which is introduced by the alteration of the velocity of the light when in the metal.

If $ABCD$ is the course of the light through the plate without any current, the course when the current flows will be slightly different. But the alteration in ν_1 being small the change in the phase of the light introduced by the difference of path through the metal will be too small to be taken into account, at any rate for incidence such that BC makes an angle not nearly $=\frac{\pi}{2}$ with the normal BN.



If the angle CBN is r, then the difference of phase introduced by the current owing to the change in the velocity of the light through the metal is

$$\frac{\partial}{\partial \nu_2}\left(\frac{t \sec r}{V'}\right) \delta\nu_2,$$

V' being the velocity in the metal with no current, and r being regarded as unaltered by the current.

Using the same approximations as before, this is

$$\frac{t \sec r}{V'^2} \frac{pk\phi}{f^2(1+k^2)(a+n'^2k^2)^{\frac{1}{2}}}$$

$$= \frac{\phi kt \sec r\, n'^2}{(1+k^2)(a+n'^2k^2)^{\frac{1}{2}}} \frac{V}{p^2}.$$

The fraction this is of the time of oscillation is therefore, since we have also $a = n'^2(1-k^2)$,

$$\frac{\phi kt \sec r}{n'(1+k^2)} \frac{V}{2\pi p}.$$

ϑ being the angle of incidence, this becomes

$$t\sqrt{\frac{1}{n'^2 - \sin^2\vartheta}} \frac{k}{1+k^2} \frac{\lambda Pl}{4\pi\sigma p}$$

$$= A\frac{\lambda Pl}{\sigma p},$$

and this, when P, σ and λ are in electromagnetic units,

$$= A\,\frac{\lambda P \sin\vartheta}{\sigma}.$$

Now $\lambda = \frac{C}{\sigma}$ where C is the Hall coefficient, so the change of phase is finally, where P is in volts and σ in ohms,

$$= A \sin\vartheta\,\frac{PC}{\sigma^2}\,.\,10^{-10}.$$

Now $\frac{P}{\sigma}$ is the current through unit area in ampères, so if G is the current through unit breadth of the metal strip

$$\frac{G}{t} = \frac{P}{\sigma}.$$

The change of phase is therefore

$$\frac{\sin\vartheta}{4\pi}\,\frac{k}{1+k^2}\sqrt{\frac{1}{n'^2 - \sin^2\vartheta}}\,\frac{GC}{\sigma}\,10^{-10} \text{ of a wave-length.}$$

And this effect is again too small to be measurable, since the plate to transmit any light must be very thin, and so the current G cannot be made large.

To conclude, we have shown that the effect of the current in the metal is to cause an alteration in the constants of the metal but this alteration is too small for the effect to be measurable (taking 10^{-3} of a wave-length as the minimum change of phase which can be detected). The alteration also of the plane of polarisation, or rather the major axis of the ellipse of polarisation, of reflected light is similarly too small to be detected, so that as far as any measurable effects are concerned, the metal's optical properties would not be altered by the current.

The cases considered have been worked out with certain approximations which will not be valid for some metals, silver, sodium, etc., but it is easy to see that in these cases the changes will still be of the same order of magnitude as in the cases considered. In all cases the thickness of metal through which light can be passed being only a few wave-lengths, there is no possibility of a small difference of velocity producing a large enough change of phase to admit of measurement.

The Hall Effect in Gases at Low Pressures. (Second paper.)
By HAROLD A. WILSON, B.A., D.Sc. (Lond.), Clerk-Maxwell Student, Fellow of Trinity College.

[*Read* 3 March 1902.]

The experiments described in this paper are a continuation of those described in the paper entitled "On the Hall Effect in Gases at Low Pressures" (*Proc. Camb. Phil. Soc.* Vol. XI. Pt. IV.) read to this society last October.

Measurements have been made of the Hall Effect and electric intensity in uniform positive columns in oxygen and hydrogen, and also of the variation of the Hall Effect along the discharge in air at various pressures.

I. *The Hall Effect in Uniform Positive Columns in Hydrogen and Oxygen.*

The apparatus used for the measurements in oxygen and hydrogen was identical with that described in the paper just referred to. The oxygen and hydrogen were obtained by electrolysis of a strong solution of caustic soda prepared by dissolving metallic sodium in water. They were dried by calcium chloride and then by phosphorus pentoxide. The apparatus was pumped out and refilled until the light from the discharge, when examined with a pocket spectroscope, did not indicate the presence of any impurities in the gas.

Tables I. and II. contain the results obtained in hydrogen.

TABLE I.

The Hall Effect in the Uniform Positive Column in Hydrogen.

Pressure. (p)	Hall Effect. (Z)	Magnetic Field. (H)	$\frac{Zp}{H}$
0·133 mms.	7·20 volts per cm.	44·2	$2{\cdot}16 \times 10^{-2}$
0·175	5·24	44·2	2·08 „
0·188	4·82	45·3	2·00 „
0·275	3·50	45·0	2·14 „
0·292	3·17	44·2	2·10 „
0·42	2·02	40·9	2·08 „
0·53	1·63	44·7	1·93 „
0·83	1·06	44·2	1·99 „
1·00	0·95	47·9	1·99 „
		Mean	$2{\cdot}05 \times 10^{-2}$

TABLE II.

The Electric Intensity in the Uniform Positive Column in Hydrogen.

Pressure. (p)	Electric Intensity. (X)	$\frac{X}{\sqrt{p}}$
0·25 mms.	13·9 volts per cm.	27·8
0·48	18·8	27·2
0·56	21·6	28·9
0·60	21·9	28·3
1·05	28·5	27·8
1·27	31·9	28·4
1·36	32·3	27·7
	Mean	28·0

The pressures are expressed in millimetres of mercury and the Hall Effects and Electric Intensities in Volts per centimetre. Above one millimetre pressure satisfactory measurements of the Hall Effect were not obtained.

The above results show that the Hall Effect and Electric Intensity in hydrogen at pressures below one millimetre follow the same laws as in air. The Hall Effect is given by the equation $Z = 2·05 \times 10^{-3} \frac{H}{p}$, and the electric intensity by the equation $X = 28\sqrt{p}$. The corresponding equations found for air in the previous paper were $Z = 2·48 \times 10^{-2} \frac{H}{p}$, and $X = 34·9\sqrt{p}$.

Tables III. and IV. give the results obtained in oxygen.

TABLE III.

The Hall Effect in the Uniform Positive Column in Oxygen.

Pressure. (p)	Hall Effect. (Z)	Magnetic Field. (H)	$\frac{Zp}{H}$
0·21	0·802	42·6	$3·95 \times 10^{-3}$
0·32	0·463	43·1	3·43 „
0·40	0·399	42·6	3·74 „
0·48	0·348	43·1	3·83 „
0·83	0·207	43·1	3·98 „
		Mean	$3·79 \times 10^{-3}$

TABLE IV.

The Electric Intensity in the Uniform Positive Column in Oxygen.

Pressure. (p)	Electric Intensity. (X)	$\frac{X}{\sqrt{p}}$
0·139	10·3	27·6
0·31	15·5	27·9
0·37	16·3	27·0
0·40	17·2	27·1
0·44	17·7	26·6
0·64	20·7	25·9
0·85	24·3	26·4
	Mean	26·9

The Hall Effect and Electric Intensity in oxygen are therefore given by the equations $Z = 3{\cdot}79 \times 10^{-8} \frac{H}{p}$, and $X = 26{\cdot}9\sqrt{p}$.

In the previous paper it is shown that in a uniform part of the discharge

$$Z = \tfrac{1}{2} HX(k_2 - k_1),$$

k_2 and k_1 being the velocities of the negative and positive ions respectively due to one volt per cm. Consequently for hydrogen we have

$$k_2 - k_1 = \frac{2Z}{HX} = \frac{2 \times 2{\cdot}05 \times 10^{-2}}{28} p^{-1{\cdot}5} = 1{\cdot}47 \times 10^{-3} p^{-1{\cdot}5}.$$

This must be multiplied by 10^8 to get $k_2 - k_1$ in cms. per second, so that

$$k_2 - k_1 = 1{\cdot}47 \times 10^5 p^{-1{\cdot}5} \frac{\text{cms.}}{\text{sec.}}.$$

The value of $k_2 - k_1$ found in air was $1{\cdot}42 \times 10^5 p^{-1{\cdot}5}$. For oxygen in the same way

$$k_2 - k_1 = \frac{2 \times 3{\cdot}79 \times 10^{-8} \times 10^8}{26{\cdot}9} p^{-1{\cdot}5} = 2{\cdot}82 \times 10^4 p^{-1{\cdot}5}.$$

It thus appears that $k_2 - k_1$ in the uniform positive column in oxygen is only about one-fifth of $k_2 - k_1$ in hydrogen or air.

II. *The Variation of the Hall Effect along the Discharge in Air.*

The apparatus used for measuring the Hall Effect in different parts of the discharge is shown in the accompanying figure.

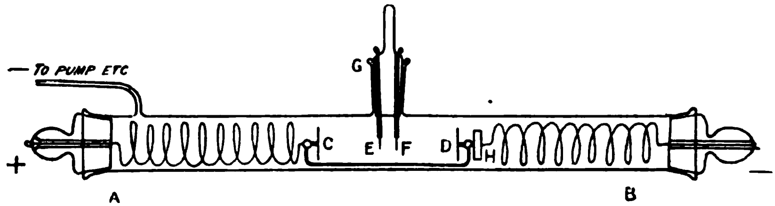


Diagram of Apparatus.

It consisted of a glass tube AB, 58 cms. long and 4·5 cms. in diameter, provided with a glass stopper at each end. At the centre of the tube a side tube G was sealed on and also provided with a glass stopper which carried two electrodes E and F for measuring the Hall Effect.

The discharge was passed between two aluminium disks C and D, each 3 cms. in diameter, which were supported by a framework of thin glass rods which kept them at a constant distance apart. Flexible spirals of copper wire connected these electrodes with platinum wires sealed through the ends of the stoppers at A and B. A coil of thin sheet iron H was fixed to the frame carrying the electrodes and enabled it to be moved along the tube by means of a small electro-magnet. In this way the electrodes E and F could be brought into any portion of the discharge between C and D. The electrodes E and F were 1·40 cms. apart, and were connected through a commutator to an insulated quadrant electrometer. The discharge between C and D was produced by means of a battery of small secondary cells and an adjustable resistance and telephone were included in the circuit. The resistance and number of cells used were adjusted until a sensibly steady discharge was obtained with the kathode nearly but not quite covered by the negative glow. Under these conditions the Hall Effect and distribution of potential in the tube are nearly independent of the current.

A pair of circular coils each of about 15 cms. radius were arranged symmetrically one above and one below the tube and separated by a distance equal to the radius of either. A current, measured by means of a Weston Ammeter, could be passed through these coils, and this produced a very uniform magnetic field near E and F in a direction parallel to the axis of the stopper G.

When making a measurement of the Hall Effect the stopper G was first rotated until the electrometer indicated that E and F were at the same potential. The magnetic field was then applied and reversed, and the change in the P. D. between E and F on reversing the field was measured by the electrometer.

The results obtained are shown in the accompanying diagrams Nos. 1, 2, 3 and 4 (pages 396, 397). An examination of the curves in these diagrams shows that the variation of the Hall Effect along the discharge is very similar to that of the electric intensity[1].

In the previous paper it is shown that in a uniform part of the discharge

$$Z = \tfrac{1}{2} HX (k_2 - k_1).$$

According to this equation Z is proportional to X so that the curves for Z and X would be exactly similar if this equation held true in all parts of the discharge.

In the striated positive column the Hall Effect falls to an almost zero value between the striae, whereas the electric intensity only falls to a comparatively small extent.

It appears therefore that the difference between k_2 and k_1 in the dark spaces between the striae is very small. It is probable that when a molecule is ionised a free electron or negative corpuscle is split off so that the negative ion is of very small mass compared with the positive ion; $k_2 - k_1$ is therefore at first very large. After a short time however the negative corpuscle probably becomes attached to a neutral molecule, so that the two ions are then of almost equal mass and $k_2 - k_1$ is very small. It seems therefore probable that ions are formed largely in the striae, and that by the time they get into the dark spaces, the negative ions which were corpuscles at first have become attached to molecules.

This view is in agreement with the idea that each stria and its adjacent dark space constitute a to some extent independent portion of the discharge analogous to that formed by the negative glow and Faraday dark space.

In conclusion I wish to say that my best thanks are due to Prof. J. J. Thomson for his kindly interest and advice during the carrying out of these experiments in the Cavendish Laboratory.

[1] For curves showing the variation of the electric intensity see H. A. Wilson, *Phil. Mag.*, June, 1900.

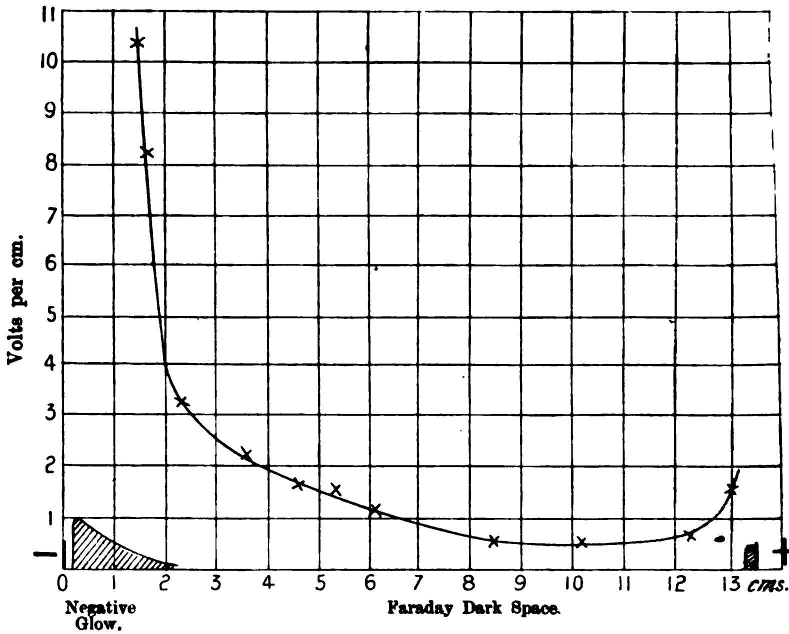


DIAGRAM No. 1. Discharge in Air. Pressure 2·4 mms. Magnetic Field 29·4.

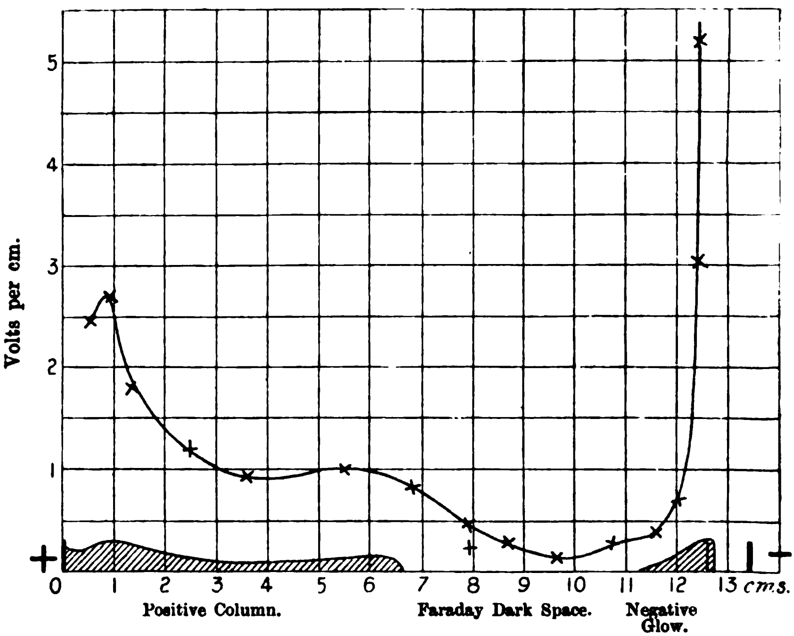


DIAGRAM No. 2. Discharge in Air. Pressure 0·5 mm. Magnetic Field 22·1.

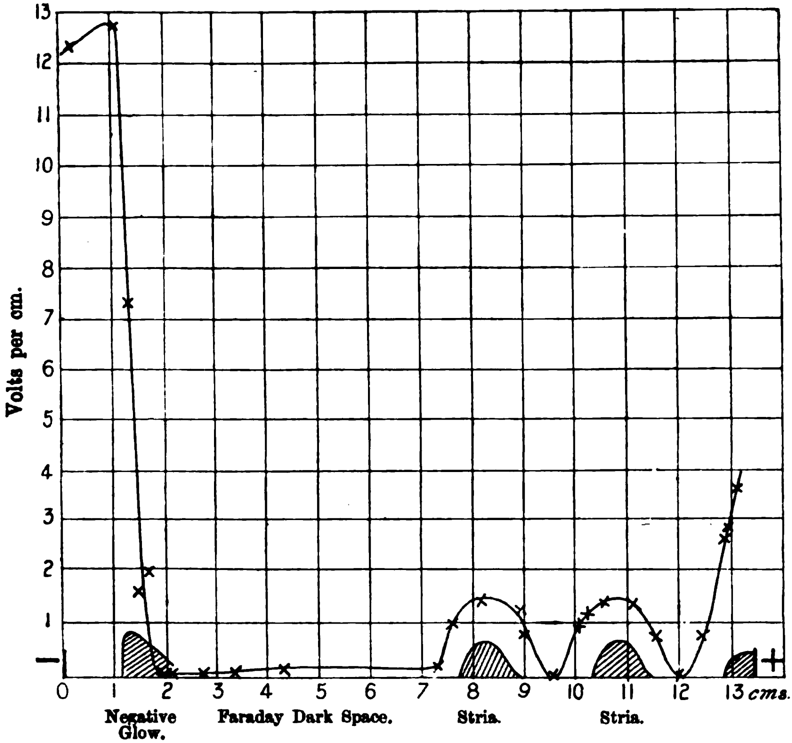


DIAGRAM No. 3. Discharge in Air. Pressure 0·3 mm. Magnetic Field 29·4.

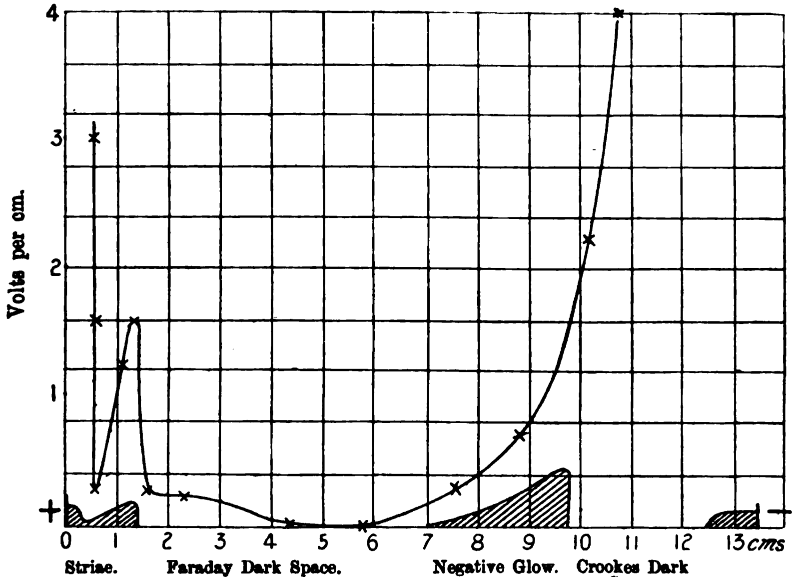


DIAGRAM No. 4. Discharge in Air. Pressure 0·11 mm. Magnetic Field 22·8.

On the Coefficient of Mutual Induction for a circle and a circuit with two parallel sides of infinite length. By G. F. C. SEARLE, M.A., Peterhouse, University Lecturer in Physics, and Demonstrator in Experimental Physics.

[*Read* 3 March 1902.]

§ 1. Maxwell, in the chapter on Circular Currents in his *Treatise on Electricity and Magnetism,* has shewn how to express, in the form of a series, the coefficient of mutual induction for two circular circuits, whose axes meet in a point at any angle θ, the successive terms of the series containing the factors

$$P_0(\cos\theta),\ P_1(\cos\theta),\ \ldots,$$

where $P_n(\cos\theta)$ is the Legendre coefficient or "surface zonal harmonic" of the nth order. In the present communication, by using a process in principle identical with that employed by Maxwell, I obtain the coefficient of mutual induction when one of Maxwell's circles is replaced by a circuit having two infinitely long parallel sides, the shortest distance between the sides being finite. A telephone circuit with its pair of parallel wires is a practical approximation to such a circuit. The series in which the result is expressed involves the two angular coordinates, which determine the direction of the axis of the circle relative to the two parallel sides, in the form of "surface sectorial harmonics," these functions playing the same part in the present problem as the Legendre coefficients play in Maxwell's problem.

§ 2. It may be useful to give a general explanation of the principle of the method before proceeding to the detailed calculation for the problem in hand.

If we take two systems, S, T, of matter, attracting according to the law of the inverse square, of which T is symmetrical round a straight line, we can apply the method to calculate the potential energy of T in the field of S provided that the least distance, s, from the origin of coordinates to any "particle" of S exceed the greatest distance, t, from the origin to any particle of T, and hence I shall not restrict the explanation to systems giving rise to magnetic force.

§ 3. Let TOT', Fig. 1, be the straight line about which the system T is symmetrical, and let O be taken as the origin of co-

ordinates for the two systems S and T. Then the semi-infinite straight line OT is called the "axis" of the system T; the "axis" thus extends in *one direction only* from O. Now let the potential due to T be expressed at any point K on its axis by the series

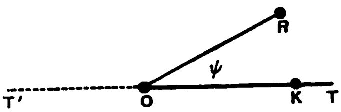


Fig. 1.

$$V_0 = \frac{g_0}{r} + \frac{g_1}{r^2} + \frac{g_2}{r^3} + \dots (r > t) \dots\dots\dots\dots\dots\dots (1),$$

where $OK = r$. Then by Legendre's theorem the potential at any point R on a radius OR, which makes an angle ψ with the axis OT, is

$$V = \frac{g_0 P_0 (\cos\psi)}{r} + \frac{g_1 P_1 (\cos\psi)}{r^2} + \dots (r > t) \dots\dots (2),$$

the series (1) and (2) being "absolutely" convergent if $r > t$ where t is the greatest distance from O of any particle of the system T.

But (2) is the potential at (r, ψ) due to a system, Q, of singular points of orders 0, 1, 2, ..., and moments $g_0, g_1, g_2 \dots$ placed at O, every particle of each singular point lying on the axis OT. Thus, at all points outside the sphere $r = t$, the potentials due to T and to Q are equal.

Now, if W be the potential energy of T in the field of S, the potential energy of S in the field of T is also W. But, when $s > t$, so that every particle of S lies outside the sphere $r = t$, centred at the origin O, the potential energy of S in the field of T is equal to the potential energy of S in the field of Q. This last is equal to the potential energy of Q in the field of S.

Considering the system Q, let $m, m', \dots$ be the masses of the particles which form the whole system of singular points, and let $h, h', \dots$ be their distances from O in the direction OT. Then the potential due to Q, at any distant point on OT, is

$$V_0 = \Sigma \frac{m}{r-h} = \Sigma m \left(\frac{1}{r} + \frac{h}{r^2} + \frac{h^2}{r^3} + \dots\right)$$

$$= \frac{\Sigma m}{r} + \frac{\Sigma mh}{r^2} + \frac{\Sigma mh^2}{r^3} + \dots \dots\dots\dots\dots\dots\dots (3).$$

But the values of V_0 given by (1) and (3) must be equal. Hence, equating coefficients of the powers of $1/r$ in the two series, we obtain

$$\Sigma m = g_0, \quad \Sigma mh = g_1, \quad \Sigma mh^2 = g_2, \ \&c. \dots\dots\dots (4).$$

§ 4. To find the potential energy of Q in the field of S, I follow a method suggested by § 131 *b* in Maxwell's chapter on Spherical Harmonics. We have simply to multiply the mass of each particle of Q by the potential at that particle due to S. That is, if U denote the potential due to S, we must find the sum $\Sigma m U$.

Now, whatever be the form of the system S, we know that at any point for which $r < s$, where s is the least distance of any particle of S from the origin, the potential can be expanded in the "absolutely" convergent series of spherical harmonics

$$U = Y_0 + rY_1 + r^2 Y_2 + \ldots (r < s) \quad \ldots\ldots\ldots\ldots \quad (5),$$

when Y_0, $Y_1 \ldots$ are functions of the two angular coordinates θ, ϕ employed to fix the direction of the radius vector r.

Hence when the angular coordinates of the axis OT are θ, ϕ, the potential due to S at a point on OT at a distance h from O, is

$$U = Y_0 + hY_1 + h^2 Y_2 + \ldots,$$

so that

$$W = \Sigma m U = \Sigma m (Y_0 + hY_1 + h^2 Y_2 + \ldots)$$
$$= Y_0 \Sigma m + Y_1 \Sigma mh + Y_2 \Sigma mh^2 + \ldots.$$

Hence by (4)

$$W = g_0 Y_0 + g_1 Y_1 + g_2 Y_2 + \ldots \quad \ldots\ldots\ldots\ldots\ldots \quad (6).$$

By what has been proved in § 3, this series expresses the mutual potential energy of the systems S and T.

We see that any term of the series, as $g_n Y_n$, is obtained by multiplying g_n by the value of Y_n corresponding to the direction of the axis of T.

§ 5. It is easily shewn that, under the condition $s > t$, imposed by § 3, the series (6) found for W is absolutely convergent. For since $s > t$, we can take a length q such that $s > q > t$, and then each of the series

$$Y_0 + qY_1 + q^2 Y_2 + \ldots,$$

$$g_0/q + g_1/q^2 + g_2/q^3 + \ldots,$$

is absolutely convergent. Hence, using $|\ |$ to denote the *numerical* magnitude of a quantity,

$$|Y_{n+1}/Y_n| < 1/q, \qquad |g_{n+1}/g_n| < q,$$

so that

$$|Y_{n+1} g_{n+1}/(Y_n g_n)| < 1.$$

The last result shews that the series for W is absolutely convergent if the least distance from O of any particle of S exceed the greatest distance from O of any particle of T.

When linear electric currents are in question, the magnetic shells, by which they can be replaced, may be of any forms provided they are bounded by the circuits. In this case the series for W is convergent, if the least distance from the origin of any part of the wire belonging to S exceed the greatest distance of any part of the wire belonging to T.

§ 6. We are now able to attack the problem of finding the coefficient of mutual induction between the circle and the circuit with two parallel sides of infinite length. Denoting the circle by T and the other circuit by S, and the coefficient of mutual induction by M, the potential energy of T in the magnetic field of S is $-M$, when the currents in S and T are each of unit strength. Thus, if W denote the potential energy in this case

$$M = -W \quad \text{.........................} \quad (7).$$

§ 7. Let the diagram (Fig. 2) represent lines drawn in a plane which cuts at right angles the two parallel wires forming

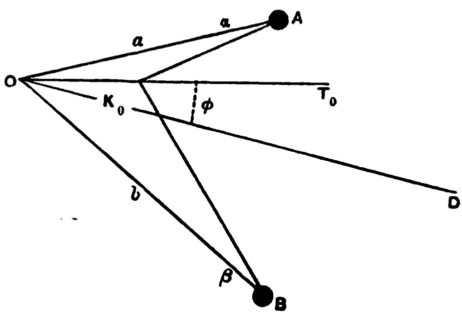


Fig. 2.

the infinite sides of S. Let A, B be the sections of the wires Taking any point O in the plane of the paper as origin, let $OA = a$, $OB = b$. Let the angle AOB be 2γ and let OD bisect it. Let OT_0 be the projection on the plane of the paper of OT, the axis of the circle, and let $T_0OD = \phi$, and let the angle between OT and a straight line parallel to the wires be θ. Thus θ is the co-latitude, and ϕ the longitude, on a sphere, whose polar axis is parallel to the long wires. Let K be any point on OT and K_0 its projection. Then if $OK = r$, the coordinates of K are r, θ, ϕ. If we denote OK_0 by ρ, then the coordinates of K_0 in the plane of the paper are ρ, ϕ.

We must now expand the magnetic potential due to S in a series involving ρ and ϕ. If the current in S, as seen from O,

appear to circulate in a direction *contrary* to that of the hands of a watch, then the magnetic potential at K_0 due to unit current is $2 . AK_0B$. Denoting the angles OAK_0 and OBK_0 by α and β we have for the magnetic potential

$$U_0 = 2 . AK_0B = 2(2\gamma + \alpha + \beta).$$

On account of the infinite length of the two parallel sides of S, the potential has the same value at all points on a straight line through K_0 parallel to the infinite sides.

Now from the triangles OAK_0, OBK_0,

$$\sin\alpha = \frac{\rho}{a}\sin(\alpha + \gamma - \phi),\quad \sin\beta = \frac{\rho}{b}\sin(\beta + \gamma + \phi).$$

Hence, expanding (Todhunter, *Plane Trigonometry*, Chapter XXI.),

$$\alpha = \frac{\rho}{a}\sin(\gamma - \phi) + \frac{1}{2}\frac{\rho^2}{a^2}\sin 2(\gamma - \phi) + \frac{1}{3}\frac{\rho^3}{a^3}\sin 3(\gamma - \phi) + \dots,$$

$$\beta = \frac{\rho}{b}\sin(\gamma + \phi) + \frac{1}{2}\frac{\rho^2}{b^2}\sin 2(\gamma + \phi) + \frac{1}{3}\frac{\rho^3}{b^3}\sin 3(\gamma + \phi) + \dots.$$

Thus, for unit current in S,

$$\begin{aligned} U_0 &= 2(2\gamma + \alpha + \beta) \\ &= 2\left[2\gamma + \rho(u_1\cos\phi - v_1\sin\phi) + \tfrac{1}{2}\rho^2(u_2\cos 2\phi - v_2\sin 2\phi) + \dots\right] \quad \text{............ (8)}, \end{aligned}$$

where $\qquad u_n = \sin n\gamma\left(\frac{1}{a^n} + \frac{1}{b^n}\right),\quad v_n = \cos n\gamma\left(\frac{1}{a^n} - \frac{1}{b^n}\right).$

But the potential at K is equal to the potential at K_0, the projection of K. It then only remains to express (8) in terms of r, θ, ϕ, the coordinates of K, by substituting for ρ its value $r\sin\theta$. We thus obtain for the potential at (r, θ, ϕ) due to unit current in S,

$$\begin{aligned} U = 2\,[2\gamma &+ r\sin\theta(u_1\cos\phi - v_1\sin\phi) \\ &+ \tfrac{1}{2}r^2\sin^2\theta(u_2\cos 2\phi - v_2\sin 2\phi) + \dots] \quad \text{......... (9)}. \end{aligned}$$

We have thus obtained the expansion contemplated in (5) in terms of the Sectorial Harmonics

$$r^n\sin^n\theta\cos n\phi,\quad r^n\sin^n\theta\sin n\phi.$$

Comparing the coefficients of r^n in (5) and (9) we have

$$Y_n = \frac{2}{n}\sin^n\theta(u_n\cos n\phi - v_n\sin n\phi) \quad \text{......... (10)},$$

except when $n = 0$. In this case, $Y_0 = 4\gamma$.

§ 8. Turning now to the circle, we have to find the magnetic potential at points on its axis, due to unit current, in the form

$$V_0 = \frac{g_0}{r} + \frac{g_1}{r^2} + \frac{g_2}{r^3} \dots .$$

We notice at once that $g_0 = 0$, since at a great distance the current acts as a small magnet; the leading term consequently varies inversely as the square of the distance. Hence the term Y_0 will not appear in the expression for M.

I now suppose that the centre of the circle coincides with O. If c be the radius of the circle, and if the current in it appear from a point T, on the "axis" of the circle, to circulate in the *same* direction as the hands of a watch, then the magnetic potential at a point on the axis, for unit current, is

$$\begin{aligned} V_0 &= -2\pi\left(1 - \frac{r}{\sqrt{r^2 + c^2}}\right) \\ &= -2\pi\left(\frac{1}{2}\frac{c^2}{r^2} - \frac{1\,.\,3}{2\,.\,4}\frac{c^4}{r^4} + \frac{1\,.\,3\,.\,5}{2\,.\,4\,.\,6}\frac{c^6}{r^6} + \dots\right) \quad (r > c) \dots (11). \end{aligned}$$

Comparing (11) with (1) we have

$$\left.\begin{aligned} & g_0 = g_2 = g_4 = \dots = 0 \\ & g_1 = -2\pi\,.\,\frac{1}{2}c^2,\ \ g_3 = 2\pi\frac{1\,.\,3}{2\,.\,4}c^4,\ \ g_5 = -2\pi\frac{1\,.\,3\,.\,5}{2\,.\,4\,.\,6}c^6 \end{aligned}\right\} \dots\dots (12).$$

§ 9. Remembering that $M = -W$, where W is here the mutual potential energy when a unit current flows in each coil, we have by (6), (10) and (12)

$$\begin{aligned} M &= 4\pi c\left[\frac{1}{2}c\sin\theta\,(u_1\cos\phi - v_1\sin\phi)\right. \\ &\qquad\left. - \frac{1\,.\,3}{2\,.\,4}\cdot\frac{1}{3}c^3\sin^3\theta\,(u_3\cos 3\phi - v_3\sin 3\phi) + \dots\right] \\ &= 4\pi c\,\Sigma\,(-1)^{m-1}\frac{(2m)!}{2^{2m}\,m!\,m!\,(2m-1)}\sin^{2m-1}\theta \\ &\qquad\times\left[\cos(2m-1)\phi\sin(2m-1)\gamma\left(\frac{c^{2m-1}}{a^{2m-1}} + \frac{c^{2m-1}}{b^{2m-1}}\right)\right. \\ &\qquad\left. - \sin(2m-1)\phi\cos(2m-1)\gamma\left(\frac{c^{2m-1}}{a^{2m-1}} - \frac{c^{2m-1}}{b^{2m-1}}\right)\right], \end{aligned}$$

where m ranges from 1 to ∞, and $c < a$ and $c < b$.

The simplest case is that in which the origin lies in the plane of the two straight wires and is midway between them. We then have $b = a$ and $\gamma = \frac{1}{2}\pi$, so that

$$u_{2m-1} = (-1)^{m-1}\frac{2}{a^{2m-1}}, \quad v_{2m-1} = 0,$$

and then, if $c < a$,

$$M = 8\pi c\left[\frac{1}{2}\frac{c}{a}\sin\theta\cos\phi + \frac{1.1}{2.4}\frac{c^3}{a^3}\sin^3\theta\cos 3\phi + \frac{1.1.3}{2.4.6}\frac{c^5}{a^5}\sin^5\theta\cos 5\phi + \dots\right].$$

By § 5, these series are convergent provided that the radius of the circle is less than the shortest distance from its centre to either of the infinite wires.

If we introduce the additional limitation that $\phi = 0$, so that now a diameter of the circle lies on a line cutting both long wires at right angles, the last expression for M reduces to

$$M = 8\pi \operatorname{cosec}\theta\,\{a - \sqrt{a^2 - c^2\sin^2\theta}\}, \qquad (c < a)$$

as may be seen by expanding the square root $(a^2 - c^2\sin^2\theta)^{\frac{1}{2}}$.

APPENDIX [*Added* 14 April 1902].

The following method, of obtaining the expression

$$W = g_0 Y_0 + g_1 Y_1 + g_2 Y_2 + \dots$$

for the mutual potential energy of the systems S and T, seems preferable to that given in §§ 3, 4 of the foregoing paper.

I will consider first the case in which every part of S is further from O than any part of T, so that $s > t$.

Let the potential due to T be expressed at points on its axis OT (Fig. 3) by the series

$$V_0 = \frac{g_0}{r} + \frac{g_1}{r^2} + \frac{g_2}{r^3} + \dots. \qquad (r > t)$$

Then the potential due to T at a point H, which lies, at a distance R from the origin, on a radius making an angle ψ with OT, is

$$V = \frac{g_0}{R} + \frac{g_1 P_1(\lambda)}{R^2} + \frac{g_2 P_2(\lambda)}{R^3} + \dots, \qquad (R > t)$$

where $\lambda = \cos\psi$.

Now let there be at H a single particle, m, of the system S. Then the potential energy of T in the presence of m is mV or

$$g_0\frac{m}{R}+g_1\frac{mP_1(\lambda)}{R^2}+g_2\frac{mP_2(\lambda)}{R^3}+\dots \qquad (R>t)$$

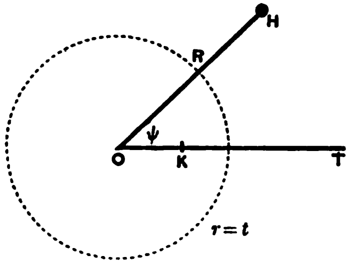


Fig. 3.

But the potential due to m at a point K on OT, for which $r<R$, is m/HK, or, when HK is expanded in ascending powers of r by the fundamental expansion formula of zonal harmonics,

$$m\left[\frac{1}{R}+r\frac{P_1(\lambda)}{R^2}+r^2\frac{P_2(\lambda)}{R^3}+\dots\right]. \qquad (R>r)$$

If there be any number of particles in S, then, provided they be all further from O than any part of T, the potential energy of T in the presence of S is given by

$$W=g_0\Sigma\frac{m}{R}+g_1\Sigma\frac{mP_1(\lambda)}{R^2}+g_2\Sigma\frac{mP_2(\lambda)}{R^3}+\dots, \qquad (s>t)$$

while the potential due to S, at a point on OT near the origin, is

$$U=\Sigma\frac{m}{R}+r\Sigma\frac{mP_1(\lambda)}{R^2}+r^2\Sigma\frac{mP_2(\lambda)}{R^3}+\dots, \qquad (s>r)$$

the summation in each case including all the values of R and λ necessary to take account of every particle of S.

Comparing the last two expressions, we see that if the potential due to S, at points on OT near O, be denoted by

$$U=Y_0+rY_1+r^2Y_2+\dots, \qquad (r<s)$$

then the mutual potential energy of S and T is

$$W=g_0Y_0+g_1Y_1+g_2Y_2+\dots,$$

the series for W being absolutely convergent provided $s>t$.

If every part of S be nearer to the origin than any part of T, and if the potential due to S, at distant points on OT, be denoted by

$$V = \frac{y_0}{r} + \frac{y_1}{r^2} + \frac{y_2}{r^3} + \dots$$

and the potential due to T, at points on OT near the origin, by

$$U_0 = G_0 + G_1 r + G_2 r^2 + \dots,$$

then we can shew, in a similar manner, that the mutual potential energy of S and T is

$$W = G_0 y_0 + G_1 y_1 + G_2 y_2 + \dots.$$

Prof. T. J. I'A. Bromwich has kindly pointed out to me that the connexion between the equations (1) and (2) requires a word of comment. The point is that if S_n be the sum of the series (2) to n terms, the quantities

$$\mathrm{Lt}_{\cos\psi=1}(\mathrm{Lt}_{n=\infty} S_n) \text{ and } \mathrm{Lt}_{n=\infty}(\mathrm{Lt}_{\cos\psi=1} S_n)$$

are not necessarily equal unless the series (2) be *uniformly convergent*. It is however easily shewn that this condition is satisfied provided r be definitely greater than t.

If M be any particle of T, whose polar coordinates with respect to OT are R, ϕ, then the potential at distant points on OT is expressed by a series, whose nth term is $\Sigma M R^n P_n(\cos\phi)/r^{n+1}$ or g_n/r^{n+1}, the summation including all the particles of T. Hence $|g_n| \ngtr t^n \Sigma |M|$. When T is a system of electric currents, flowing in circles whose common axis is OT, it can be shewn that

$$|g_n| \ngtr 4\pi t^{n+1} \Sigma |i|,$$

where t is the greatest distance from O to any wire of the system.

If now we take two lengths k, l, such that $k > t$ and $l > k$, then $|g_n P_n(\cos\psi)/l^{n+1}| < |g_n/k^{n+1}|$ for all values of $\cos\psi$, the extreme values ± 1 included. But, by the inequalities just mentioned, the series $g_0/k + g_1/k^2 + \dots$ is *absolutely* convergent, and hence when $r \nless l$ the series (2) is *uniformly* convergent for all values of $\cos\psi$. Thus, provided r be definitely greater than t, we may put $\cos\psi = 1$ either before or after the summation.

Notes on Semper's Larvae. By K. RAMUNNI MENON, B.A., Christ's College; Assistant Professor, Presidency College, Madras.

[*Received* 3 March 1902.]

While examining the Madras Plankton during the months of September and October last I was fortunate enough to come across some specimens of the Anthozoan larvae originally described by Semper in 1867, and since then in greater detail by É. Van Beneden (1) As some of the stages obtained on the present occasion have not, as far as I know, been previously described, and further appear to me to be of some value in determining the phylogenetic relations of the Zoantheae, the group to which these larval forms undoubtedly belong, a few observations have been embodied in the following notes. I regret I have not been able to consult Van Beneden's recent volume on the Anthozoan larvae; I cannot thus ascertain if any of these stages or observations are really new.

I must here express my most grateful thanks to Prof. A. G. Bourne for enabling me to obtain a regular and abundant supply of Plankton material; and to Mr A. E. Shipley, of Christ's College, for communicating these notes for publication.

Of Semper's first larva—named Zoanthella, by Van Beneden—I obtained half-a-dozen specimens of a very early stage, and after a prolonged search succeeded in getting one of a much later stage. This was carefully preserved and cut into sections. Of the young specimens one was preserved and cut; the others were kept in sea-water with a view to obtain later stages. This attempt proved a failure, as after a period of over a month the two which survived were only a little more advanced than the earliest stage.

The single specimen of the latest stage obtained measured nearly 8 mm. in length, and was about 3 mm. thick in the middle. It has a pyriform body with a circular mouth placed at the narrow anterior end (fig. 1); a small ventral lip projects forwards below the mouth, but there is no dorsal lip. I have not seen the aboral aperture mentioned by Semper, and do not believe any such exists in this larva. The whole surface of the body is covered by small cilia. The peculiar, iridescent, ventral, longitudinal band of long cilia, stretching from near the mouth to about a quarter of the animal's length from the aboral end, beats in a slow, rhythmical manner from side to side and apparently plays only a subsidiary

part in the locomotion of the animal. The animal has a gray colour, with dark brown patches, especially at the aboral end.

After going through the sections, I have little to add to Van Beneden's most excellent description. The stomodaeum is short and its walls are thrown into folds. Except near the lower end of the stomodaeum there is no correspondence between the ridges and grooves and the chambers and mesenteries. Though Van Beneden believes in the existence of a ventral siphonoglyph I have not been able to find any indication of it. There is no difference in length between the dorsal and ventral regions of the stomodaeum. It is only a comparison of the arrangement of the mesenteries of this form with that of other Zoantheae that justifies the use of the terms *dorsal* and *ventral*.

There are twelve mesenteries, six large and complete, and six small and incomplete. The three pairs of macromesenteries differ considerably in size: their arrangement is the same as that described by Van Beneden. The differences between the micromesenteries are not quite so marked. The dorso-lateral pair seem to be the largest; of the two remaining pairs, the dorsal pair are slightly smaller than the ventro-lateral. This is different from the account given by Van Beneden, according to whom the dorso-lateral are the largest and the ventro-lateral the smallest, the dorsal pair being thus intermediate in size. The order given above, though differing from Van Beneden's, agrees with what obtains in Semper's second larva. A diagram of the arrangement of the mesenteries of *Zoanthella*, with the differences between the micromesenteries slightly exaggerated, is given in fig. 3. The probable order of their development is also indicated in the same diagram. It has been objected to thus deducing, from the size of the mesenteries, their order of development that the incomplete mesenteries are not necessarily later formations than the complete ones, but that their incompleteness may be due to other causes, McMurrich (2). This objection certainly does not hold good in the case of *Zoanthina*, and most probably does not do so in the case of *Zoanthella*. It seems to be perfectly reasonable to infer the order of the mesenteries in these forms from their size until actual facts are brought forward to disprove it.

The longitudinal muscles of the mesenteries, as far as they can be made out, are arranged in the manner described by Van Beneden. The dorsal pair of micromesenteries and the ventral pair of macromesenteries have their longitudinal muscles on the faces turned away from the dorsal and ventral median chambers respectively. The remaining macromesenteries have their muscles on their ventral faces, and the micromesenteries on their dorsal faces. I have not been able to detect any transverse muscles on the mesenteries or in the body wall.

The macromesenteries have well-developed mesenterial filaments. Gland cells and occasionally the larger kind of nematocysts are seen in them.

The deeper layer of the ectoderm is in places closely packed with the larger kind of nematocysts. It is very rarely that a whole nematocyst, with capsule and thread, is seen. What one generally finds is a number of threads loosely and irregularly coiled, lying in large oval spaces in the deeper parts of the ectoderm; their capsule is visible. These threads are very conspicuous structures in transverse sections. The smaller nematocysts are very numerous in the superficial layer of the ectoderm. The colour of the larva is due to the numerous pigment cells in the ectoderm. The mesogloea is a homogeneous membrane, thicker in some places than in others. It contains cell-elements of two kinds, small, clear, ovoid cells and large, round, granular cells. Of these the former seem to be derived from the similar cells which lie here and there in the ectoderm, immediately next to the mesogloea, just in the same way as the large, granular cells are derived, as is indeed stated by Van Beneden, from the deeply staining cells which lie here and there on the inner face of the mesogloea. The cells of the mesogloea lie in spaces which are often seen without the cells. They do not form strands, as in *Zoanthina*.

The endoderm is a layer of great thickness. It is thin in the region of the stomodaeum, but increases in thickness aborally. In sections passing a little below the lower end of the stomodaeum the endoderm forms large masses almost filling up the mesenteric chambers, with their rounded inner ends projecting into the coelenteron. The endoderm covering the faces of the mesenteries forms a comparatively thin layer. About a quarter of the animal's length from the aboral end the endoderm almost fills up the coelenteric cavity, a small irregular space in it representing the much reduced coelenteron. Still lower down the cavity disappears altogether. The endoderm is highly vacuolated and contains numerous scattered nuclei. This condition of the endoderm is interesting inasmuch as it shows that the explanation suggested by G. C. Bourne of a similar condition of the endoderm in *Euphyllia*, viz., that the great development of the vacuolated endoderm filling up the coelenteron is connected with the presence of a large number of Zooxanthellae, is not applicable to all such instances. The endoderm covering the dorsal and ventral faces of the macromesenteries as well as the whole of the endoderm covering the micromesenteries, consists of large granular cells or masses of cells with one or more nuclei. It stains very deeply. Similar deeply-staining cells often form a core to the large vacuolated masses projecting into the coelenteron. The

mesenteries, on account of the presence of these cells, form most conspicuous structures in transverse sections. The cells of the mesenterial filaments agree with those of the lining of the stomodaeum. The filaments consist of radially arranged, elongated cells and have a rounded outline.

The youngest stage obtained of *Zoanthella* was a sterrogastrula of about 1 mm. in length. It had a vermiform body with rounded ends, and was slightly bent towards the ventral side. In addition to the covering of small cilia there was the characteristic longitudinal band of long cilia. It had a yellowish colour with irregular dark brown patches (fig. 4).

In this stage there is a circular mouth placed at one end of the body. A section of this stage is shown in fig. 5. The ectoderm is a well-defined layer, and consists of radially arranged columnar cells. Besides the ordinary cells, cells with granular contents and nematocysts are also present. The nuclei which are round or elongated are arranged in the outer and middle parts of the ectoderm. Below this nucleated layer there is a zone which remains uncoloured in stained sections. On the ventral side of the transverse section there is a shallow groove which lodges the long cilia. At the bottom of the groove the nuclei are round and lie close together.

The ectoderm is separated from the inner mass by a thin, structureless lamella. This lamella is very thin throughout and in the region of the ciliated band appears to be almost absent. Within this mesogloea there is a syncytium which completely fills up the interior of the gastrula. The syncytium contains numerous scattered nuclei, yolk spherules and large and small vacuoles, some of which are in the living larva filled with fat globules. The vacuoles become larger towards the mouth. Immediately below the mouth there is a shallow quadrangular space into which the mouth opens. The space is lined by a continuation of the ectoderm, and is thus a short stomodaeum. The colour of the larva seems to be due to the coloured globules in the inner mass. Whether this syncytial mass has been developed secondarily from the simple wall of a coeloblastula as in *Manicina*, or, as seems more probable, is the inner portion of a solid morula the outer cells of which became differentiated to form the ectoderm, cannot of course be decided till the earlier stages are examined.

The definitive endoderm is formed from the syncytium in a manner which has been described for *Manicina*, *Ceriactis*, and other genera. This is well seen in an older larva of which a section is represented in fig. 6. The ectoderm does not differ from that of the preceding stage. The mesogloea remains very thin. Neither here nor in the earlier stage is there any folding of the mesogloea to indicate the commencement of a mesentery.

The syncytium presents a new appearance. The central portion has separated off from the peripheral and lies in a central cavity into which the stomodaeum opens. In sections passing through the oral half of the larva this central portion is not seen; in this region, therefore, the mass has already broken down, giving place to a cavity which occupies the whole of the interior of the larva. The vacuolated peripheral portion of the syncytium containing a little yolk becomes the endoderm. The central portion, which is also vacuolated and contains yolk, has quite disappeared in the following stage, having, no doubt, been resorbed. In this stage, a section of which is shown in fig. 7, the central cavity is lined by a vacuolated endoderm layer containing scattered nuclei and small rounded granular masses of yolk. The endoderm thus apparently does not at any stage consist of regularly arranged cells like the ectoderm, but is from the beginning a vacuolated syncytium which in later stages becomes enormously developed and fills up the greater part of the coelenteron.

The coelenteron thus arises by the splitting of the inner mass and becomes definitely established by the breaking down and resorption of part of that mass.

In the stage represented in fig 7, along two opposite lines of the coelenteron, the nuclei of the endoderm lie close together, and here the endoderm (as far as can be made out from the longitudinal section) projects in the form of two irregular ridges into the coelenteron. They are, no doubt, the beginnings of the first pair of mesenteries. The mesenteries are thus formed at an early stage and before the others. The stomodaeum is quite well marked and its epithelium is of the same nature as the outer ectoderm. I have not noticed in the ectoderm of the earlier stages the pigment cells found in the late stage.

I obtained several specimens of Semper's second larva which, I believe, is the one named *Zoanthina* by Van Beneden. The youngest specimen (fig. 8) measured about $\frac{3}{4}$ mm. and the oldest over 3 mm. in length. I have not found any specimens of this larva associated together in the curious manner described by Van Beneden—younger stages occurring within the body of an older stage.

The form of the larva varies from time to time. Its usual appearance as it lies on the bottom of the glass vessel is that shown in fig. 8. Often it assumes a more elongated club-shaped or subcylindrical form. The colour varies; it may be light-brown or dark-brown or slightly yellow, or the larva may be almost colourless. The colour seems to depend to some extent on the amount of coloured yolk in the endoderm. Light streaks running along the outside of the larva mark the attachments of the mesenteries, which are also made evident by shallow external grooves.

As in *Zoanthella* there is a uniform covering of small cilia; but what characterises this form is a ring of especially long cilia attached to the bottom of a groove running round the body at about a third of its length from the oral end. This circular band is not so markedly iridescent as the longitudinal band of *Zoanthella*. The cilia are directly orally and work together rhythmically as if they formed a rhythmically vibrating membrane (the movement can be best compared to that of the umbrella of a jelly-fish). The ciliated band pulsates very rapidly for a few seconds, then stops for a short interval, and then pulsates again. The result of this movement of the cilia is that the larva darts through the water, aboral end directed forwards, for a few millimeters, then rests on the bottom of the vessel, and then darts through the water again—its orbit thus consisting of a number of small arcs. These movements were best observed in the youngest specimen, which was remarkably active. The older specimens, when not lying on the bottom of the vessel, suspend themselves in the water, the circular band of cilia moving very slowly and only occasionally. The movement of this ciliated band thus differs from that of the ciliated bands of an ordinary trochosphere.

The mouth is a quadrangular opening. I have not found the oral papillae mentioned by McMurrich. The wall of the stomodaeum is thrown into longitudinal folds and grooves which correspond to the mesenteric chambers and mesenteries. They become less marked towards the mouth. There are five ridges projecting into the lumen of the stomodaeum and corresponding to the dorsal and the two pairs of lateral primary mesenteric chambers; there are four grooves opposite the attachments of the dorsal and lateral pairs of mesenteries to the stomodaeum. The ventral region is occupied by one large groove, the siphonoglyph. These structures can be made out in the living larva as well as in sections, and are thus quite natural.

The inner organisation agrees in all essential respects with that of *Zoanthella*. A siphonoglyph can be distinctly made out in larger specimens. It lies, as is usually the case in the Zoantheae, on the side of the stomodaeum, which is on that account designated ventral. In earlier stages the siphonoglyph appears to be wanting; its lower end is not produced into a lappet.

There are twelve mesenteries, six large and six small, having the same arrangement as in *Zoanthella*. The ectoderm has, in addition to ordinary cells, gland cells, some of which have coarse granules and others clear, non-staining, reticulate contents. Pigment cells with brown or yellow granules are present, but are not so numerous as in *Zoanthella*. There are two kinds of nematocysts (fig. 9), the larger of which are much more elongated relatively than the larger nematocysts of *Zoanthella*. These occur in large

numbers in the outer and middle parts of the ectoderm and are not confined to the deeper layer. Below the nucleated layer of the ectoderm there is a zone of granular material which is closely applied to the mesogloea and which remains colourless in stained preparations. From the layer immediately outside the mesogloea which contains most of the nuclei belonging to the granular zone, conspicuous strands of granular material, with nuclei here and there, proceed inwards through canals in the mesogloea. They can, often, be traced right across the mesogloea into small granular masses lodged in depressions on its endodermal side. Cords of the same material are also seen ascending into the mesogloea of the mesenteries.

The mesogloea is a well-developed homogeneous structure. It is thicker in some places than in others. Its inner border is very irregular, being often raised into broad conical processes projecting into the endoderm between the mesenteries. It does not show any lamination, even where it is thickest. It is pierced by a network of spaces in which the above-mentioned strands of granular material are lodged. Small granular masses are seen in sections of the mesogloea and are no doubt sections of these strands. In some specimens—of presumably a later stage—these cords are hollowed out to form canals. These canals, with bridges of granular substance running across them, are seen in the mesogloea of the mesenteries, both large and small, as well as in the parietal mesogloea opposite the attachments of the mesenteries. In the earlier stages of the larva the mesogloea is extremely thin and does not contain these elements. But their connections in the later stages render it almost certain that they are derived chiefly from the ectoderm. Similar ectodermal strands and canals have also been described in *Zoanthus* by Hertwig.

The longitudinal muscles of the mesenteries are much better developed than in *Zoanthella*. Their arrangement is the same as that already described for that form.

The endoderm is for the most part a vacuolated layer, containing numerous scattered nuclei and yolk masses of different shapes and sizes. Round the stomodaeum it forms a thin layer; its parietal portion is much thicker and increases in thickness aborally. The yolk masses which are abundant in some specimens are comparatively few or absent in others. Curiously enough the youngest larva had hardly any yolk in it. In the peripheral layer of the endoderm there are often seen small or large coarsely granular yellow particles. They are conspicuous in specimens with little yolk and are probably the remains of the broken-down and absorbed yolk spheres. The endoderm is not so highly vacuolated and does not fill up the coelenteron or encroach on it to the same extent as in *Zoanthella*.

The mesenterial filaments are well developed and differ from those of *Zoanthella*. They are much convoluted, so that in transverse sections the ectodermal portion is seen, sometimes in the form of kidney-shaped bands closely pressed against the middle of the free edge of the mesentery, sometimes lying on one side or the other, and often broken up into parts separated by intervening regions of endoderm cells. In young specimens, however, the filaments have the same rounded outline as in *Zoanthella*.

The youngest specimen of *Zoanthina* has only six mesenteries. They are all complete and have mesenterial filaments. The filaments are simpler than in the stage described previously, the mesentery ending in a simple, rounded border. The lateral mesenteries are much larger than the dorsal and ventral pairs, which differ from each other only slightly. The six mesenteric chambers are approximately all equal. Fig. 10 represents a section of this stage and indicates the probable order of the development of the mesenteries. The mesenteries and chambers are quite well-developed round the stomodaeum. The stomodaeum is almost rectangular in section, and is not thrown into folds.

In the next stage there are, as in the earliest, six complete mesenteries with mesenterial filaments. In sections passing through the stomodaeum, as well as for a distance below it, and in sections passing through the aboral region, these six mesenteries and these alone are present. But in sections of the intermediate region four small and incomplete mesenteries are also present. The dorsal and ventral primary chambers have no micromesenteries, but each of the remaining chambers has a micromesentery in it. The dorso-lateral pair of micromesenteries are slightly larger than the ventro-lateral pair and must therefore have preceded them in development. Fig. 11 is a diagrammatic representation of this stage. The parietal endoderm is richer in yolk than in the youngest larva. The mesogloea is thicker, especially in the mesenteries.

In the next stage there are twelve mesenteries, having the arrangement shown in fig. 12. Two new micromesenteries have appeared in the dorsal chamber, thus bringing the number of mesenteries up to twelve. When it is remembered that there is a considerable difference in size between the smallest and the largest specimens of this stage, and almost all the specimens obtained belonged to this stage, there can be no doubt that the twelve-mesentery stage is a prolonged and the most characteristic stage in the life-history of this form. This receives additional interest from the fact that comparative anatomy long ago reduced the mesenterial arrangements of the Zoantheae to two fundamental types, each consisting of twelve mesenteries. The twelve-

mesentery stage thus forms the starting-point of the genera of the Zoantheae.

I have not obtained later stages than this. There can be no doubt, however, that new mesenteries are formed, as stated by McMurrich, in the two mesenteric chambers lying on the two sides of the ventral chamber.

In the development of the Zoantheae, then, we have a sterrogastrula in which a central cavity is developed by the breaking down of the inner portion of a syncytium. Two mesenteries are developed at first, dividing the coelenteron into a dorsal and a ventral chamber, and in each of these two new mesenteries make their appearance. There is thus a stage with six mesenteries and six mesenteric chambers. Thus far the order of development of the mesenteries is the same as what is usually described for other Actiniaria. Whether this stage is only a transient stage, rapidly passing into the next, can be best determined by direct observation. The difference in the size of the macromesenteries suggests that the dorsal and ventral pairs must have been formed at about the same time and not long after the lateral; the micromesenteries too were presumably formed within short intervals, and this seems to be the actual case, as the specimen with ten mesenteries does not differ very much in size from the smallest specimen with twelve mesenteries. On the other hand, there is a considerable difference with regard to size between the specimen with six mesenteries and that with ten. Though these inferences cannot be of much value, based as they are on solitary examples, they render it very probable that there is a considerable interval between the six-mesentery stage and the next. The macromesenteries were most probably formed at a very early stage—this is actually the case with the lateral macromesenteries as seen in *Zoanthella*. The micromesenteries are not formed till the larva has reached a much more advanced stage, and then they are formed pretty much at the same time.

The development of the dorsal pair of micromesenteries would convert the six-mesentery stage into the *Edwardsia* stage. While, according to Van Beneden, they arise after the dorso-lateral pair of micromesenteries, my sections induce me to believe that they are the last of the three pairs to be formed. In any case they are not the first; so that even if an eight-mesentery stage were to turn out to be of some duration, it would not be an *Edwardsia* stage.

On the whole it is reasonable to conclude that the characteristic stage in the development of the Zoantheae is the stage with six macromesenteries, and that this stage is as characteristic of the group as the *Edwardsia* stage is of the Hexactiniae.

With the twelve-mesentery stage we practically reach the adult microtype.

With regard to the relation of the Zoantheae to other groups, Van Beneden, after a most careful study of Semper's first larva, came to the conclusion that the Zoantheae are not related to the Actiniaria through *Edwardsia*, but that they form a group quite distinct from the others. McMurrich, Boveri, and others, on the other hand, consider that the Zoantheae are allied to the other groups and derive them with the others from a common *Edwardsia* stage. Goette in a recent contribution is inclined to unite them with the Cerianthеae and the Antipatharia, and to derive them from a common hexamerous type, while all the other Anthozoa are derived from a common octomerous type.

It seems impossible to maintain that the Zoantheae are derived from an *Edwardsia* type. Whatever the relations of this group to the other groups associated with it by Goette may be there can be no doubt that it is widely separated from the Actiniaria which are derived from the *Edwardsia* type. As the six-mesentery stage is the characteristic stage in their development, the Zoantheae have to be derived from a hexamerous type. In as much as the six mesenteries of this stage are homologous with the first six mesenteries of the *Edwardsia* stage, the two types must be considered to have had a common line of descent. While the Zoantheae branched off from the main stem at an early stage (the six-mesentery stage), the other groups did not branch off till a later stage (*Edwardsia*) when there were eight macromesenteries. In connection with this early isolation of the group it is interesting to note that the larvae of the Zoantheae have retained what must be considered a primitive feature in their longitudinal and transverse ciliated bands. These bands are vestiges of a condition common to the ancestors of the Anthozoa and the Ctenophora, and characterised by the presence of longitudinal bands of long cilia in addition to a uniform covering of small cilia. In the Ctenophora these bands became modified to form the eight rows of swimming-plates. In the larvae of the Zoantheae they were reduced to a single band, or to their oral ends which fused together to form a circular transverse band.

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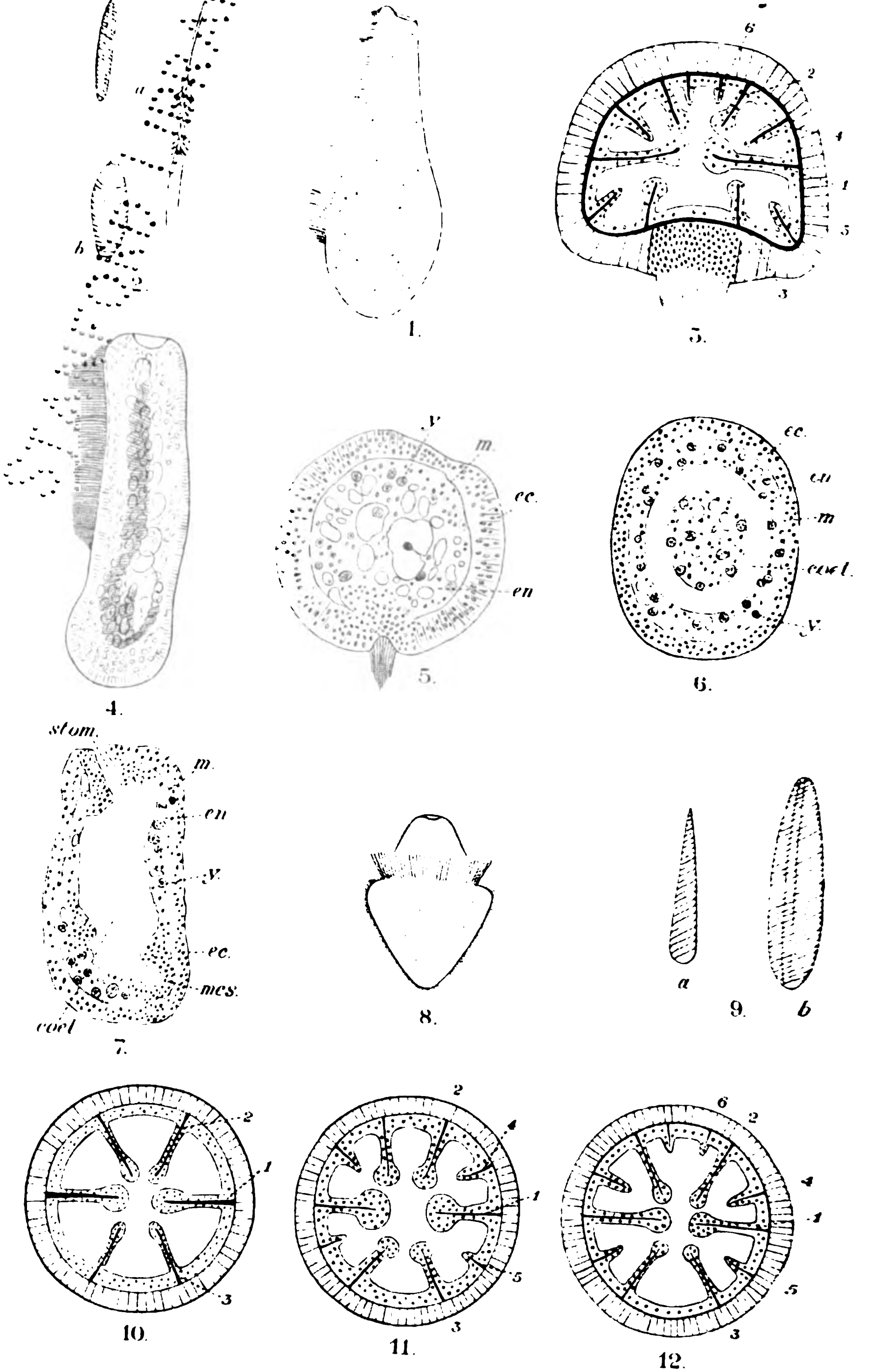

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EXPLANATION OF PLATE IV.

FIG. 1. Semper's first larva. × 5.

FIG. 2. Nematocysts of same. *a.* small × 330; *b.* large × 200.

FIG. 3. Transverse section of same (diagrammatic). 1—6, first six pairs of mesenteries.

FIG. 4. Sterrogastrula of same. × 50.

FIG. 5. Transverse section of 4. *ec.* ectoderm; *m.* mesogloea; *en.* endoderm mass; *vac.* vacuoles; *y.* yolk. The cilia of the ectoderm are not shown in this or in any of the drawings.

FIG. 6. Transverse section of a stage later than 4. Letters as before. *coel.* coelenteron.

FIG. 7. Longitudinal section of a still later stage. Letters as before. *stom.* stomodaeum; *mes.* mesentery.

FIG. 8. Youngest specimen of Semper's second larva. × 30.

FIG. 9. Nematocysts of an older larva. *a.* small × 1000; *b.* large nematocyst × 800.

FIG. 10. Transverse section (diagrammatic) of 8. 1—3, mesenteries (macro.).

FIGS. 11 and 12. Transverse sections of two later stages. 1—6, the six pairs of mesenteries (macro. and micro.).

PROCEEDINGS

OF THE

Cambridge Philosophical Society.

On a Definite Integral. By Prof. T. J. I'A. BROMWICH, M.A., St John's College.

[*Read* 5 May 1902.]

The following integral presented itself to me in the first place as a good illustration of the theory of reducing two quadratic forms to canonical types. But I find that it has been treated by the late Mr Black and that his solution was published from his papers by Prof. Hill in 1897 (*Camb. Phil. Trans.* Vol. XVI. p. 219); according to Prof. Hill the integral in question is of some importance in the theory of statistics. I hope that the alternative investigation given below may not prove uninteresting.

The integral to be evaluated is

$$\int_{(n)} Ve^{-U}\, dx_1 dx_2 \ldots dx_n,$$

the limits of integration being $-\infty$ to $+\infty$ for all the n variables $x_1, x_2, \ldots, x_n$; while U, V are quadratic forms[1] containing these n variables and a constant x_0. In order that the integral may be finite it is necessary and sufficient that U should be positive for all real values of the variables; that is, U is a *positive, definite* form.

Let U_0, V_0 be the parts of U, V respectively which do not contain x_0; then, clearly, U_0 is also definite and positive. Hence we can find *real* linear functions of $x_1, x_2, \ldots, x_n$ (say $y_1, y_2, \ldots, y_n$) such that

$$U_0 = \Sigma y_r^2, \quad V_0 = \Sigma c_r y_r^2, \qquad (r = 1, 2, \ldots, n)$$

[1] So far as the *form* of U, V is concerned, x_0 is on the same footing as the variables $x_1, x_2, \ldots, x_n$; it is only in the *integrations* that x_0 is distinguished from the rest as a constant.

where $c_1, c_2, \ldots, c_n$ are the roots of the determinantal equation in λ, namely,

$$|\lambda U_0 - V_0| = 0.$$

It is to be observed that any number of the quantities $c_1, c_2, \ldots, c_n$ may be equal without modifying the reduced forms of U_0, V_0; for the quadratic U_0 is definite, and so the invariant factors of the determinant are *linear* (Weierstrass, *Berliner Monatsberichte*, 1858, p. 207, or *Werke*, Bd. I. p. 233; particularly § 4).

When the new variables y are substituted in U, it will take the form

$$\Sigma y_r^2 + 2x_0 \Sigma d_r y_r + kx_0^2, \qquad (r = 1, 2, \ldots, n)$$

where the constants d_r, k depend on the original coefficients of U and on the substitution giving $y_1, \ldots, y_n$ in terms of $x_1, \ldots, x_n$; but at present we do not need to determine d_r and k explicitly. Now put

$$z_r = y_r + d_r x_0, \qquad (r = 1, 2, \ldots, n)$$

and then

$$U = \Sigma z_r^2 + lx_0^2,$$

where l is a new constant whose value is required subsequently. To find l, suppose that the original expressions for U, V were

$$U = \Sigma a_{rs} x_r x_s, \quad V = \Sigma b_{rs} x_r x_s, \quad (r, s = 0, 1, 2, \ldots, n)$$

and write for brevity

$$u = |U| = |a_{rs}|, \qquad (r, s = 0, 1, 2, \ldots, n)$$

$$u_0 = |U_0| = |a_{rs}|, \qquad (r, s = 1, 2, \ldots, n)$$

so that u_0 is the minor of a_{00} in the determinant u.

Now we have determined $y_1, y_2, \ldots, y_n$, so that

$$U_0 = y_1^2 + y_2^2 + \ldots + y_n^2,$$

and the determinant of these coefficients is unity. Hence, by a well-known theorem,

$$u_0 = M^2,$$

where

$$M = \frac{\partial(y_1, \ldots, y_n)}{\partial(x_1, \ldots, x_n)}.$$

Similarly, by considering U, we find

$$u = lN^2,$$

where

$$N = \frac{\partial(x_0, z_1, \ldots, z_n)}{\partial(x_0, x_1, \ldots, x_n)} = \frac{\partial(y_1, \ldots, y_n)}{\partial(x_1, \ldots, x_n)} = M.$$

Hence

$$l = u/u_0,$$

which gives l in terms of known quantities.

Introducing $z_1, z_2, \ldots, z_n$ in V in place of $x_1, x_2, \ldots, x_n$, we shall have

$$V = \Sigma c_r z_r^2 + 2x_0 \Sigma f_r z_r + p x_0^2, \qquad (r = 1, 2, \ldots, n)$$

where the quantities f_r, p are certain new constants, of which we only need to find p. Now px_0^2 is the value of V corresponding to the values

$$z_1 = 0, \quad z_2 = 0, \ \ldots, \quad z_n = 0,$$

or to

$$\frac{\partial U}{\partial z_1} = 0, \quad \frac{\partial U}{\partial z_2} = 0, \ \ldots, \ \frac{\partial U}{\partial z_n} = 0.$$

But, since $z_1, z_2, \ldots, z_n$ are connected with $x_1, x_2, \ldots, x_n$ by a linear substitution whose determinant M is *not* zero[1], it follows that the last set of equations are equivalent to

$$\frac{\partial U}{\partial x_1} = 0, \quad \frac{\partial U}{\partial x_2} = 0, \ \ldots, \ \frac{\partial U}{\partial x_n} = 0.$$

If these are solved for $x_1, x_2, \ldots, x_n$ we find

$$u_0 x_r - u_r x_0 = 0, \qquad (r = 1, 2, \ldots, n)$$

where u_r is the minor (with proper sign) of a_{0r} in the determinant u. Substituting these values in V, we have

$$px_0^2 = V(x_0, \ u_1 x_0/u_0, \ u_2 x_0/u_0, \ \ldots, \ u_n x_0/u_0),$$

or

$$pu_0^2 = V(u_0, \ u_1, \ u_2, \ \ldots, \ u_n)$$
$$= \Sigma b_{rs} u_r u_s. \qquad (r, s = 0, 1, 2, \ldots, n)$$

We can now evaluate the integral proposed, for we have, from the definition of M,

$$\int_{(n)} Ve^{-U} dx_1 \ldots dx_n = \int_{(n)} (Ve^{-U}/M)\, dz_1 \ldots dz_n,$$

where the sign of M is supposed to be *positive*; and since, for our present purpose, M is given only as $u_0^{1/2}$, this assumption merely requires that the square-root shall be positive. As already remarked, M cannot vanish and so no difficulty can arise on this account.

The range of the variation of each of the variables z is from $-\infty$ to $+\infty$, and so the original integral becomes

$$u_0^{-\frac{1}{2}} \int_{(n)} (\Sigma c_r z_r^2 + 2x_0 \Sigma f_r z_r + px_0^2) \exp\left[-\Sigma z_r^2 - lx_0^2\right] dz_1 \ldots dz_n.$$

[1] For we have seen that $M^2 = u_0$; and u_0 is the determinant of a *definite* quadratic form, and cannot, therefore, vanish.

When this is expanded as the sum of a number of partial products, each term appears as the product of a number of known integrals, such as

$$\int_{-\infty}^{+\infty} e^{-s^2}ds = \pi^{1/2}, \quad \int_{-\infty}^{+\infty} se^{-s^2}ds = 0, \quad \int_{-\infty}^{+\infty} s^2e^{-s^2}ds = \tfrac{1}{2}\pi^{1/2},$$

and so the value of the whole integral is

$$u_0^{-\frac{1}{2}}\pi^{n/2}e^{-lx_0^2}\left(\tfrac{1}{2}\Sigma c_r + px_0^2\right). \qquad (r = 1, 2, \ldots, n)$$

All the constants in this are known with the exception of Σc_r; now $c_1, c_2, \ldots, c_n$ are the roots of an equation in λ which, when expanded, takes the form

$$u_0\lambda^n - (\Sigma b_{rs}\xi_{rs})\lambda^{n-1} + \ldots = 0, \quad (r, s = 1, 2, \ldots, n)$$

where ξ_{rs} is the minor (with proper sign) of a_{rs} in u_0, so that ξ_{rs} is a second minor of u.

Hence $$u_0\Sigma c_r = \Sigma' b_{rs}\xi_{rs},$$

and substituting we have, finally, the value

$$u_0^{-5/2}\pi^{n/2}e^{-ux_0^2/u_0}\left[\tfrac{1}{2}u_0\Sigma' b_{rs}\xi_{rs} + x_0^2\Sigma b_{rs}u_r u_s\right],$$

where the accented Σ indicates the omission of the zero suffixes.

This result agrees, save as to notation, with Mr Black's.

On the Unit of Classification for Systematic Biology. A Reply to Mr BERNARD. By J. STANLEY GARDINER, M.A., Fellow of Gonville and Caius College.

[*Read* 5 May 1902.]

In a paper on the above subject (*Proc. Camb. Phil. Soc.* Vol. XI. p. 268, 1901) Mr Bernard proposes a method of classification on a novel system by *localities*. The specimens of a collection are to be divided into the *forms* for each locality, and each *form* is to be termed *X. loc.* 1, 2, 3, 4, 5 *etc.* in accordance with the number of *forms* in that locality.

I may say at the outset that Mr Bernard's scheme has many merits, which have been very ably put forward, but I must at the same time remark that it is open to several objections, so serious that, in my view, its ultimate adoption is impossible. It is, however, eminently desirable that the method should be tested on a few genera of corals, and it is hoped that the authorities of the British Museum with their immense collections will publish their next few catalogues of corals in this form. In any case such catalogues, considering variation in a thorough and unique manner, as it is necessary they should, cannot fail to be of the highest scientific interest and importance.

In the first place no attempt has been made by Mr Bernard to define accurately what is meant by the term *form*. Each form is said to be "an aggregate of structural characters regarded in the abstract." Now *form* is taken as the *unit* of classification, and for scientific purposes, as Mr Bernard proposes to use it, must be capable of exact definition. Is it, however, more fitted for accurate determination than *species* or *variety*? So far as I have seen in corals the specimens of any collection have three classes of characters. Supposing that fifteen characters—a purely arbitrary number—may be perceived in the skeleton of any specimen, about five will belong to the genus, including in this class the family and group characters as well, five to the species and five to the specimen. The generic characters can admittedly be

in general perceived when a number of specimens be examined, and need not be dealt with. The other two classes of characters differ in that the one shows discontinuous or specific variability, and the other continuous or normal variability[1]. The former class serves to define the species, while a scientific unit for the second class can only be found in each several specimen. The coralla in their characters of the last class intergrade into one another in every conceivable way. One or more of these characters may become fixed in any set of specimens, and hence of specific importance. In the same way characters, specific in most species, may become continuously variable in others. Generally, however, the interchange between these two groups of characters is extremely rare in corals, and affects only a limited number of the characters.

The term *variety* serves for the different groups in a species, which show in their specific characters discontinuous variation. The separation of these from species depends on the examination of a not inconsiderable number of specimens as well as the insight of the individual worker. Their determination requires the examination of thousands of specimens rather than tens, a task of Herculean proportions in the Madreporaria. If the presence in corals of the three classes of characters mentioned above be generally recognised, the fault of the systematist can only lie in the confusion of species and varieties. Mr Bernard's *form*, scientifically determined, is almost the same—so far as I conceive it from his paper—as a *variety*. If it be the same, it is based on a scientific foundation, but if it is meant to be any grade below this, it is surely devoid of such groundwork. The only real base of a lower order is the sum of the characters of each specimen, the foundation adopted by most systematists. It appears to me that the free recognition of the ascertained, probable and possible facts of variation would do more to clarify systematic biology than the adoption of additional, *purely* artificial standards of other orders.

I may for a moment return to Mr Bernard's proposal in its aspect of practical utility, leaving for the moment completely out of account its scientific bearing. I would ask whether, supposing a large collection of corals be not known to the investigator to come from the same locality, the *forms* would be easier to determine than in a similar-sized collection from mixed localities. It appears to me that the difficulties of ascertaining our *forms*—unless we mean by the term *species*, *varieties* or *specimens*—will be even harder than that of determining *species*, as the latter is

[1] *Vide* "Materials for the Study of Variation" (1894) and "Heredity, Differentiation, and other Conceptions of Biology," *Proc. Roy. Soc.* Vol. LXIX. 1901, and numerous other papers by W. Bateson, F.R.S.

more difficult than marking out *genera*, or *genera* than *families*, etc. The arduousness of our task of investigating the *species* is only thrown back a stage further with increase of intricacy.

The present method does not in truth fit in with all our requirements, but I deny that Mr Bernard is in any way providing us with a better. His method takes account of locality alone. The apparent supposition is that the species in any locality will not be able—on account at least of failure of opportunity—to breed with those of other of Mr Bernard's localities. The anticipation is that the species of diverse localities may by natural selection, *acting on dissimilar variations*, have formed and be forming new species. Thus the method would discover in different areas the lines of the evolution. These views are certainly not supported in the Atlantic and Indo-Pacific Oceans by the distribution of those marine organisms, which possess free-swimming larvae. The instances, too, of other forms (not possessing such larvae), varying in dissimilar characters in different regions, are extremely scarce. Further, so far as we know, the normal variability in any one locality, where the conditions of life are luxuriant and diverse, is as great as over the whole of either of our two oceans. That this is the case in corals, I am convinced by my experience of Madreporaria in the Maldives, where nearly every variety of habitat (in which they can dwell) may be found. I may perhaps point to the indubitable normal variability of *Flabellum rubrum* (= *Flabellum irregulare*, etc.), as well as to the probable specific variation of this same coral in a single channel in the Philippines[1].

"To arrange the organic kingdom in the order of its evolution" and "to arrange each particular form in its order of development above the forms from which it can be derived and below those to which it has itself given rise" are Mr Bernard's ideals of classification. "The facts will themselves reveal the true species in process of time." Once more we seem to be brought back to the mire of a bygone age. We are to deal only with facts, while to a future generation the power of working miracles is to be given. All classification—in its broadest sense—is at present and must, so far as we can foresee, remain theory, but it is to be brought into the realms of creed. From the nature of the case there can be no finality of form to our classification so long as the branches and twigs of our tree of life

[1] *Vide* Semper, *Zeit. f. wiss. Zool.* Bd. xxii. pp. 242 *et seq.* (1872), and papers by the author on the Cape of Good Hope corals in *Marine Investigations in South Africa*, and on variation, protandry and senescence in *Flabellum rubrum* and other corals in *Proc. Camb. Phil. Soc.*, both *in the Press*.

In the above paragraph I am dealing with the facts as at present recognised, but it is necessary to observe that the presence of specific variation in the Madreporaria is not as yet generally known.

continue to grow. Zoology as a science commenced with the doctrine of evolution, a generalisation of facts. Species in the older sense are not, but the meaning of the term is clear to all. It is not unscientific to call each twig of our tree by a separate name. In nature each twig is different from its neighbour, and is many times reduplicated. Each replica varies somewhat, but the main characters of the twig are repeated in all. New twigs may perhaps be formed by the splitting of old twigs, but the more general method—certainly in corals—is by the bursting of buds.

In conclusion I would venture to remark that Mr Bernard's method of classifying *forms* would receive a more scientific foundation, if it were built on a correlation of his *forms* with the physical conditions of their environment, *i.e. on habitat,* as influencing variability rather than on locality. In any case the new unit appears to me to be quite unnecessary—even if scientifically accurate (which, I consider, it is not)—and harmful, in that if adopted, it would tend to obscure facts and the reasonable deductions therefrom. It would prevent for all time the rearing up of our superstructure on a firm foundation, and is calculated to materially hinder enquiry—the offspring of speculation—into the means by which our tree sprung up and grows.

If Mr Bernard cannot satisfactorily classify his forms, I may perhaps venture to point out that he has not yet come to the limits of enquiry into the structure of the Madreporaria. The Porifera were formerly classified practically entirely by the form of their skeleton, and the examination of their soft parts—a far more difficult task—has yielded valuable results of the highest scientific importance. The Hexactiniae, from which the Madreporaria are almost certainly derived, are necessarily classified entirely by means of the polyp-structure. For the Madreporaria the skeleton—a later and an entirely extra-mural structure, as shown by Mr Bourne, whose observations I can fully confirm—is alone examined. The most careful investigation of the corallum is desirable, so as if possible to trace how far and the means by which our living forms may have been evolved from the fossil. Yet by this means we cannot ever hope to get more than a short way back.

Ought not rather every character, especially such as be specifically variable, to be taken into account in systematic biology? Mr Bernard makes no attempt to put this into operation, and yet admits the proposition:—"And here in passing I should like to remark that I am only developing the teaching of my honoured friend and teacher Prof. Ernst Haeckel, who 30 years ago in his *Biologie der Kalk Schwämme* insisted that classification was worthless unless based upon profound morphological study. It is the

neglect of this teaching which has made modern Systematic Zoology what Dr Dohrn calls it, an Augean stable." We know not at present whether the soft parts of corals are even specifically variable or not[1]. Until they be examined, Mr Bernard's system of classification is not proved to be in any way more necessary—let alone desirable or scientifically admissible—for the Madreporaria than for any other division of the animal kingdom.

[*Note.* The question of species and varieties, which has been to some extent discussed in the foregoing article for corals, has been ably considered in Crustacea by Mr L. A. Borradaile in a paper "On Varieties in Marine Crustaceans," *Fauna and Geography of the Maldive and Laccadive Archipelagoes*, Vol. I. Pt. II. pp. 183 *et seq.* (1902).]

[1] In this connection *vide* "On the Species of the Genus *Millepora*," by Sidney J. Hickson, *Proc. Zool. Soc.* pp. 246—257, 1898.

On Radio-active Rain. By C. T. R. WILSON, M.A., F.R.S., Sidney Sussex College.

[*Read* 5 May 1902.]

As the experiments of Elster and Geitel (*Physik. Zeitschr.* II. p. 590 and III. p. 76 and p. 308) and of Rutherford and Allen (*Physik. Zeitschr.* 1st March 1902) have shown, a negatively charged body exposed in the atmosphere becomes radio-active, apparently indicating the presence of some radio-active substance in the atmosphere; it occurred to me to test whether any of this radio-active substance is carried down in rain.

For this purpose I have recently on several occasions boiled down freshly fallen rain to dryness and tested the residue for radio-activity.

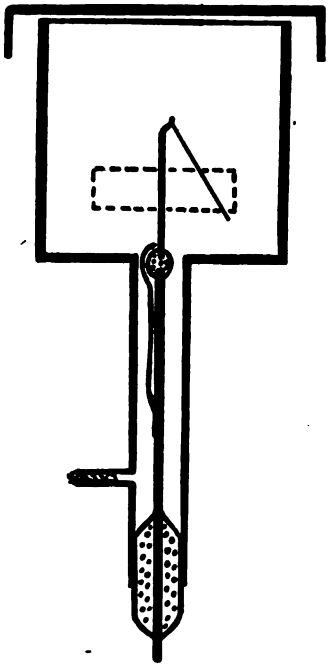
Rain collected both at Peebles and at Cambridge has been found to impart radio-activity to the vessel in which it has been evaporated.

The radio-activity was detected by means of the increase in the ionisation of the air within a small vessel of which the top, or in other experiments the bottom, was of thin aluminium or of gold leaf, the other walls being of brass. The ionisation within this vessel was measured by the same method as was used in experiments on the spontaneous ionisation in air and other gases, and described in the *Roy. Soc. Proc.* Vol. LXVIII. p. 151.

In the Peebles experiments the vessel was cubical, the length of each edge being 5 cms., and the top was of thin aluminium ·00032 cms. in thickness. The apparatus is shown in the figure.

The brass rod passing through the vertical tube was insulated from it by a sulphur plug and was kept at constant potential. Fixed on its upper end by means of a small sulphur bead was a thin brass wire with a narrow clean-cut gold leaf attached. This brass wire and gold leaf formed a leaking system of very small capacity. It could be brought to the same potential as the supporting rod by means of a contact-maker, consisting of a piece of the balance spring of a watch, soldered to the supporting rod and bent at the top so that it might make contact with the brass wire of the leaking system without touching the sulphur. The contact-maker was worked from outside by a magnet.

Glass windows in the front and back of the apparatus enabled the position of the gold leaf to be read by a microscope of which the eye-piece was provided with a micrometer scale. The time



taken by the gold leaf to travel over an exact number of scale divisions was observed, under normal conditions and when the vessel in which the rain had been evaporated was placed inverted over the ionisation apparatus as shown in the figure. With this apparatus the movement of the gold leaf due to the spontaneous ionisation was at the rate of about 11 divisions per hour.

In the experiments made at Cambridge the apparatus used for detecting radio-activity differed from that just described, mainly in being inverted, the bottom being closed by a sheet of gold leaf. The rain was evaporated in a platinum bowl, which was cooled by floating on water and then placed below the ionisation apparatus. The bowl was a shallow one, wider than the ionisation apparatus, so that the surface on which the rain had been dried could be brought close up to the gold leaf bottom of the detecting apparatus.

The general result of these experiments has been the same at both places. In all cases in which a convenient quantity of

rain, snow or hail has fallen in the course of half an hour, and in which this has been immediately evaporated to dryness, quite a marked indication of radio-activity has been shown by the residue. From about 50 c.c. of rain-water a radio-active residue was in all these cases obtained strong enough, after traversing the aluminium or gold leaf, to increase the rate of leak to four or five times its normal value.

In several of these experiments the rain-water was filtered; the radio-activity is not due to the visible solid particles in the rain-water.

The showers on which tests for radio-activity were made fell on the dates given below:—

March 29. (Peebles) about 10 A.M. Fine but copious rain.
April 3. (Peebles) 9.0 to 9.25 A.M. Rain and wet snow.
April 22. (Cambridge) about 12.30 P.M. Rain.
May 2. (Cambridge) 11.50 A.M. to 12.5. Rain.
May 3. (Cambridge) noon. Heavy shower beginning as hail.

In the Peebles observations the rain was collected in a large porcelain vessel (developing tray) placed in an open space in a garden; in the Cambridge experiments, in a large glass funnel placed on the roof of the Cavendish Laboratory.

As already stated, in all these cases considerable radio-activity was detected, and the variations in the nature of the precipitations did not seem to influence much the magnitude of the effect obtained.

The radio-activity obtained by the evaporation of rain disappears in the course of a few hours, falling to less than half its initial value in one hour.

From the evaporation of distilled water or of tap-water or of rain-water which had stood for some hours no radio-activity was obtained. Nor did such water when exposed for some hours in the open air and then boiled down to dryness yield any radio-activity.

On an Attempt to detect the Ionisation of Solutions by the action of Light and Röntgen Rays. By J. A. CUNNINGHAM, St John's College, 1851 Exhibition Research Scholar.

[*Received* 15 May 1902.]

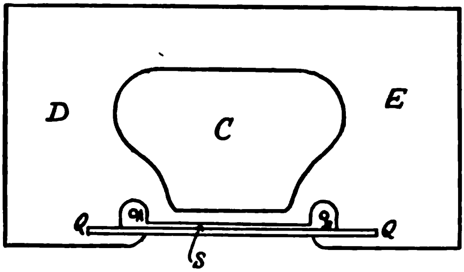
Solutions are in so many respects analogous to gases, that it seemed quite worth while enquiring experimentally whether the analogy would extend to the production of ions by ultra-violet light and Röntgen Rays. There seemed room for hope that some of the energy absorbed by a substance with an absorption spectrum might be traced in this investigation. Some information might also be obtained on the nature of photochemical action.

Arrhenius (*Wien. Ber.* 96 (2), p. 831, 1887) has shown that the conductivity of a film of silver chloride, or bromide, is doubled or trebled by exposure to sunlight. He measured the change of conductivity for illumination with light of different wave-lengths.

It was thought desirable to look for the effect in Carey Lea's solution of colloidal silver. Considerable difficulty was at first experienced in preparing the solution. A strict adherence to the methods described by Carey Lea was not sufficient to obtain a soluble precipitate so long as ordinary chemicals were used. It was not until the very purest chemicals were procured that satisfactory results were obtained. The water used was redistilled and condensed in a block-tin tube. Other solutions at several different dilutions tested for the effect were:—silver nitrate, ammoniacal solutions of silver chloride, uranyl nitrate $\{UO_2(NO_3)_2\}$, and Hoffmann's violet. These were selected as being typical members of several different classes of solutions which might be expected to be affected by radiation.

After many attempts to design a suitable electrolytic cell which would expose a thin layer of the solution to the rays, the one shown in the figure was finally adopted. The body *DE* of the cell was made of paraffin cast under the exhausted receiver of an air-pump in order to expel air bubbles. Two vertical holes *A* and *B* contained the platinised wire electrodes, and a carefully planed recess *S* was cut into the front of the

paraffin until it reached these holes, so that when the quartz plate QQ was fastened over this recess it left a flat chamber with



HORIZONTAL SECTION THROUGH ELECTROLYTIC CELL.

two lateral expansions for the solution under examination. A large cavity C was left in the paraffin to permit artificial cooling by means of a water circulation. For experiments with Röntgen Rays a thin sheet of ebonite was substituted for the quartz plate.

The source of light chiefly employed was an alternating arc between carbon terminals in the cores of which thick iron wires were inserted. By means of two quartz lenses the light from this arc was made to converge upon the film of solution under investigation. In some cases the cell was simply exposed to a bright sky at the Laboratory window. For Röntgen radiation an ordinary X-Ray tube driven by a small induction coil was employed.

The resistance of any given solution was measured by Mr Fitzpatrick's modification of Kohlrausch's method (*Brit. Ass. Report*, 1886, p. 328). The cell was standardised by making a measurement of its resistance when filled with $\frac{N}{50}$ KCl solution.

After some rather rough preliminary work, all the solutions were made up and kept in a dark-room. The cell was also filled there, and placed in a carefully constructed light-tight box, through which projecting wires were permanently sealed. These were readily connected with the electrodes of the cell by means of mercury cups.

Even with this last most improved form of cell and with the assistance of a water circulation maintained only a few millimetres behind the illuminated layer (and the paraffin prepared as above is very transparent to ordinary visible light), it was found impossible to keep the temperature constant before and after turning on the light. The temperature of the solution was measured by the change in resistance of a very thin platinum

wire placed in the thin layer of liquid parallel to and half-way between the electrodes.

As has been said it was hoped that a distinct and readily measurable effect would have been obtained. But after deducting the ordinary temperature effect, it was found that the increase of conductivity of the solutions experimented with, due to illumination of an arc particularly rich in ultra-violet light, never amounted to as much as one per cent. of the conductivity of the given solution prepared and kept in the dark. It was greatest for solutions of ammoniacal silver chloride, uranyl nitrate, and colloidal silver, but the actual figures obtained could not be relied upon.

With Röntgen Rays there was a distinct small effect in the following order:—strong solutions of Hoffmann's violet (3·7 per cent. increased conductivity) down to very dilute solutions (3·44 per cent.); dilute solution of colloidal silver (0·8 per cent.); ammoniacal solution of silver chloride (0·25 per cent.); silver nitrate, and uranyl nitrate (no measurable change).

These experiments were carried out in the Cavendish Laboratory at the beginning of the year 1900—1901, and I am indebted to Professor Thomson and to Mr Whetham for many valuable suggestions.

Note on the Dispersive Power of Running Water on Skeletons: with particular reference to the Skeletal Remains of Pithecanthropus erectus. By W. L. H. DUCKWORTH, M.A., Jesus College.

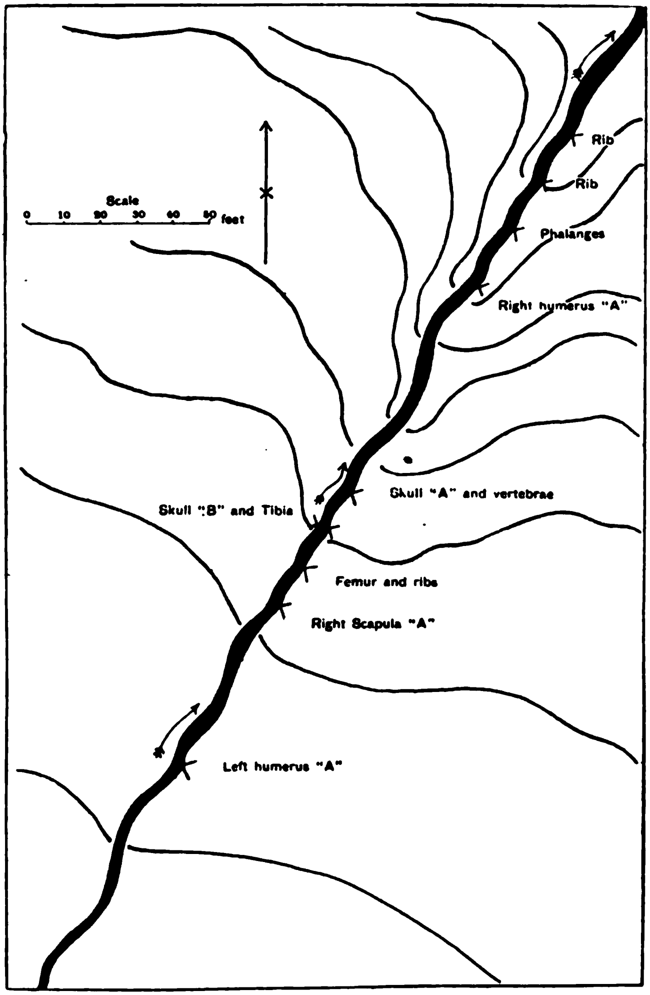
[*Read* 19 May 1902.]

Among the objections raised against the acceptance of Dr Dubois' view as to the nature of the fossil bones found by him in Java and ascribed to an animal form intermediate between the apes and Man (*Pithecanthropus erectus*), there was one which disputed the community of origin of the several remains: it was urged in fact that since the distance separating the calvaria (skull-cap) and the femur was 48 ft. 9 in. (15 metres), the two bones could not have belonged to the same individual. Now it is very important, if not essential, for Dr Dubois' theory that the two bones should be regarded as having formed part of the same skeleton, and the objection was met by the response that experience would shew that the distance by which they were separated is not too great to preclude the possibility of their possessing a common origin in a single skeleton. It must be further explained that the remains were discovered in the bank of a river even now of considerable size, and that Dr Dubois suggests that crocodiles probably played a part in securing the dispersion of the bones of many of the animals which perished in the much larger Pliocene representative of the modern Solo river.

The object in view in the present account is to suggest that a stream of much smaller volume than the Solo river is capable of dispersing remains of skeletons over a distance considerably greater than the fifty feet or so required by Dr Dubois' theory. Incidentally two other points are illustrated by the specimens used in demonstration of this proposition.

In the northern part of Carnarvonshire there is a large marshy tract of upland some hundreds of acres in extent, situated immediately to the south-east of Penmaenmawr. This marshy plateau is drained by several mountain streams, the general direction of which is roughly east by north. While walking over this eastern versant in the spring of 1901, I noticed a number of bones of animals dispersed along the line, and in the bed, of

one of these streams, at that time of very small dimensions, though probably melting snow in winter, or prolonged rains,



In the map the stream is shewn flowing upwards from left to right, and the thin lines are contours indicating the slope.

would have increased it temporarily. At the time in question, the width of the stream was about four feet.

The first bone I picked up was the right humerus of a horse,

and I at once noticed in it indubitable signs of the disease known in human pathology as Osteo-arthritis and vulgarly as "Rheumatics." Struck by the reflection that one of the bones found by Dubois was also a pathological specimen, and that Virchow had discussed the possibility of osteo-arthritis being the cause of the disease in that case (cf. *Zeitschrift für Ethnologie*, 1895), I determined to investigate the remainder of the bones, and in particular, to endeavour to determine how far such fragments of the skeleton could be dispersed by such a comparatively small stream as I had before me.

Examination of the bones indicated that two, and only two animals, shewn by the characters of the bones to be small horses, had perished here. There seems no doubt that the animals had been mired in the marsh during the preceding winter, the state of the bones suggesting this limit of time. Furthermore, as numerous ponies roam in a semi-wild state over the neighbouring hills and marshy tracts, there is no reason to suppose that the animals had been specially brought to this spot to be destroyed.

The investigation was further simplified by the discovery that of the two animals, one was a young individual (between three and four years) and the other (the rheumatic one) aged. Thus the identification of the several remains was rendered much more easy than would otherwise have been the case.

The remainder of my examination resolved itself into pacing the distances between the various bones, and I contented myself with the observations embodied in the accompanying map; this shows the position of the more important parts of the skeleton of the older animal, which I have denoted by the letter "A"; the positions of one or two bones of the other pony "B" are also indicated, as well as some ribs and digital bones (phalanges) whose ownership was not determined with certainty.

I. The important point brought out by the observations is that the two humeri of the animal "A" were separated by a distance of 153 feet along the bed of this small stream. In comparison with this, the distance of 15 metres (48 feet 9 in.) demanded by Dr Dubois for the Javan bones can be granted without difficulty.

II. A few other points seem worthy of notice in this place. Firstly, the possibility of dispersion by wild animals was here excluded: the bones exhibit no signs whatever of having been gnawed by dogs, which are almost the only animals that could be suggested as responsible. Foxes are excluded for the same reason. Secondly, it will be noticed that the ribs have been carried furthest down-stream, no doubt owing to their lightness.

Again, if the distance of 153 feet from humerus to humerus should appear to prove too much, it must be mentioned that the course of the stream runs down at an angle of about 8° on the

average; the small volume of water at work would thus be to some extent compensated.

III. The foregoing notes present the chief points of interest in connection with the subject in question. Two further remarks appear appropriate with regard to the bones themselves.

In the first place, the bones shew that the shoulder-joint of the pony "A" was in an advanced stage of the disease called Osteo-arthritis. Such an advanced stage of this condition is rare, though not unknown, in the horse. The subject derives a special interest from the point of view of the relation of disease to diet, whether in Man or the horse. A purely vegetable diet would not seem to be an efficient safeguard against the onset of Osteo-arthritis. This subject has been ably discussed by Dr Balfour in the *Edinburgh Medical Journal* (Feb. 1870, p. 713), where special reference to Osteo-arthritis in the horse is made. Again, Virchow (*loc. cit. Z. für Ethnologie*) describes a not dissimilar condition in bones of the extinct cave-bear (*Ursus spelaeus*) under the name of Hohlen-gicht. Evidence of similar conditions in early Tertiary ungulates was given by Professor Marsh at the International Congress of Zoologists at Leyden in 1895. Secondly and lastly, the water of this particular stream and marsh seems to have contained sufficient acid (whether vegetable or other) to lead to disintegration of the bones in certain places. This effect must be very carefully distinguished from those of Osteo-arthritis. In the latter, the joint-surfaces and the neighbouring parts of the bones will be affected, and in such a way that erosion and eburnation (polishing) of the joint-surface is found accompanied by bony outgrowths around the margins of the latter. Erosion due to acidity of the water first affects the very thinnest parts of the bones and is accompanied of course by nothing in the way of exostosis. The scapula of "A" shews both conditions very excellently, the joint region affording evidence of Osteo-arthritis as already said, and the blade of the scapula being perforated in its thinnest part by the solvent action of the water.

Reflexion and transmission of Light by a Charged Metal Surface. By P. V. BEVAN, B.A., Trinity College.

[*Read* 5 May 1902.]

As there appears to be an essential difference between positive and negative charges of electricity with regard to the matter associated with the charges, it seems probable that some difference should be observable in the effect of charging a metal mirror positively and negatively on light reflected from the mirror.

We can suppose that a charged metal consists of the metal itself in the ordinary condition with a layer on its surface of corpuscles which are associated with the electrical charge. In the case of negatively charged metals these corpuscles would be of the nature of cathode particles, while with positively charged surfaces the corpuscles would be molecules from which a negative particle had been abstracted. The equations for the metallic medium will now be altered as this layer will consist of these corpuscles in greater abundance than throughout the substance of the metal, so that for the case of reflected light we have to consider three media, the air, the layer containing the charge and the body of the metal itself.

We can consider the layer with the charge to be very thin indeed, as compared with the wave-length of light, so that in our equations we can finally put the thickness of this layer $= 0$, keeping the surface density finite.

Suppose in the charge layer we have n' charges per unit volume, which we can suppose freely moveable in the direction parallel to the surface, but not in the direction normal to the surface. Let the axis of Z be normal to the metal, Ox and Oy in the surface.

Then the equations we may take for a corpuscle are, if ξ, η, ζ represent its displacement,

$$m\ddot{\xi} = eX,$$

$$m\ddot{\eta} = eY,$$

$$m\ddot{\zeta} = eZ - a^2\zeta,$$

supposing a harmonic force acting in the normal direction resisting any motion of the corpuscles out of the charge layer.

If we suppose the forces are periodic and vary as $e^{\iota pt}$, we have

$$\xi = -\frac{eX}{mp^2},$$

$$\eta = -\frac{eY}{mp^2},$$

and
$$\zeta = -\frac{eZ}{mp^2 - a^2},$$

as far as the external forces are concerned, we shall therefore have a flux of ions

$$e\frac{d\xi}{dt}, \text{ etc.}$$

And the equations for the charge layer are

$$\left.\begin{aligned} \frac{dX}{dt} - \frac{n'\iota e^2}{mp} X &= V\left(\frac{\partial N}{\partial y} - \frac{\partial M}{\partial z}\right) \\ \frac{dY}{dt} - \frac{n'\iota e^2}{mp} Y &= V\left(\frac{\partial L}{\partial z} - \frac{\partial N}{\partial x}\right) \\ \frac{dZ}{dt} - \frac{n'\iota e^2 p}{mp^2 - a^2} Z &= V\left(\frac{\partial M}{\partial x} - \frac{\partial L}{\partial y}\right) \end{aligned}\right\}.$$

We have also the magnetic equation

$$\frac{d\mathbf{H}}{dt} = -V \operatorname{curl} \mathbf{E}.$$

If we suppose the forces vary as

$$e^{\iota(lx+my+nz+pt)},$$

we have
$$\left.\begin{aligned} X\left(p - \frac{n'e^2}{mp}\right) &= V(mN - nM) \\ Y\left(p - \frac{n'e^2}{mp}\right) &= V(nL - lN) \\ Z\left(p - \frac{n'e^2}{mp^2 - a^2}\right) &= V(lM - mL) \end{aligned}\right\} \quad\text{.............(1)},$$

and
$$\frac{p}{V}(L, M, N) = -(mZ - nY,\ nX - lZ,\ lY - mX).$$

We have therefore
$$\left.\begin{aligned} \Sigma Ll &= 0 \\ \Sigma LX &= 0 \end{aligned}\right\} \quad\text{..........................(2).}$$

We get

$$X\left(p-\frac{n'e^2}{mp}\right)=\frac{V^2}{p}\{-m(lY-mX)+n(nX-lZ)\}$$

$$=\frac{V^2}{p}\{\Sigma l^2 . X-l\Sigma lX\},$$

$$Y\left(p-\frac{n'e^2}{mp}\right)=\frac{V^2}{p}\{\Sigma l^2 . Y-m\Sigma lX\},$$

$$Z\left(p-\frac{n'e^2}{mp}\right)+Z\left(\frac{n'e^2}{mp^2}-\frac{n'e^2}{mp^2-a^2}\right)=\frac{V^2}{p}\{\Sigma l^2 . Z-n\Sigma lX\}.$$

So that, multiplying by L, M, N and adding, we obtain

$$NZ\left(\frac{n'e^2}{mp^2}-\frac{n'e^2}{mp^2-a^2}\right)=0.$$

As we suppose a^2 is not $=0$ we must have, for the propagation of waves without change of type, $N=0$ or $Z=0$.

If $Z=0$ we have

$$lX+mY=0, \text{ from (1)},$$

so that the electric force is perpendicular to the direction of propagation.

Consider light incident normally on the metal. l and $m=0$.

Writing $$p-\frac{n'e^2}{mp}=A,$$

we have $$AX=-VnM,$$

$$AY=\quad VnL,$$

and therefore $$A-\frac{V^2}{p}n^2=0,$$

or $$n^2=\frac{Ap}{V^2}$$

$$=\frac{1}{V^2}\left(p^2-\frac{n'e^2}{m}\right).$$

The velocity of propagation is

$$\frac{p}{n}=\frac{p}{\left(p^2-\frac{n'e^2}{m}\right)^{\frac{1}{2}}}V \text{......................} (3).$$

For normal incidence we get the same expression for the velocity if we take $N=0$.

We have then in the metallic charged layer a wave transmitted with the velocity given by (3).

Let us consider then the case of reflection at this layer, the layer being on the surface of the ordinary metal to which the equations

$$(a + \iota b)\frac{d\mathbf{E}}{dt} = V \operatorname{curl} \mathbf{H},$$

$$\frac{d\mathbf{H}}{dt} = -V \operatorname{curl} \mathbf{E}$$

apply.

Suppose the charge layer is in thickness h, which we shall finally make extremely small.

Consider incident light polarized in the plane xz, the angle of incidence being 0.

In the air we have

$$Xe^{\iota(nz+pt)} + X'e^{\iota(-nz+pt)},$$

where X' is complex and represents the amplitude of the reflected light.

In the charge layer we have

$$X_1 e^{\iota(n_1 z+pt)} + X_1' e^{\iota(-n_1 z+pt)}.$$

And in the metal, which we suppose thick enough for the first surface only to be effective in affecting the light,

$$X_2 e^{\iota(n_2 z+pt)}.$$

The conditions to be satisfied are the continuity of tangential electric and magnetic force at the two surfaces $Z = h$ and $Z = 0$.

The electric conditions give

$$\begin{aligned} Xe^{\iota nh} + X'e^{-\iota nh} &= X_1 e^{\iota n_1 h} + X_1' e^{-\iota n_1 h}, \\ X_1 + X_1' &= X_2. \end{aligned}$$

The magnetic equations give

$$\begin{aligned} nXe^{\iota nh} - nX'e^{-\iota nh} &= n_1 (X_1 e^{\iota n_1 h} - X_1' e^{-\iota n_1 h}), \\ n_1 (X_1 - X_1') &= n_2 X_2, \end{aligned}$$

whence

$$\frac{X'}{X} = e^{2\iota nh}\frac{(n - n_1)(n_1 + n_2)e^{\iota n_1 h} + (n + n_1)(n_1 - n_2)e^{-\iota n_1 h}}{(n + n_1)(n_1 + n_2)e^{\iota n_1 h} + (n - n_1)(n_1 - n_2)e^{-\iota n_1 h}}.$$

If in this expression we put $n_1 = n$, we get the case for reflection at an ordinary metal surface giving

$$\frac{X_0'}{X} = \frac{n - n_2}{n + n_2}.$$

If we put $h = 0$, we also obtain the same ratio.

Now $$n_1^2 = \frac{1}{V^2}\left(p^2 - \frac{\sigma}{h}\frac{e}{m}\right).$$

Since, if σ is the surface density of the charge layer, $\sigma = n'he$, and n_1 is given in (3).

$n_1^2h^2$ is equal to h multiplied by a factor which is finite when $h=0$, so that expanding the exponentials in $\frac{X'}{X}$, and retaining only first powers of n_1h, we obtain

$$\frac{X'}{X} = \frac{(n-n_2) + 2\iota h\,(nn_2 - n_1^2)}{(n+n_2) + 2\iota h\,(nn_2 + n_1^2)}.$$

Terms of the order n_1^2h are all that we need retain, so that we have, making $h=0$,

$$\frac{X'}{X} = \frac{n - n_2 + 2\iota \frac{\sigma}{V^2}\frac{e}{m}}{n + n_2 - 2\iota \frac{\sigma}{V^2}\frac{e}{m}}.$$

Now in the metal with which n_2 is associated, we have the equations

$$(a+\iota b)\frac{d\mathbf{E}}{dt} = V \operatorname{curl} \mathbf{H},$$

$$\frac{d\mathbf{H}}{dt} = -V \operatorname{curl} \mathbf{E},$$

where $$a + \iota b = \nu^2 (1 - \iota k)^2,$$

ν and k being Drude's coefficients.

We have then, as the disturbance in the metal varies as $e^{\iota(n_2 z + pt)}$,

$$n_2^2 = \frac{p^2}{V^2}(a+\iota b) = \frac{p^2}{V^2}\nu^2(1-\iota k)^2,$$

and $$n \text{ in the air } = \frac{p}{V}.$$

So that we have $$\frac{X'}{X} = \frac{1 - \nu(1-\iota k) + \iota\phi}{1 + \nu(1-\iota k) - \iota\phi},$$

where $$\phi = \frac{2\sigma}{pV}\frac{e}{m},$$

$$\frac{X'}{X} = \frac{1 - \nu + \iota(\phi + \nu k)}{1 + \nu - \iota(\phi + \nu k)}.$$

We observe therefore that there will be a change of phase in the reflected light, and the terms indicating the part of this change due to the charge, that is, the terms dependent on ϕ, involve σ the surface density and the ratio $\frac{e}{m}$. Should the effect

of the charge be measurable, we have here a method of measuring the ratio $\frac{e}{m}$ for the charged particles which are associated with the charge on a conductor.

If we put the ratio of the amplitudes in the form $Re^{\iota\lambda}$, then the change of phase as a time is

$$\frac{1}{p}\tan^{-1}\frac{2\nu(\phi+\nu k)}{1-\nu^2+(\phi+\nu k)^2}.$$

The change of phase when the metal is uncharged is

$$\frac{1}{p}\tan^{-1}\frac{2\nu^2 k}{1-\nu^2(1-k^2)}.$$

Now $$\phi=\frac{2\sigma}{pV}\frac{e}{m},$$

V is 3.10^{10}, p we may take as 3.10^{15}, so that ϕ is of the order

$$2\sigma\frac{e}{m}.10^{-26},$$

where e and σ are in electrostatic units. $\frac{e}{m}$ is of the order 3.10^{17} in electrostatic units, and ϕ is therefore of the order $\sigma.10^{-8}$. ϕ may therefore be considered small compared with νk.

The difference between the change of phase due to the charged metal and that due to the uncharged metal is to the first order in ϕ,

$$\frac{1}{p}\cdot\frac{2\nu\{1-\nu^2(1+k^2)\}}{1+2\nu^2(k^2-1)+\nu^4(k^2+1)^2}\phi.$$

And so the fraction of the wave-length that the phase is changed due to the charge is

$$\frac{4\pi\nu\{1-\nu^2(1+k^2)\}}{1+2\nu^2(k^2-1)+\nu^4(k^2+1)^2}\phi,$$
$$=A\phi.$$

$$\left.\begin{array}{ll}\text{For Silver} \ldots\ldots\ldots & A=-\ \cdot127,\\ \quad\text{Copper}\ldots\ldots\ldots & A=-\ \cdot65,\\ \quad\text{Bismuth} \ldots\ldots & A=-1\cdot26,\\ \quad\text{Platinum}\ldots\ldots & A=-1\cdot0,\\ \quad\text{Potassium} \ldots & A=-\ \cdot175,\\ \quad\text{Mercury} \ldots\ldots & A=-\ \cdot705.\end{array}\right\}$$

Now $$\phi=\frac{2\sigma}{pV}\frac{e}{m}.$$

For sodium light

$$p = 3 \,.\, 10^{15},$$

$$V = 3 \,.\, 10^{10},$$

and $$\frac{e}{m} = 3 \,.\, 10^{17}, \text{ for negative particles,}$$

so that $$\phi = \tfrac{4}{3}\, 10^{-8} \sigma \text{ approximately.}$$

To obtain therefore a change of phase of $\frac{1}{100}$ of a wave-length σ must be $\frac{3}{4}\, 10^{6}$ in electrostatic units.

If a condenser consisting of two metallic films deposited on the sides of a thin plate of glass or mica be used, and light be transmitted through one of them and reflected at the second, the conditions for a large surface density could be obtained—if the reflecting surface be charged negatively, the effect of the positive charge on the first film could be neglected if $\frac{e}{m}$ for the positive metal be small compared with the same quantity for the negative charge. Suppose the glass between the films 10^{-4} cms. in thickness, P the difference of potential of the films. Then

$$\sigma = \frac{10^4 \,.\, PK}{4\pi}, \text{ say } 5 \,.\, 10^3 P,$$

so that for the change of phase of $\frac{1}{100}$ of a wave-length P must $= 300$ about, in electrostatic units. Or in volts $P = 90{,}000$. It seems probable that the condenser would break down under this difference of potential.

The effect therefore on the light is of such a small order that with the present methods of detecting difference of phase it would probably escape detection. The above theory makes no pretence at representing accurately the state of things actually existing, but it may be regarded as sufficiently near the truth to deduce the order, at least, of the effect considered.

The Histology of the Endosperm during Germination in Tamus communis *and* Galium Tricorne. By WALTER GARDINER, M.A., F.R.S., *Fellow and Bursar of Clare College*, and ARTHUR W. HILL, M.A., *Fellow of King's College.*

[*Read* 17 February, *Received* 12 June 1902.]

In a communication laid before the Royal Society in 1897[1], a description was given of the phenomena which accompany germination in the Endosperm of *Tamus communis*. We have recently re-examined the endosperm of *Tamus* and also investigated the histology of the endosperm of *Galium Tricorne* by way of comparison, and we hope to study the germination of other thick-walled seeds in the same way as opportunity occurs.

The macroscopic features of the germination of *Tamus* prove to be of some interest, and as only a few figures of the stages of germination have been published before by Bucherer and others[2], it seems well to describe shortly the external features of germination before proceeding to consider the changes which take place in the endosperm. If one of the small, round, seeds be examined the micropyle can be distinguished as a tiny brown point. Beneath it is situated the somewhat ovoid embryo, which is found to be lying in a radial position, with its more pointed or cotyledonary end directed towards the centre of the seed, and its blunt end, which is occupied by the radicle, lying just internal to the micropyle[3] (Plate V, Fig. 3). In a thin section through the micropyle the actual pore is visible, surrounded by the strongly-thickened and suberized walls of the integuments.

On the commencement of germination, the radicle begins to grow outwards and, owing to pressure from within, a little semi-circular area of the testa around the micropyle breaks away from the rest of the seed coat and gets pushed upwards by the emerging radicle; and since at one point it remains attached to the seed coat, it has the appearance of a little lid having the peg-like micropyle in the centre (Fig. 1). Through the opening so formed the radicle grows out (Fig. 2), followed soon afterwards by the

[1] Gardiner, W., *Proc. Roy. Soc.*, 1897, p. 105, figs. 1 and 3.
[2] Bucherer, *Bibliotheca Botanica*, Heft 16, Le Maout and Decaisne (Eng. ed.), pp. 794, 795.
[3] Cf. Solms Laubach, *Bot. Zeit.*, 1878, pp. 65—82.

lateral plumule, which is covered and protected by the cotyledonary sheath. After the primary root has attained some length the first leaf of the plumule commences to develope, and, pushing aside the lobes of the cotyledonary sheath, grows upwards with its tip sharply bent over (Fig. 4). The tips of the young leaves and the apex of the plumule are well provided with multicellular hairs, whose cells apparently contain oil.

At about the same time the tissue just at the junction of the first leaf and the cotyledon divides and forms a small bulb or tuber, which is epicotyledonary in origin[1] (Fig. 5). As germination proceeds the reserve materials of the seed are gradually transported and re-stored as starch in the thin walled cells of the rapidly-enlarging tuber (Fig. 6). The primary root is replaced by adventitious roots springing from the tuber, which by their shortening is pulled down deeper into the ground; and finally the first leaf grows out into the air and develops a lamina not unlike that of the mature leaf of this plant (Fig. 7).

A microscopical examination of sections of the ripe seed of *Tamus* shews that the hard brown testa consists of an outer layer of flattened cells, within which comes a second layer of deeply-pitted thick-walled cells, elongated in the tangential direction; but in the vicinity of the micropyle this layer is more prominent, and the cells, which here are elongated in the radial direction, shew a larger number of pits than the corresponding cells in other parts of this layer. The little beak-like persistent micropyle is formed by cells of this character. Within this layer of thick-walled cells comes another single layer, which is succeeded by the thick-walled endosperm tissue (Fig. 3).

The cells of the endosperm have thick and unpitted walls, and are arranged more or less radially with reference to the centre of the seed. They are filled with aleurone grains, oil, and some small crystals, which, together with the cellulose of the walls, form the chief reserve materials of the seed. No starch occurs in the ripe seed, but soon after germination has commenced a little may be seen in the cells around the embryo, just under the testa. If a section of the young germinating embryo is examined a large number of raphide-containing cells will be seen, which were not visible in the resting condition. These raphides, which apparently consist of Calcium oxalate, occur in the cortical tissue in special cells, which speedily become larger than those around them. The chief seat of deposition of these crystals is the upper portion of the cotyledon and the cotyledonary sheath. In older seedlings they are found in some abundance in the cortical tissues of the roots and leaf. As the depletion of the endosperm proceeds and the seedling increases in size the cotyledon enlarges considerably

[1] Cf. Bucherer, *loc. cit.*, Le Maout and Decaisne, *loc. cit.*

and encroaches upon and displaces the tissue of the endosperm. As we have seen the seedling absorbs the reserve materials contained in the seed, but, instead of using them entirely for purposes of growth, re-stores them again in part as starch in the thin-walled epicotyledonary tuber, which very early makes its appearance at the base of the developing cotyledon (Fig. 6).

In the short sketch of the germination of the seeds of *Tamus* already given, the relations of the embryo to the endosperm have been briefly described, and we may now proceed to deal with the histology of the endosperm during germination.

The cells of the endosperm vary considerably in size. Those immediately beneath the testa are large and elongated in the radial direction, but in the centre of the seed they are much smaller and arranged in an irregular manner. In all cases the walls are unusually thick, and are composed of a hard and horny variety of cellulose, which gives very little reaction with the dyes usually employed for a microchemical examination of the cell wall.

With watery solutions of both methylene blue and Congo red, only a very slight staining of the walls takes place, and with the former reagent the staining is practically confined to the middle lamellae of the walls of the more peripheral cells of the endosperm. In most of the walls a middle lamella is not evident, but it can be distinguished as a fine line in the walls of the loosely-packed and rounded cells in the centre of the seed, and also in the walls of the cells around the embryonic cavity. The cell walls remain practically unstained with safranin, but with a solution of iodine in potassium iodide they are stained yellow, and after the action of iodine solution and sulphuric acid take on a brownish coloration. Sections of the endosperm stain crimson violet with alkali-alizarin, but after being heated with 2 °/₀ sulphuric acid for two hours they are not stained at all with this reagent[1].

From these reactions it seems probable that the walls are composed of a reserve cellulose like that described by Gruss from the seeds of *Phoenix* and other plants, and not of a cellulose compound similar to that of the ordinary cell walls of the plant.

The connecting threads, which have been briefly described in the communication previously mentioned[2], occur in small or large groups throughout the walls and shew a well-marked median node at the middle lamella. Towards the periphery of the seed, where the cells are larger than those in the centre and shew distinct end and side walls, the connecting threads are seen to be distributed somewhat differently, for in the longer side walls numerous groups of threads occur scattered at intervals, whilst in

[1] Gruss, "Studien über Reserve-cellulose," *Bot. Cent.*, 70, 1897, p. 242. The cell walls of the endosperm of *Galium* and *Phoenix* gave a similar reaction.

[2] Gardiner, *Proc. Roy. Soc.*, 1897.

the shorter end walls one large group of threads occurs, which occupies the whole of the wall[1] (Fig. 3). Thus, though the walls are unpitted, the connecting threads are arranged in groups. If sections of a young germinating seed are examined the cotyledon is seen to have increased in size, and abundant evidence of enzyme action is afforded by the partially broken-down walls of the surrounding endosperm cells.

A careful examination of the walls of the cells in the immediate vicinity of the cotyledon, which walls moreover are crowded with threads, shews that the ferment effects an entrance by only a few of the threads, and, proceeding from the interior of the cell outwards, works along them towards the middle lamella, where it quickly becomes more active and rapidly dissolves the reserve cellulose in the neighbourhood of the lamella, thereby forming large cavities broadly fusiform in shape, which shew their broadest diameter at the lamella, and taper away to the edge of the wall on either side[2] (Fig. 8 (*y*)).

In this region, owing no doubt to the mucilaginous character of the lamella and adjoining layers of the wall, the enzyme is more vigorous and effective than on the younger and more horny layers of the cell membrane. As the ferment action proceeds these cavities break into one another, and so the whole wall becomes disorganized and in its altered and mucilaginous condition now stains with the dyes employed for demonstrating the 'connecting threads.' The tracks of the threads can, however, still be seen after the cell walls have become disorganised (Fig. 8).

If sections of a seed are examined, which has been allowed to germinate for a longer period and in which the cotyledon has enlarged so considerably inside the seed as to have displaced the greater part of the endosperm, it is found that the ferment attacks the cell walls of the more peripheral portions of the endosperm in a somewhat different manner to that just described, for whereas we have shewn that the ferment dissolves the walls of the rounded cells at the centre of the seed, more particularly in the *region of the lamella,* in the older walls it commences its attack at the inner or free edge of the wall, and proceeds outwards, in a V-shaped manner towards the middle lamella (Fig. 9). These differences in the method of the ferment action on the cell walls appear to be due to some differences in the composition of the 'reserve-cellulose' in different parts of the endosperm.

The amount of endosperm tissue undergoing dissolution at any given time is comparatively small, since the action of the ferment is localized to the layers of cells immediately surrounding the

[1] Cf. Gardiner, *Proc. Roy. Soc.*, 1897, Fig. 1.

[2] Cf. Hill, *Ann. Bot.*, vol. xv. 1901, fig. 13, Pl. XXXII. p. 596. Also Gardiner, *loc. cit.* fig. 3.

cotyledon; but as the cotyledon increases in size, the sphere of action of the ferment gradually extends towards the periphery of the seed until all the tissue has been attacked and the cotyledon completely fills the whole of the space previously occupied by the endosperm.

The progress of the ferment action in the case of the germination of the seed of *Tamus* may be described as *centrifugal* with reference to the embryo, since the ferment, which apparently proceeds from the cotyledon, affects first those walls of the endosperm cells which are in immediate contact with it, and, as the cotyledon enlarges *pari passu* with the disorganization and absorption of the endosperm, progressively extends its sphere of action into deeper layers.

At any given stage of germination the ferment action is practically confined to a narrow zone of endosperm tissue, some three or four cells deep, immediately outside the cotyledon.

The seed of *Galium Tricorne*, which was chosen as another example of a thick-walled seed for comparison with *Tamus*, offers quite a different type of ferment action during germination.

Certain points regarding the general anatomy of the seed may first be described. The seeds of *Galium Tricorne*, which are closely invested by the dry pericarp, have, as is well known, the form of hollow spheres, slightly flattened on the ventral side, and on this side also, a pore is left which opens into the internal cavity. The curved embryo lies buried in the endosperm, with its radicle directed towards the basal portion of the seed[1] (Fig. 11).

The endosperm is composed of thick-walled cells whose contents stain a deep brown colour with a solution of iodine in potassium iodide. Besides protoplasm and nuclei the cells contain aleurone grains, which, together with the cell walls, constitute the principal reserve materials. The cells do not shew any very definite shape or arrangement, though they tend to radiate outwards from the embryo. The walls are of varying thicknesses and are irregular in outline, and thus shallow pits occasionally occur, more particularly in those parts of the endosperm remote from the embryo.

The thick and irregularly-pitted walls of the endosperm are richly provided with connecting threads, which are usually arranged in barrel-shaped groups, though scattered threads also occur. The threads, which are often beautifully curved, shew a well-marked median node. No distinction can be drawn between the groups of threads which occupy the pit-closing membranes and those which are found in the thick parts of the walls, for often the pits are only on one side of the cell wall (Fig. 16 (y)). In such cases

[1] Lubbock, "On Seedlings," vol. II., figs. 439—441. Le Maout and Decaisne, p. 483. Engler and Prantl, *Pflanzen-familien*, IV. 4, p. 150.

the accompanying group of threads presents the appearance of the so-called "pit-threads[1]"—"aggregirte Plasmaverbindungen"—on the one side, and of the "wall threads"—"solitäre Plasmaverbindungen" of Kohl[1]—on the other side of the middle lamella, and it is therefore clear (and this is also true for the endosperm of *Phoenix*) that the distinction which has been made between these two forms of threads in thick-walled endosperms is not a valid one, since all gradations from the "pit" type to the "wall" type may occur in the endosperm tissue of one and the same seed.

The composition of the reserve cellulose in *Galium* as in *Tamus* is rather a matter of conjecture. Yet sections of the fresh endosperm give fairly definite results, which tend to prove that the walls are composed of a substance of the nature of pecto-cellulose. With weak watery methylene blue, all the walls take up the stain, but those just under the testa stain more deeply. With Congo red, on the other hand, all the walls are stained pale pink, with the exception of those just beneath the testa, which are scarcely coloured by the stain, and a similar effect was also noticed with a solution of iodine in potassium iodide. On treating sections with iodine and sulphuric acid the cell walls turned a bright greenish blue.

If sections of a seed which has been allowed to germinate are examined, it is seen that the space occupied by the cotyledons has become considerably enlarged, and the endosperm cells lying around this cavity have become somewhat crushed and depleted of their contents. The walls of the cells also in this region appear thinner and more mucilaginous than those situated at a greater distance from the developing embryo. In sections treated with methylene blue or Congo red these differences are very sharply marked, for with the blue the cells around the cotyledonary cavity remain uncoloured, whereas the walls further away are fairly deeply stained, but with Congo red, on the other hand, the walls of the cells which surround the cavity, and which appear mucilaginous, stain a deep rose-pink, whilst those which are as yet unaltered only take on a faint pink coloration (Fig. 12). Similar effects to these were noticed in sections stained in safranin, and in this case the mucilaginous walls are unaffected by the stain. From such results it seems highly probable that the composite cell walls are being disorganised by an enzyme in such a way that the pectic compounds, which were seen to stain with methylene blue or safranin in the ungerminated seed, are being removed, whilst a matrix, consisting of an hydrated form of pure cellulose, which gives the characteristic reactions with Congo red and iodine and sulphuric acid, is left behind.

[1] Kohl, "Dimorphismus der Plasmaverbindungen," *Ber. d. Deut. Bot. Ges.*, Bd. XVIII., 1900, p. 364, Taf. XII.

Examination of the cell walls about the region where this change is in progress shews that the action of the ferment on the walls does not proceed in a centrifugal manner, as was seen to be the case in *Tamus*; for instead of the ferment originating in the embryo and proceeding outwards to the endosperm, it appears to originate in the endosperm and proceed inwards towards the embryo, and so work in a *centripetal* manner. This method of attack, which can easily be followed with the help of suitable reagents, such as Congo red or picric aniline blue, gives a very peculiar and characteristic appearance to sections of the endosperm of germinating seeds (Fig. 13), for the walls which are attacked shew stained mucilaginous areas on their sides towards the periphery, *i.e.* away from the centre of the seed, whilst the sides towards the centre retain their sharp contours and shew little sign of disorganisation (Figs. 14 and 15).

The relation between the progress of the ferment action and distribution of the "connecting threads" is not always very clear, for though in some cases a boring out along certain of the threads can be observed, yet in many others the dissolution of the walls appears to take place without any obvious connection between them.

Cell walls on which the ferment has just commenced its attack frequently shew a jagged edge, owing to the ferment forming little V-shaped disorganized areas, which are at first separated from each other by thin slips of the unaltered cell wall (Fig. 14), and in such cases the action appears to take place along the canals of certain of the connecting threads, but when the disorganization is more advanced the mucilaginous portions of the wall are usually found to be bounded by rounded surfaces of the unaltered cell wall, and it is then difficult to make out that the ferment action proceeds by means of the threads (Fig. 15). The disorganization of the wall gradually extends until only a few islands of cells situated in a mucilaginous matrix remain, shewing the original composition of the wall, and these finally disappear, leaving an uniform wall of hydrated cellulose in which groups of connecting threads can still be clearly seen (Fig. 14).

When germination has proceeded for some time sections of the endosperm shew that the centripetal action of the ferment has commenced in cell walls at some considerable distance from the embryonic cleft, especially in cells under the testa (Figs. 13 and 14). The progress of the ferment in these distant walls is essentially the same as that just described, and sections treated with methylene blue shew that there is a very large unstained area around the embryonic cleft. With the vigorous growth of the embryo the cotyledons increase in size and continue to encroach upon the endosperm until the reserve materials stored in the cells and cell walls are completely absorbed, and, as soon as this has been

accomplished, the cotyledons are withdrawn from the depleted seed and unfold in the air, where they perform the functions of the ordinary green leaves[1].

In making a comparison between *Tamus communis* and *Galium Tricorne* it is interesting to note the great difference between the behaviour of the two seeds during germination. The composition of the cell walls appears also to be different in the two cases. In *Galium*, judging from microchemical reactions, it approaches more nearly to a pecto-cellulose, but in *Tamus* the walls appear to be composed of the so-called "reserve-cellulose," very similar to that described by Gruss[2] in the endosperm of *Phoenix sylvestris*.

It is not unlikely that the character of the enzyme action during the germination of these two seeds has some relation to the differences in the composition of their cell walls.

In the case of *Tamus* the mode of attack of the enzyme, which apparently is secreted by the cotyledon, is, with reference to the embryo, *centrifugal*, whilst in *Galium*, on the other hand, it is *centripetal*; in this latter case it is the cells of the endosperm which appear to contain the enzyme, or perhaps rather a zymogen which gives rise to an enzyme. It is possible that the formation of the enzyme or zymogen may be determined by a stimulus chemical or otherwise proceeding from the embryo.

On germination of the seeds the enzyme is liberated in the cells and commences to attack the walls and work towards the embryo in a centripetal manner. In both cases it was noticed that although in the first instance the walls were frequently entered by a ferment along the course of a thread, their subsequent disorganization took place without any definite relation to the connecting threads in the cell wall.

Strasburger[3], who has recently examined the endosperm of *Tamus* in connection with his work on "connecting threads," forms a somewhat different conclusion. He confirms the results which had been previously obtained[4], but he considers that the *chief function* of the threads is to serve as the passage ways by which the enzymes enter and travel through the walls and effect their solution during germination. Further, he attempts to prove that it is the threads which traverse the thickness of the wall (the "solitary threads" of Kohl[5]) rather than those found in the pit-

[1] Cf. Lubbock, *loc. cit.*

[2] Gruss, "Studien über Reserve-cellulose," *Bot. Cent.*, 70, 1897, p. 242.

[3] Strasburger, "Ueber Plasmaverbindungen pflanzlichen Zellen," *Jahr. f. Wiss. Bot.*, Bd. xxxvi. Heft 3, pp. 535—538.

[4] Gardiner, *Proc. Roy. Soc.* 1897.

[5] Kohl, *Ber. d. Deut. Bot. Ges.* 1900. Dimorphic threads had been previously described in the paper in the *Phil. Trans. Roy. Soc.* 1883, in the endosperm of *Bentinckia*, *Howea*, *Lodoicea* and *Phytelephas*, with figures. Cf. also Gardiner, *Proc. Roy. Soc.* 1897, p. 104.

closing membranes, (the "aggregated threads" of Kohl) which are concerned with the translocation of the cellulose-dissolving enzymes of the seed. In support of this view he brings forward the case of the endosperm of *Phoenix reclinata*, which, he says, possesses pit threads only; and here he finds that enzymes do not travel by means of the threads, but attack the thick cell walls which are devoid of any threads and leave the pit-closing membranes unaltered.

But an examination of the endosperm of species of *Phoenix*[1] by our methods proved as Pfurtscheller discovered that both pit and wall threads occur[2], though the latter are usually found only in special parts of the endosperm, viz. in the walls of cells near the periphery of the seed and of those immediately surrounding the embryo.

In the course of germination the thick walls undoubtedly appear to be attacked by the ferment and dissolved independently of the threads, but further examination seems desirable to determine whether the initial stage of the conduction of the ferment is or is not similar to that of *Tamus*, where it would appear that the contagion of ferment action having been once communicated by the threads, the subsequent rapid disorganisation and solution extends over the whole area of the wall independently of them.

The pit threads are still visible after enzyme action has proceeded for some time, but we do not find that the pit-closing membrane remains unaltered, since it swells considerably and finally becomes disorganised like the rest of the wall.

A similar state of things was noticed in the endosperm of *Galium*, where well-marked threads can be seen in the cell walls around the cotyledonary cleft, though the walls have been so altered by the enzyme that they no longer give the reactions of the normal unaffected walls of the seed.

The manner of disorganization of the cell walls of the endosperm of *Phoenix*, which seems to have so little relation to the presence of threads, appears to us to offer additional proof that the connecting threads are not concerned primarily with the translocation of enzymes, for those portions of the thick cell walls, which do not possess "connecting threads," can be dissolved by ferment action in the absence of pores by which it might be conveyed into the substance of the walls. Where, however, threads are abundantly distributed throughout the walls they do no doubt afford convenient channels along which the ferment may pass, and by means of which it may reach the older layers of the cell wall, but it seems

1 The species used were *P. sylvestris* and *P. dactylifera*.

2 Pfurtscheller, *N. Jahrb. KK. Franz-Jos.-Gymn.*, *Wien*, 1883, p. 63. Kohl, *loc. cit.*, p. 367, mentions that "solitary threads" occur in the endosperm of *Phoenix*.

certain that this function of the connecting threads in ripe endosperm tissue must be one of only a secondary character.

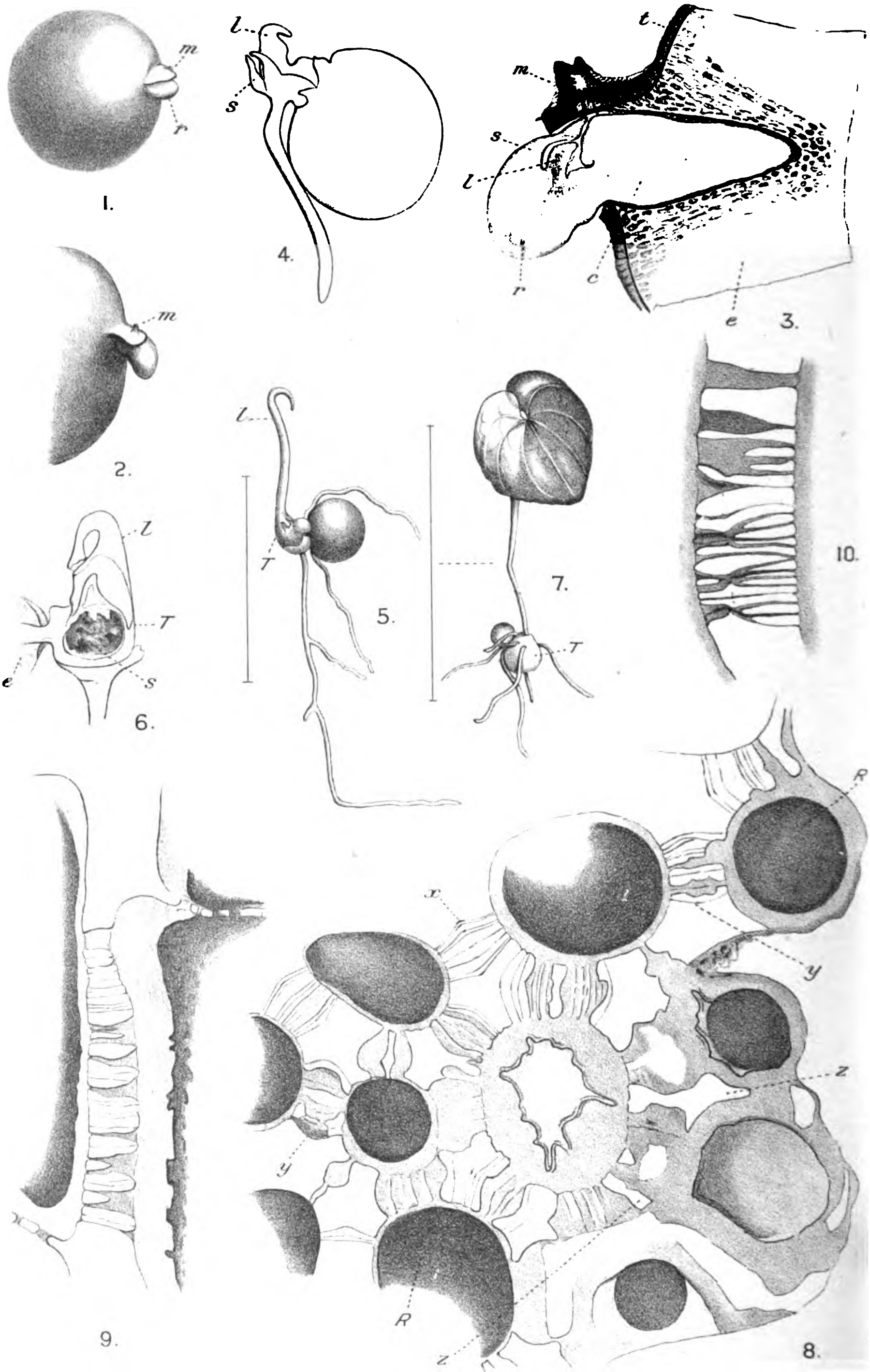
We consider, moreover, that there is at the present time, and, in the light of existing researches, insufficient ground for laying stress on the differences between the pit and wall threads or of assigning to either of them any special or distinctive function although it is not improbable that some such distinction does as a matter of fact exist.

As to the principle or master function of the connecting threads in the endosperm, there can be little doubt that the opinion already set forth[1], and to which Strasburger[2] takes exception, is the true view of the case, namely, that the "connecting threads" are primarily essential for the conduction of food and stimuli during the development of the endosperm and the seed, and that in such seeds as those of *Tamus* and *Galium* the period during which their functions are discharged with greatest perfection is "*inter vitam*" and not "*post mortem.*" Endosperms such as that of *Phoenix rupicola* which possess the two forms of threads lend additional support to this conclusion, since the wall threads are found in large numbers at those parts where they would naturally be most useful, for they occur not only in the walls of the peripheral endosperm tissue, where they would serve as channels of communication between the developing seed and the parent plant, but also in the walls of the cells which immediately surround the embryo itself.

The conclusions to which we are led by a study of the germination of the seeds of *Tamus* and *Galium* are that although the ferments can attack and dissolve the thick cell walls of the endosperm, without any necessary relation to the "connecting threads," yet that in the initial stages the penetration of the enzyme may be effected by means of the threads, which thus afford a means of reaching the internal parts of the wall.

Secondly, that the connecting threads are concerned mainly and primarily with the conduction of food and stimuli from the parent plant to the developing embryo and endosperm of the seed, and that any further use to which they may be put during the germination of the seed must be regarded as only of secondary significance.

[1] Gardiner, *Proc. Roy. Soc.* 1897, p. 107.
[2] Strasburger, *loc. cit.*, p. 536.



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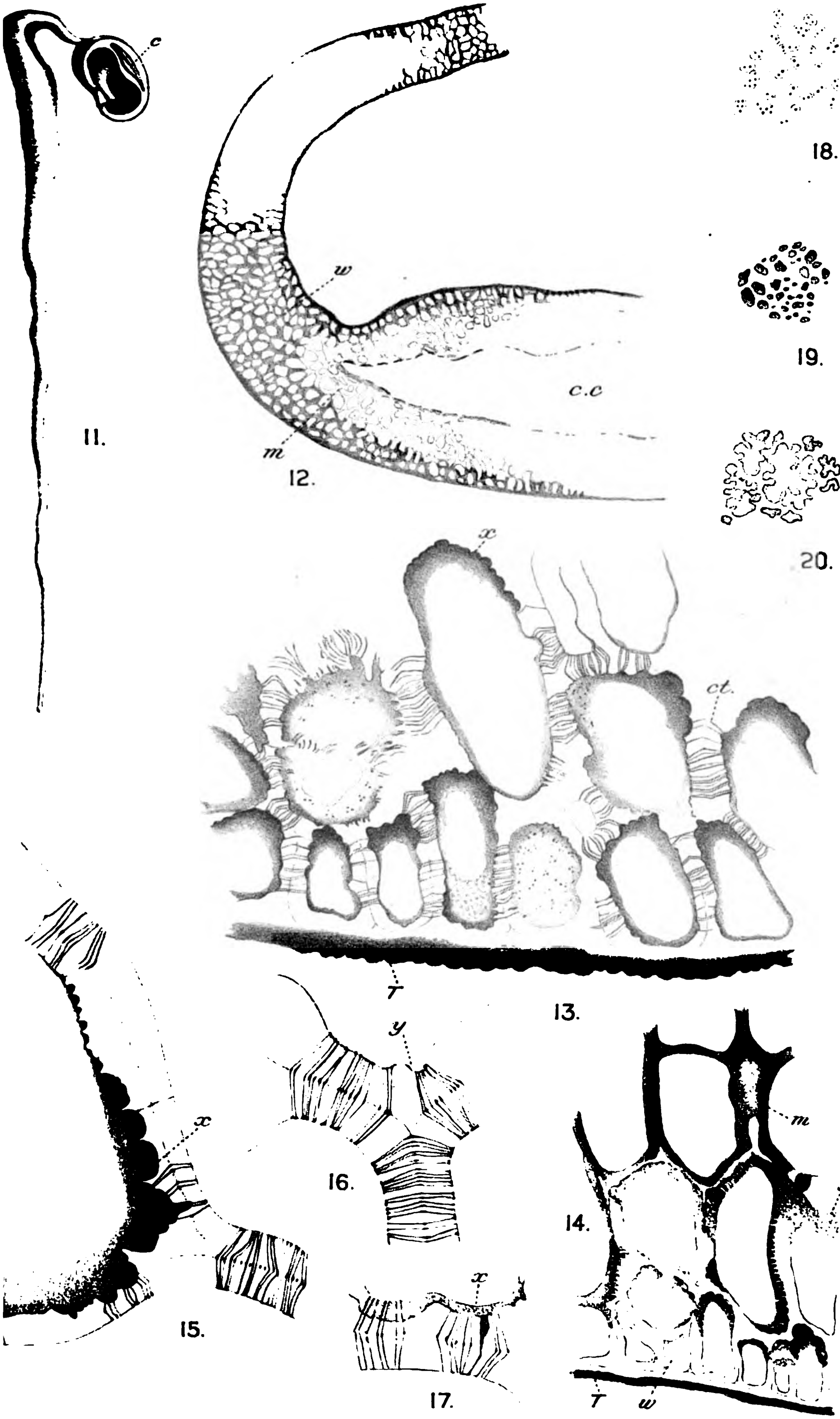


Edwin Wilson, Cambridge

EXPLANATION OF PLATE V.

FIGURES 1—10, *Tamus communis.*

FIG. 1. Seed of *Tamus* at the commencement of germination, the radicle has just pushed open the lid-like portion of the testa around the micropyle. *m.* micropyle; *r.* radicle.

FIG. 2. A slightly older stage than Fig. 1. The micropyle lid, *m.*, seen from above.

FIG. 3. The embryo and seed in section as in fig. 2 shewing the testa composed of thick-walled cells; the 'lid' surmounted by the micropyle and the radially arranged endosperm cells. In the embryo the cotyledon (*c.*) embedded in the endosperm (*e.*), the first leaf (*l.*) covered by the cotyledonary sheath (*s.*) and the extruded radicle (*r.*) are seen. *t.* testa. *The line* (*c.*) *does not reach the cotyledon.*

FIG. 4. A more advanced stage of germination. The first leaf (*l.*) of the plumule has pushed aside the two lobes of the cotyledonary sheath (*s.*) and commenced to grow upwards.

FIG. 5. The young seedling shewing the formation of the tuber (*T.*) at the base of the first leaf; the first leaf possesses a sharply incurved tip.

FIG. 6. A young epicotyledonary tuber full of starch seen in section, *s.* starch, *e.* endosperm, *T.* the base of the cotyledon.

FIG. 7. An adult seedling in its first year, shewing the seed, to which the large tuber (*T.*), with its adventitious roots, still remains attached, the long petiole and expanded lamina of the first leaf. The dotted line marks the level of the soil.

FIG. 8. A section of the endosperm near the centre of a germinating seed; the cells are here rounded and possess thick walls, which are well provided with connecting threads as at *x.* Various stages of the action of the ferment on the wall can be seen. As soon as an entrance into the wall has been effected along one or more of the threads the ferment proceeds to dissolve the wall, especially in the region of

the middle lamella as at *y*, until only a few islands *z* of the original wall remain. The parts of the wall undergoing solution become swollen. They are indicated by shading. Cell contents *R.* are seen in some cells. Mag. 460.

FIG. 9. A cell wall from the peripheral part of the endosperm of a seed in an advanced stage of germination. The ferment here attacks the younger layers of the cell wall and gradually works inwards. Mag. 460.

FIG. 10. As fig. 9. The action of the ferment along the course of separate threads is more clearly seen. Mag. 750.

FIGURES 11—20, *Galium Tricorne.*

FIG. 11. A young seedling with the cotyledons still embedded in the endosperm. The seed has been cut in half to shew its hollow nature and the position of the cotyledons (*c.*).

FIG. 12. A transverse section of the endosperm of a recently germinated seed. The section has been treated with safranin, which stains the unaltered cell walls (*w.*) but leaves unstained the walls (*m.*) of the cells surrounding the cotyledonary cleft (*c.c.*), which have been altered by ferment action.

FIG. 13. A section of the endosperm of a germinated seed, shewing the centripetal action of the ferment on the cell walls. The attacked portions of the wall are shaded. *T.* testa. The ferment appears to attack the walls with but little relation to the numerous 'connecting threads' (*ct.*) for it does not usually work along the threads, but the cell wall in process of dissolution shews a crenulated edge where the ferment action is taking place. Mag. 350.

FIG. 14. A section of the endosperm of a seed in an advanced stage of germination, stained with Congo red to shew the effect of the ferment action on the composition of the cell wall. The dye stains the attacked portions of the wall (*m.*) (shaded in the figure) and leaves the original wall (*w.*) unstained. The centripetal action of the ferment and its method of attacking the cell wall are well seen. *s.* a piece of a wall in surface view. *T.* testa. Mag. 180.

FIG. 15. A piece of a partially attacked endosperm wall more highly magnified, shewing groups of 'connecting threads,' the crenulated edge (x) of the unaltered wall shews the method of ferment action. Mag. 750.

FIG. 16. 'Connecting threads' in the unpitted cell walls of the endosperm, at (y) the wall is seen to possess a shallow pit on one side only. Mag. 750.

FIG. 17. A small piece of cell wall in process of disorganization shewing two groups of threads. In one case the ferment has bored out a single thread x. Cf. fig. 15. Mag. 750.

FIG. 18. A piece of the wall of an endosperm cell in surface view, shewing that the connecting threads, which together form a large group, are arranged in little groups of twos or threes each in a slight pit. Mag. 1000.

FIG. 19. The action of the ferment on the wall in surface view, shewing small areas undergoing change, cf. with fig. 18. Mag. 1000.

FIG. 20. A more advanced stage of ferment action, the small areas have become merged together with the spread of the sphere of action of the ferment, only a skeletal framework of unaltered wall being left. Mag. 1000.

Regeneration in Samia ailanthus. By H. H. BRINDLEY, M.A., St John's College.

[*Received* 16 June 1902.]

The subject of regeneration in Lepidoptera has in only a few cases received systematic and experimental study. Réaumur[1] seems to have been the first to investigate the effects of mutilating the larval legs on those of the imago, but the number of cases he described, as well as those of Mélise[2] and Watson[3], who made some observations on *Sericaria* and *Dicranura* respectively, are too few to throw much light on the extent to which the features of regeneration in lepidopterous insects compare with what is known regarding regeneration in other Insecta and in Arthropods generally. Newport's[4] experiments on *Aglais* and the recent work of Chapman[5] on *Liparis* are much fuller, and furnish at least an indication that the results of a particular injury to the appendage of an insect with complete metamorphosis are much less constant than is the case in those Insecta and other Arthropoda which attain sexual maturity through a series of ecdyses. It is now fairly well established that the regeneration of an appendage of an Arthropod is a highly specialised process[6]. The new growth may be apparently the close counterpart of the normal, or it may be an alternative structure of specialised form, such as the 4-jointed tarsus of Blattidae and Phasmidae and apparently the few jointed antenna of some Collembola, or in exceedingly rare cases (two or three Malacostraca and doubtfully some Insecta) the new appendage may assume the form of another appendage with which it is in serial homology[7]. Setting aside the last-mentioned exceptional

[1] *Mém. de l'Acad. des Sci.*, 1712, p. 223, and 1718, p. 263, and *Mémoires sur les Insectes*, 1734, p. 365.

[2] *Ann. de la Soc. Entom. de Belgique*, 1879, XXII., C. R., p. xcii.

[3] *Entomologist*, 1891, XXIV., p. 108.

[4] *Phil. Trans.*, 1844, and *Mag. Nat. Hist.*, 1847, XIX., p. 145.

[5] *Entomol. Record*, 1900, XII. pp. 141 and 177.

[6] Brindley, for summary and bibliography, *P. Z. S.*, 1897, p. 903, and 1898, p. 924; Morgan, "Regeneration," New York, 1891; and Przibram, *Arbeit. Zoolog. Institute*, Wien, 1899, XI., Heft 2; *Archiv f. Entwickelungsmech.*, 1902, XIII., Heft 4; and *Zool. Anz.* 1902, XXIV., no. 661, for records later than *P. Z. S.* cited.

[7] Bateson, "Materials for Study of Variation," 1894, and *P. Z. S.*, 1900, p. 268; Herbst, *Archiv f. Entwickelungsmech.*, 1896, II. p. 544, and Przibram, *Zool. Anz.* XIX., 1896, p. 424.

instances of homoeosis, there is among Arthropod animals a high degree of constancy of form and feature in the regeneration after particular injuries; *i.e.*, if regeneration occurs at all, the new limb, though at least at first of small size, is well-formed and apparently an efficient organ, and in some cases it is known that the mechanism of regeneration is so specialised that neither the stage at which injury was inflicted or the degree of injury, so far as it is possible to gauge the latter accurately, make no profound difference to the form assumed by the regenerated limb. The above statements however appear to receive but partial support from those insects in which the post-embryonic development includes complete pupation, though the number of experiments hitherto made is very small and these have been confined to the Lepidoptera. Newport amputated the 3-jointed larval leg in its basal or second joint just before pupation and described the results for about 15 cases. In the imagos there was much variation in the condition of the injured limbs. In all cases femur, tibia, and tarsus could be distinguished, but the number of tarsal joints varied considerably. In all, however, the terminal claw of the tarsus was present. This fact and the drawings which illustrate his paper suggest that the reproduced tarsus in all these cases should be regarded as representing the whole of the normal tarsus, rather than for instance that a 3-jointed tarsus should be considered as equivalent to three particular joints of the normal tarsus. However this matter be regarded, it remains that the tarsus is sufficiently represented to bear the normal termination, the claw: so that his observations illustrate the tendency, seen so widely in the regenerated limbs of Arthropods, to produce the proper terminal structures, though normality in the number of joints may be absent. Chapman has extended the range of the enquiry by amputation of the larval leg in the penultimate instar before pupation, and secured uniformity of method by always amputating the basal joint and performing the operation immediately after an ecdysis. In the imago the claw apparatus is present in the six or seven cases this author describes and figures, though there is considerable variation in the number of joints, and in one instance the imaginal limb is only two rounded joints bearing a dwarf claw. Dr Chapman informs me that he has made several hundred mutilations in different instars with the view of observing the progress made from stadium to stadium, and it is to be hoped that the results of these, the first at all extensive experiments, will be published by him, as to my great regret I have so far had no opportunity of taking advantage of his very kind suggestion that I should examine his material myself.

The following observations were made while I was unaware that Dr Chapman was examining the same subject, and were

suggested by Newport's result that there is great want of uniformity in the regeneration of the limb of a pupating insect after a particular injury. Hence it seemed desirable to obtain a number of cases of the results of particular injuries in particular instars, especially as Newport's cases were too few to permit any general conclusion being arrived at. I am indebted to Miss F. Durham for kindly placing at my disposal a large number of caterpillars of *Samia ailanthus* which were being reared in the Balfour Laboratory, Cambridge, in 1899 and 1900. The results were unfortunately somewhat limited, in consequence of the very large number of failures to emerge from the pupa, an event which had apparently nothing to do with the mutilations inflicted, for, as frequently happens under artificial conditions, the mortality was very high in the pupal state among the whole set of larvae in the laboratory. The experiments made were as follows.

(*A*) Instar of body-length 2·0 to 2·5 cm. and of pale yellow colour turning to white. This is probably the instar but one before pupation. The injury inflicted was amputation of the third leg in its basal joint by scissors. Unfortunately only four imagos were obtained. The imaginal leg was well developed as regards femur and tibia, but in one case the tarsus was 3-jointed and in the other three 4-jointed, and in only one case (one of the 4-jointed tarsi) were the terminal claws well developed. In the other three they were entirely absent, and the tarsal joints were not well formed though their articulations were distinct. In one of those with a 4-jointed tarsus the pupal period was 300 days (August to July) and in the other two 63 days (August to October), while the case with a 3-jointed tarsus was 307 days in pupa. The single case in which the claws were developed was one of the 63 days' pupas. Hence, though the number of cases is far too small for certainty, there is a suggestion that the length of time spent in pupa does not affect the degree of regeneration.

(*B*) Instar of body-length 3·5 to 4·0 cm., and spinning about 8 days after mutilation, so that probably the instar was the one which pupates, though I was unable to ascertain this point with certainty. The injury was amputation of the posterior leg in the basal joint. The average number of days spent in pupa was 61. Eleven imagos were obtained, and the leg was regenerated in all, though in a very variable manner. The femur was well developed in about half the cases, and stunted in the others. Tibia and tarsus were sometimes distinguishable, but the tibia and femur, and tibia and tarsus respectively were apparently fused together in certain cases, while the tarsus when recognisable had never more than four joints. In about half the cases the claws were more or less distinctly suggested, as was also a terminal lobe apparently representing the palpal apparatus.

(*C*) The last larval instar immediately before pupation, and all cases had spun by next morning. (Amputation during spinning seemed to disturb the larvae very slightly, they almost immediately resumed the process of enclosure in the leaf selected, and in no case did mutilation seem to delay the act of pupation.) The injury was amputation of the basal joint of a posterior leg. Twenty-seven imagos were obtained, after a pupation averaging about 36 days. The injured leg was usually a flattish stump, and in the majority of cases there was a projection from this, so that the new growth may possibly be regarded as a femur plus an outgrowth representing tibia and tarsus. In only three cases was there a clear suggestion of a tarsus, and in one of these it seemed to consist of four joints. There was in no case any indication of the claw apparatus.

(*D*) The instar as in (*C*), but the injury was amputation of a posterior leg in the second (or middle) joint. Eighteen imagos were obtained, and the injured limb was better formed than in (*C*). The femur was usually very short and broad, but in nearly all cases it seemed possible to distinguish tibia and tarsus. In two or three cases the apparent tarsus was not divided into joints, but in most one or two articulations were more or less indicated. In one case only were both the terminal claws well formed.

It must be understood that the above interpretations of the regenerated limb claim only an approximate accuracy, as though the larva from its large size is an easy one to injure in exactly the desired manner, the imaginal legs are so covered with hairs that there is often much difficulty in making out the articulations, especially as the regenerated limb is always dwarf and liable to be broken in attempts to remove the hairs by brushing. But the observations made may be taken as at least emphasizing the great variability of result arising from apparently the same degree of injury, and as confirming the statements of Newport and Chapman in this respect for other Lepidoptera. The want of uniformity of result stands out in great contrast from what is known in the case of an insect with "direct" development; *e.g.* in *Stylopyga* (*Periplaneta*) *orientalis* out of several thousand legs bearing the characteristic 4-jointed tarsus of regeneration, only 8 were found with the tarsus at all malformed. There is much suggestion that the mechanism of regeneration is less highly specialised in an insect whose post-embryonic development includes metamorphosis than in one which proceeds by simple ecdyses to maturity. In this connection it is natural to remember the greater liability of the long legs of an orthopterous form to accidental injury compared with the short legs of the lepidopterous larva, and the possibility that a more specialised mechanism of reproduction has arisen in the case of greater need for such.

The results obtained under (*B*), (*C*) and (*D*) above differ from those of Newport and Chapman in two respects, viz. the terminal claw apparatus was as a rule not present in the regenerated limb, as it was characteristically in their cases, and in *Samia* there was no case in which the tarsus was regenerated with the normal number of joints, while Chapman has figured at least three cases of a 5-jointed tarsus in *Liparis*. It is quite possible that generic differences exist in these respects. The experiments on *Samia* tend to suggest that the earlier the instar injured the more the imaginal limb approaches the normal in form and size. As regards the comparative results of injuring a leg to a greater or lesser extent at the same larval stage, the results obtained under (*C*) and (*D*), which it was hoped would throw light on the point, were attended with so much variability, and were, owing to the numerous deaths in pupa, so few in total number, that all that can be said is that there is at least a suggestion that an injury at the onset of pupation is, if confined to the middle joint of the leg, followed by the production of a rather better formed imaginal leg than if it includes removal by amputation through the basal joint. So far as it goes, there is here some evidence in favour of Gonin's conclusion[1], based on anatomical examination, that just before pupation only the extremity of the developing pupal leg projects into that of the larva, and that amputation of the latter at its base is therefore likely to injure the pupal leg more extensively than section through the middle joint. Chapman's observations on *Liparis* were undertaken in part with the object of examining the evidence for Gonin's view, and he arrives at a conclusion unfavourable to this author, but I am unable to agree with the interpretation he places on his results with *Liparis* in this respect, though, as stated above, my own experiments on *Samia* are too few to do more than suggest that Gonin's view is the correct one.

[1] *Bull. d. l. Soc. Vaudoise d. Sci. Natur.*, 1894, sér. 3, xxx., p. 122.

Some Notes on Variation and Protandry in Flabellum rubrum, *and Senescence in the same and other Corals.* By J. STANLEY GARDINER, M.A., Fellow of Gonville and Caius College.

[*Read* 19 May 1902.]

CONTENTS.

About a year ago I received for determination some hundreds of specimens of *Flabellum* from the Biological Department of the South African Museum, Cape Town. Of these eight belong to *F. pavoninum*, while the remainder—over 500 in number—are referable to *F. rubrum*, of which *F. cumingii*, *elongatum*, *crassum*, *crenulatum*, *elegans*, *spheniscum*, *profundum*, *irregulare* and *transversale* are synonyms. Mr Forster Cooper and I also dredged a number of specimens in the Maldives, which mostly—being especially carefully preserved—have been used in the subsequent part of this investigation. Further, for the species, I have with the assistance of Prof. Jeffrey Bell and Mr H. M. Bernard examined the British Museum collection.

The account of the species, anatomy, and development of the Cape of Good Hope collection of *Flabellum* forms the subject of a memoir, *in the press*, among the "Marine Investigations in South Africa," published by the Department of Agriculture of the Cape Colony. These Notes form a supplement to that paper, founded on a renewed investigation of special points in my Maldivan and in the British Museum specimens in addition to those from the Cape of Good Hope.

Semper procured specimens of *F. rubrum* in the Philippines, and described them in 1872 under the name of *F. irregulare*[1]. He dredged his specimens in the Lapinig Channel, off Bohol, in the centre of the group from 6—10 fathoms. With them were also obtained a large number of specimens of *F. stokesi*—described

[1] *Zeit. für wiss. Zool.*, Bd. XXII., pp. 242—245, Pl. XXVI., figs. 7—17.

as new under the name of *F. variabile*[1]—and *Blastotrochus nutrix*[2]. The latter genus is one which I can consider as in no way different from *Flabellum*. It was founded by Milne-Edwards and Hamie[3] on a misconception, the universal belief at that day and in Semper's time being that the coral skeleton was formed by the endoderm, whereas it is really a deposit outside the ectoderm and hence morphologically completely outside the coral polyps. The wall of the coral of *Flabellum* is *epitheca* and not *theca*, so that hence there is no polyp tissue external to it, from which budding could occur.

The Cape of Good Hope specimens belong to *F. rubrum*, but have about 2 per cent. of their number intermediate towards *F. stokesi* and *F. nutrix*. The Maldive specimens mostly belong to Semper's *F. stokesi*, but some to the same author's other species. The first three cycles of septa in all are equal and fuse in the axial fossa by trabeculae from their edges. The fourth cycle is smaller, but commonly some of its septa fuse to the others by trabeculae, this character being generally more marked in *F. stokesi*. The Maldive specimens of each of the so-called species as compared with the Cape specimens show an increased length of the basal scar. The breaking off of the corallum takes place along one of the lines of growth, and probably the growth is greater with increase of temperature, better food-supply, etc., so that even if all corallites broke off along the same growth line—an unlikely supposition—the scar would in different regions vary in size. There is absolutely no difference in the polyp anatomies of either of the two species, but the presence of wings along the end of the calicle in one and of spines in the other is a fairly definite difference. The British Museum forms show, however, intermediates in this character and also in the size of the scar. Out of about 600 the total number of intermediates (in the strictest sense) was 17, of which 4 alone were intermediate in both characters. It is hence evident that *F. rubrum* and *F. stokesi* represent variations of the same species.

Doubt only remains as to the so-called *Blastrotrochus nutrix*. I had only one specimen from the Maldive, which was absolutely identical with Semper's description of the species. It was quite similar in the polyp anatomy to the other forms, but in the corallum closely approached to *F. rubrum*. The so-called buds are approached in many of the Cape specimens of *F. rubrum*. Neither in these nor in the Maldive specimen is there any connection with the tissues of the large polyp, and it is doubtful whether they may not have been largely derived from ova, rather

[1] *Loc. cit.* pp. 245—251.
[2] *Loc. cit.* pp. 237—241.
[3] *Ann. des Sc. nat.*, 3e sér., t. IX., p. 281 (1848), and Cor. II. p. 87 (1857).

than from the cutting off of masses of the central polyp by its advancing epithaeca. The presumption at present—from the large number of specimens dredged by Semper—is that these small polyps were formed in the latter manner. With knowledge of some of the facts alone—and none of the possible causes—this form must be deemed to represent a third variation.

The polyps appear to give reliable characters of the species of the genus. In consideration of this and the definite existence of about 3 per cent. of intermediates, the three so-called species must be deemed to be varieties of a single species. This is the first suggested case of discontinuous variability in the Madreporaria, and it is particularly interesting from the fact that all the three forms were found both in the Maldives and Philippines in each case in the same habitat. According to the rules of nomenclature the species should be termed *F. rubrum*, the two varieties being var. *stokesi* and var. *nutrix*. The names of the type and its varieties exemplify the singularly unfortunate and inelastic character of these rules. The naming of a type-form among the above three forms is obviously undesirable. Further the form, originally described, merely represented an exceptional case of normal variability, probably brought about by environmental or habitative influences, the usual operative cause among corals[1].

The largest specimens of *F. rubrum* are characterised by having 24 equal septa fusing to one another by trabeculae in the centre of the corallum, forming a false columella. In development there are at first 6 septa, which subsequently join together. Additional septa are formed and an additional 6 fuse by trabeculae with the first cycle. The corallite at this stage is seldom more than 2 or 3 mm. along the long axis of its calicle. Development now proceeds more slowly, and further septa only commence to fuse in the calicle, when a length of 8 to 10 mm. is attained. The scar of either of the three varieties is usually in the stage with 12 septa fusing, showing additional reason why the size of the scar is not a specific character.

It is during this pause in the development that the larger mesenteries commence to receive their generative organs. At first there is a mere slight thickening of the endoderm on the two sides of the mesenteries behind the contorted ends of their filaments. This nutritive endoderm is very granular, and cell divisions are not apparent. Small testicular acini appear in the structureless lamella between its two layers. These are at first isolated, but partially by growth and partially by the addition of

[1] Vide "Marine Crustaceans, I. On Varieties," by L. A. Borradaile, *Fauna and Geogr. Maldives and Laccadives*, Camb. Univ. Press, vol. I., Pt. II., pp. 193–8 (1902).

fresh acini an oval mass is formed. When the calicle is about 13 mm. long, fresh testicular elements cease to be added to the primary mesenteries—those which reach the stomodoeum—and further change takes place merely in the enlargement of the acini with increase of spermatozoa, their shape becoming polygonal or quite irregular. Meantime the smaller mesenteries gradually develop their testes.

When the length is 15—17 mm., all the mesenteries have testes, but only those on the larger mesenteries are as yet functional. The spermatozoa appear to escape by *temporary* ducts, all in each single acinus being shed together. The corallum, in the type variety, has at this period usually 20 septa fusing by trabeculae in the axial fossa. In 51 specimens out of 65 of this size the number of these larger septa is 20, constituting a very determinate stage. The appearance is so regular indeed that Semper considered that he had here a true case of 5-rayed symmetry, whereas the development shows clearly that the true symmetry is hexacoralline in accordance with Milne-Edwards' law. The corallum in the Cape specimens, when of this size, has 5 lines of growth which correspond probably to annual periods, and it is interesting to observe that the specimens figured by Semper exhibit the same number.

In still larger specimens of about 20 mm. in length ova commence to appear in the generative masses on their inner sides—*i.e.* towards their free edges—which are of course terminated by the mesenterial filaments. Each gonad now consists on its outer side of a series of closely set irregular sperm acini with definite open spaces in their centres, hence more or less ripe. On the inner side the acini are more rounded and less crowded, but some trace at least of the central space is visible. The ova, generally 2 to 4, appear on the inner edge near the upper end of the mass, and are at first quite minute. No further sperm acini are now added. As more and more of the older acini extrude their contents, the newer ones and the ova fill their space. The ova grow enormously, with the final result that the mass becomes entirely female, consisting of usually 2 or 3 large ova, flattened on their sides against one another and occupying the whole area of the former testis.

Together with the above changes the corallum becomes more rounded and less pointed at the ends of the axial fossa. The epitheca is considerably thickened by fresh deposition of corallum within the calicle, and indeed there seems to be everywhere a renewal as it were of the activity of the calicoblast layer. The wings of the calicle are filled in with corallum, and may be much worn down outside, perhaps leaving hollow spines, filled in life with extensions of the polyp. This change, though, is due to

extraneous, environmental causes, the polyp tissues ceasing to exercise an efficient, protective influence over the outer parts of the epitheca, consequent on its greater thickness.

With increase of size, beyond 25 mm. in length, the ova ripen. Ducts are formed through the thickened nutritive endoderm down to the ova, which subsequently escape through them. In sections along their lengths they show longitudinal striation, but they are very distinctly ducts. They correspond in fact to the structures termed "cône nutritif" and "Fadenapparut" in Actiniaria, which I would suggest are simply *oviducts*.

As the ova ripen they escape, but the place of each of the original ones (2 to 4) is taken by 2 to 4 fresh ova. The several organs on the primary mesenteries of a polyp, whose calicle is about 32 mm. long, form ovoid masses of 6 to 9 closely adpressed ova. A general increase in size of the separate organs takes place, but this, after the ripe testes have once been formed, is never proportional to the general increase in size of the polyps. An ovary with 2 or 3 ova occupies about the same space as a ripe testis, and the increase is only when it comes to consist of 6 or more ova. The area of the nutritive endoderm increases still less than that of the ovary, so that in the largest polyps it covers the generative masses to a proportionably less degree.

The largest specimen—of the type variety from the Maldives—I have examined was 40 mm. in length. A large number of the ova had escaped, many of the mesenteries having none. On most there were a number of ova, quite isolated from one another. On none of the mesenteries could I find any small ova to take the place of the escaped eggs. Indeed it seemed obvious that a critical period had been reached, after which ova cease to develope. No change was at this stage visible in the ectoderm, neither external nor of the mesenterial filaments. The whole endoderm was on the other hand devoid of fat and much vacuolated, even the nutritive part over the ovaries being but little granular. A second specimen 36 mm. long from the Cape and a third 33 mm. from the Maldives exhibited like conditions to the larger form mentioned above.

So far I have referred only to the larger or primary mesenteries, *i.e.* those which fuse with the stomodoeum. The smaller mesenteries are always of the same sex as the larger, and appear to exhibit the same changes. Their ovaries do not usually consist each of more than 3 or 4 ova.

There is no direct proof—indeed it is only a presumption—that the polyp now dies. The annual (?) lines of growth are so regular, right up to whatever size the corallite may be, that obviously growth must be a very orderly process. The largest specimen among over 600 corallites was a Cape of Good Hope

form in the British Museum 42 mm. in length. If the corallites continue to live, surely this should be exceeded in size from its locality. At the same time it is likely that in other places with less or more favourable environmental conditions the maximum may be above or below this. Thus in the Maldives Mr Forster Cooper and I dredged 8 dead forms, which averaged about 38 mm. (judging from two of the less decayed specimens brought home), the largest living form being 40 mm.

In the smaller specimens the replacement of the escaped ova by fresh ova is quite clear. Occasionally before an egg escapes, the young ova are to be seen lying between the ovary and the mesenterial filament ready to take its place[1]. Indeed at no time is there a dearth or vacuity in the ovaries, whereas in the oldest forms the ova are separated from one another, and there are no signs of fresh ova being produced nor of the gaps being otherwise filled up.

A somewhat similar phenomenon to the above is shown by a colonial coral, which I originally described from South Pacific specimens under the name of *Coenopsammia willeyi*[2]. This coral is common in the Maldives, seated on the under sides of stones of the boulder zone. All the separate polyps of a colony are of the same sex, male or female; of protandry I have seen no suggestion. Further, all the separate colonies obtained by Dr Willey at Sandal Bay, Lifu, were of the same sex. Two colonies found at Minikoi in July, 1899, were both female, as were also seven collected at Hulule, Male Atoll, in January, 1900, on the same day, hence probably from a restricted area. Other colonies I have not examined, as two or more areas are or may be mixed together in my collecting jars.

In addition the genital organs of the polyps of *Coenopsammia willeyi* in each of the above three collections are in the same state as those of other polyps of the same species from their own area. Thus in the Sandal Bay specimens the polyps were all females with ova separated from one another, but yet fairly regularly arranged. The ova are less regular in the Minikoi colonies, some having dropped out and not been replaced. In the Hulule polyps some of the ova touch one another, and are flattened on their

[1] Although I have examined a large number of specimens of Madreporaria of all kinds, I have not been able to satisfy myself as to the origin of the generative cells. The appearance in *Flabellum* suggests that they arise at the bases of the mesenterial filaments, and thence wander to their ultimate position. This, however, is not confirmed in other species, and the question requires further examination. It is interesting to observe that, should the generative cells be derived from the epithelium of the filaments, they would be ectodermic in their primary origin. For a consideration of the body layers in the Actinozoa vide "On the Anatomy of a supposed new species of *Coenopsammia* from Lifu," *Willey's Zoological Results*, Pt. IV., p. 374—5 (1899).

[2] *Willey's Zoological Results*, Pt. IV., pp. 357—380 (1899).

adjacent sides. The Minikoi specimens correspond to the last state found in *Flabellum*, and it is interesting to remark that the two colonies examined were the *only living* ones found in this locality, although dead corolla were very constantly turned up.

One part of the reef at Rotuna, some hundreds of square yards in extent, was nearly completely overgrown by a coral, which I identified as *Madrepora pulcra* Brook, var. *alveolata* Brook[1]. This is one of the staghorn-growing species of the genus with widely separated branches, and was not found save in this single area. Most of the stems were dead at the base, the coralla in some parts of the area having no living polyps at all. In three living twigs, which I examined, all the polyps of every part were female. The ova were very far from regular, some of the polyps having on their mesenteries isolated ova, others no ova at all, though they had evidently recently possessed them. All the branches over the whole area were evidently dying. My examination, which was made in 1898, was undertaken to ascertain the cause, but was without result. No trace of silting up by mud was found, nor was there marked shrinkage and decay such as results from drying up owing to undue exposure to the sun and air. The endoderm was everywhere devoid of food granules, but except in its staining properties appeared healthy. The condition again is only paralleled by that of the largest *Flabellum*, suggesting that the same operative cause may have acted in both cases.

Death of the corals on reefs is common, and has been observed by most workers thereon. In some cases it may have been due to exposure, in others to silting up or even too high a temperature, but in such not one nor two species of corals nor even genera would be affected, but all coral growth on the area. In addition there is much death, which cannot properly be assigned to environmental causes, where single colonies die or a number of colonies of a single species in a particular area. Separate dead colonies, enormous masses perhaps, are often found where neighbouring colonies of the same species, both large and small, are flourishing.

A different case—here and there observed in the Maldives—would be that where a species or genus of corals was absent in the living condition from one reef, while on the next with *apparently* absolutely similar conditions it was perchance growing luxuriantly. Living *Coeloria daedalea* was absent from the west or seaward reef of Addu Atoll off Maradu, while *Leptoria tenuis* was extremely common, forming large colonies. The reverse in these two species was the case on the seaward reef off Midu to the north-east of the same atoll. Elsewhere in the Maldive

[1] *Proc. Zool. Soc.*, 1898, p. 259.

Group these two species were found on practically every seaward reef examined by Mr Forster Cooper or myself. A more striking case was that of *Madrepora hispida* (sp. ?) at Minikoi. It was the commonest coral on the sand flats within the reef of this atoll, to the south and south-west existing as great groves, formed by branches from many colonies. To the north and north-west the same coral was extremely common. The colonies were larger, but only patches on their surfaces or the mere tips of their branches were covered with living polyps.

It is useless to quote further instances. In my notes a large number of cases of death in corals are recorded, but I did not at the time appreciate the importance nor meaning of the phenomenon. It is in most instances doubtful as to the extent of the area over which death had taken sway, and I am, except in the above instances, uncertain whether the largest and the smallest colonies of the affected species died. My impression distinctly is that practically all colonies of a species in any one area died, or that there were only the isolated deaths of individual large colonies.

Each coral block has presumably originated from a single ovum, and such a colony cannot *normally* give rise to other masses asexually. The limitations in the size of colonies—clearly visible on any reef in massive *Porites* and other massive genera—points clearly to some prohibition of their growth. Such a regular restriction must be due to some innate reason in the organisms themselves. There can be no rejuvenescence, and the operative cause is, probably, the same as that which ultimately produces the death of our forest trees. The maximum of productiveness, so far as the formation of the germs of a fresh generation is concerned, is reached, and then the parent gradually becomes less fruitful and ultimately dies. In the animal kingdom there is no close parallel, although it is a reasonable deduction that senile decay occurs in all multicellular organisms or else absolute extinction at some period or other. The phenomenon in domestic animals, a few fish (trout, etc.) and other vertebrates is known, but for comparison with these the whole colony of polyps must be regarded as a single organism.

If—as I am impelled to believe—the ripening of the generative organs of a large number of polyp colonies of the same species in a single locality or habitat, followed by the subsequent death of all these colonies, is a regular phenomenon, the consequences may be of the most wide-reaching importance in the formation of coral reefs. With this question I am not in this paper concerned. The evidence of it is at present meagre, and the case of *Flabellum*, where death appeared to depend on size or age, does not lend support to its wide-spread prevalence.

Remembering the occurrence of a ciliated larva in the Madreporaria, it is impossible to believe that the colonies of a single species in any area are of the same age, and they certainly are not of the same size, hence have not a like number of polyps. It is consequently necessary to suppose that in an area—owing probably to some change in the physical conditions of their environment—all the corals of some single species have been stimulated to ripen and dehisce their generative products at the same time, the act leading ultimately to their weakening and death. As the colonies in the cases especially examined were female there may be protandry.

The above explanation is neither complete nor quite satisfactory, but it is clearly evident that there is here an entirely different phenomenon to senescence. The only close parallel is that of the larger species of bamboo, in which death follows the act of flowering. *Every plant* of the commercial species in a district produces its fructification at the same time and dies. As widespread inconvenience commonly results in India, China and Burmah, the circumstance is of common knowledge. There are no definite experiments to show whether the offshoots of bamboos of different ages *ab ovo* fructify at the same time. It is, however, almost certain that in the country of an intelligent people they must do so, or else they would be largely exported from district to district. The cause in bamboos must be sought in some change of season, and hence there is almost a complete parallel with the above-mentioned colonial corals. The whole phenomenon is obscure, but the circumstances show that there is a phenomenon, which requires—and would almost certainly repay—special investigation[1].

The consideration of the above phenomenon arose naturally out of my investigation of the anatomy of *Flabellum*. It is yet only right to mention that my friend, Mr R. C. Punnett, had more particularly drawn my attention to the question of senescence in general.

[1] I would express my thanks to Prof. Marshall Ward for references to the phenomenon of the fructification and death of bamboos. These merely show that there are no definite experiments. For the facts I have relied on Mr Willis' and other information, which I collected in India and Ceylon.

I have discussed the question of senescence in invertebrates with several zoologists. Mr Bles has mentioned to me the case of an *Alcyonium*, but unfortunately no account of it has, so far as I am aware, been published.

Note on the Influence of Ultra Violet Radiation on the Discharge in a Vacuum Tube having a polished Zinc Electrode. By
WILL C. BAKER, 1851 Exhibition Scholar, Queen's University, Kingston, Ont.

[*Read* 19 May 1902.]

Warburg[1], from his researches upon the influence of radiation on the spark discharge between metal spheres, concluded that although ultra violet light falling on the kathode reduces the "period of delay" (Verzögerungsperiode), it does not sensibly alter the minimum potential difference required to start a spark. In the cases of zinc and brass, however, no satisfactory measurements could be made owing to complications resulting from the Hallwachs effect.

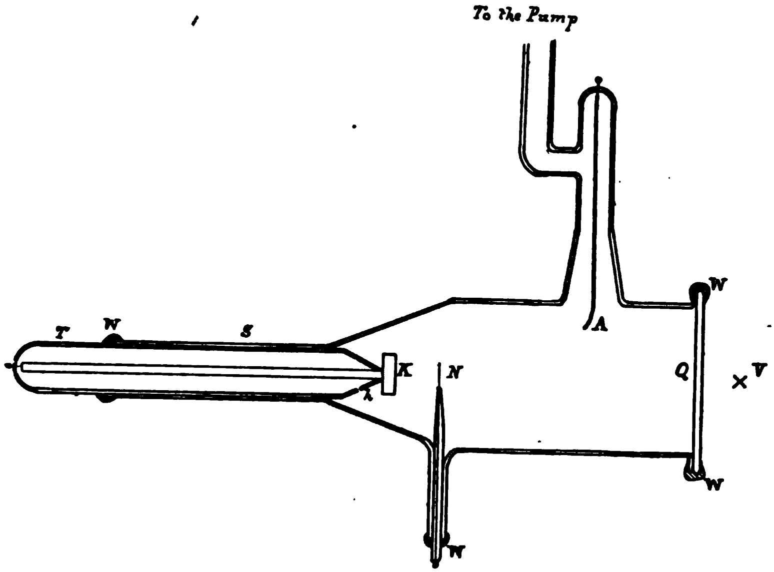
Kreussler[2] found that when a kathode of a metal showing this Hallwachs effect, is raised to a potential but slightly below that required to start a spark to an anode a millimeter away, the effect of ultra violet light falling on it is to liberate a current the magnitude of which rapidly increases as the potential difference between kathode and anode approaches the sparking value.

In view of these results Professor J. J. Thomson suggested the advisability of measuring the "negative drop" in a tube, the polished zinc kathode of which would be illuminated with ultra violet light, in order to determine whether the radiation produced any measurable change.

A tube was constructed as shown in the figure. Q is a window of quartz, 2 mm. thick, opposite which (at V) was placed the source of ultra violet light. The anode A is of copper, and is set in a side tube so as not to obstruct the radiation that was to fall on the kathode. N is a search needle, cased in glass to within 6 mm. of its point. The tube terminated in a piece of smaller tubing shown at S. The zinc kathode K was firmly fixed in a second bit of tubing T, that closely fitted inside S, and which could be slid out to permit the zinc to be polished. A small

[1] *Wied. Ann.* Bd. LIX. no. 1.
[2] *Ann. der Physik*, Bd. v. no. 10, 1901.

hole, h, gave connection between T and the main tube. The joint between S and T was made with sealing-wax (W).



The experiments were made at a pressure giving a dark space of 6 to 8 mm. The source of ultra violet light was the spark from a 6-inch induction coil—with a leyden jar across the secondary—taken between aluminium terminals. No condensing lens was used but the spark gap was placed immediately in front of the quartz window. In order to prevent induction effects from the spark gap and its leads, it was found necessary to enclose them in an earthed metal case, except for a small opening through which the rays passed. The effectiveness of the shielding will be seen from the fact that although the discharge would start in the dark for a potential difference of 555 to 557 volts, and while the radiation was falling on the kathode it would start at 511 volts—a bit of ordinary window-glass placed in front of the quartz window prevented the discharge from starting at 554 volts.

When the discharge was started an electrostatic voltmeter, whose terminals were connected to the kathode (K) and the needle (N), showed a potential difference of 375 volts, and no change could be detected in this when the rays were put on or shut off, though a change of two volts should have been easily seen had it occurred.

A D'Arsonval galvanometer was placed in circuit, but if there was any change in the current when the rays fell on the kathode it was certainly much less than 0·5 per cent. This is not surprising, as Kreussler's[1] greatest observed photo-electric current from zinc was only of the order of 10^{-8} amperes, while the current through this tube was of the order of 10^{-4} amperes.

By shunting the current from part of the cells across a liquid high resistance, and picking off the current at any required point, it was possible gradually to vary the potential difference between the electrodes in the tube. The electrostatic voltmeter connected from one end of the shunt to the anode of the tube indicated the change of potential. The following numbers give the total potential difference across the tube, *i.e.* to the electrometer readings are added the potential produced by the part of the battery not shunted. Where several readings are given they indicate different measurements.

Radiation falling on kathode,

Discharge starts	513,	513,	514.
Discharge will not start	512,	512,	512.

This discharge was very faint and could not be seen when the spark gap was in action, but it continued after the sparks were stopped. The electrometer reading fell three or four volts when the discharge was passing but rose again as soon as it was stopped. As no change in the electrometer reading could be detected on starting the sparks, when the potential across the tube was 512 volts, it was concluded that no measurable discharge passed while the light was on the kathode, and certainly the tube remained perfectly dark after the sparks had been stopped.

Next was measured the lowest potential that would start the discharge in the dark, and this was found to be 557, 558, 558. A thin glass plate was next put in front of the window and the discharge could not be got to start by means of the rays at 554 volts, but out of six or eight trials it went once at 156 and once at 155.

Coming back to a repetition of the first measurements, it was found possible to start the discharge by means of the ultra violet light at 511, 511, 510 volts, showing that the gas was gradually becoming weaker with regard to the potential required to break it down. In the above cases the discharge would not start at 509, 508 volts.

Next, the discharge was started, the rays were cut off, and then the potential was gradually reduced until the discharge ceased. The readings of the electrometer, taken immediately

[1] *Ann. der Physik*, Bd. v. no. 10, 1901.

after the discharge stopped, were 505, 504, 504, but these were always three or four volts lower while the discharge was passing. The same "weakening" of the gas, noticed above, was found here, for a later reading gave 503 volts. It is hoped that this point may be investigated before long, but at present all that can be stated with certainty is the following—:

(1) The "negative drop" was found to be the same, whether the radiation was falling on the kathode or not.

(2) No change was detected in the magnitude of the current when the kathode was illuminated or when the light was shut off. The change—for probably a real change does occur—was at least small in comparison with one of one-half per cent.

(3) A marked change in the potential required to start the discharge was produced by the action of the ultra violet light: the effect being a lowering of the required potential. In the case under examination the reduction was from 558 volts to 510 volts.

(4) By gradually lowering the potential difference after the discharge had been started and then all rays cut off, it was found possible to carry the potential to a point at least three or four volts lower than the minimum potential required to start the discharge by the action of the ultra violet light.

(5) The effect is due to the action of the radiation at the kathode and not—at least not to any appreciable extent—to a volume ionization by the rays as they pass down the path of the discharge.

This last point is shown by making the copper rod the kathode and the zinc electrode positive. A metal screen completely shielded the copper from the light. The following numbers were obtained:

Copper negative, Zinc positive.

291 cells will start the discharge in the dark;
290 cells will not start the discharge in the dark, nor when the rays are passing down the tube.

Zinc negative, Copper positive.

(*a*) In the dark,

268 cells start the discharge;
267 cells do not start discharge.

(*b*) Ultra violet light falling on the kathode,

250 cells start the discharge;
249 cells do not start discharge.

Very little importance is attached to the difference in the potentials required to start the discharge in the two directions in the dark, as the copper rod was in a narrow tube, while the zinc had plenty of space about it.

To Professor J. J. Thomson are due my best thanks for suggesting the investigation and for his ever ready advice and encouragement.

Note on a general numerical connection between the atomic weights. By C. A. VINCENT, B.A., St John's College.

[*Read* 5 May 1902.]

If a list of all the atomic weights in ascending order of magnitude be taken and the order in this list be called n, then the nth atomic weight, from $n=3$ to $n=90$ is given by the equation

$$W=(n+2)^{1\cdot 21}.$$

If the atomic weights are from Clarke's 1901 list with hydrogen as unit, then the greatest difference between the computed and determined value will not exceed 4 units, nor will the error ever be greater than 5°/$_\circ$; in 36 cases the result will not be a unit wrong and in 20 cases will not be 1°/$_\circ$ wrong; the mean error for the whole 58 elements considered is about 1·005, the mean percentage error about 1·6.

By replacing $n+2$ of the above formula by N, and taking N as indicating the order in an augmented list of the elements, the formula may be made to embrace the whole of the 77 elements now definitely known. This necessitates predicting an element between hydrogen and helium, and one between helium and lithium. No other gaps are left till after samarium, when in order to complete the list it is necessary to assume elements in various places, making 15 gaps in all. The 13 gaps introduced after samarium are in general accord with those predicted by the periodic table.

On the variation with the wave-length of the double refraction in strained glass. By L. N. G. FILON, B.A., King's College.

[*Received* 17 June 1902.]

1. It is well known that transparent isotropic substances behave under strain like crystalline bodies. Attention was first called to this by Fresnel (*Annales de Chimie et de Physique*, Vol. XX.) and by Sir David Brewster (*Phil. Trans.* 1816). Since then the phenomenon has been examined, theoretically and experimentally, by M. Neumann (*Abhandlungen der k. Acad. v. Wissenschaften zu Berlin*, 1841, II.; see also *Pogg. Ann.* Vol. LIV.), by Clerk Maxwell (*Trans. R. S. Edin.* Vol. XX. Part I.; or *Collected Papers*, Vol. I.), by Wertheim (*Annales de Chimie et de Physique*, Ser. 3, Vol. XL. p. 156) and by Kerr (*Phil. Mag.* Oct. 1888).

Of these only Wertheim appears to have made any attempt at determining experimentally how this effect varies with the kind of light transmitted. If light passes through a plate of thickness τ, which is subjected to principal stresses P, Q in its plane, these stresses being uniform throughout the thickness, then it is found that the light in traversing the plate is broken up into two rays polarized in the directions of principal stress, and the relative retardation of these rays on emergence is given by

$$r = C \times (P - Q) \times \tau,$$

where C is a coefficient depending only on the nature of the material and on the wave-length of the light used.

Wertheim, from observations of a uniformly compressed block of glass through which he passed successively (i) sodium light, (ii) white light, (iii) white light filtered through a red glass, stated the following law:

The relative retardation in air, measured in centimetres, is constant for all the colours. In other words, if r in the above equation be measured in centimetres in air, the coefficient C should be independent of the wave-length.

This leads to the conclusion that the difference of the two refractive indices is independent of the colour, *i.e.* that the double refraction due to strain exhibits *no dispersion*.

The experiments described below were undertaken with the intention of testing the exactness of this law throughout the whole of the visible spectrum, Wertheim's result being derived from only three kinds of light, viz. sodium, neutral tint, and red glass viewed by transmission, which latter is hardly homogeneous.

2. The apparatus employed in the experiments was as follows. A glass beam *AB* (Fig. 1), about 30 cms. long, 3·65 cms. height and 2·9 cms. thick, was placed on two knife-edges *C, D,*

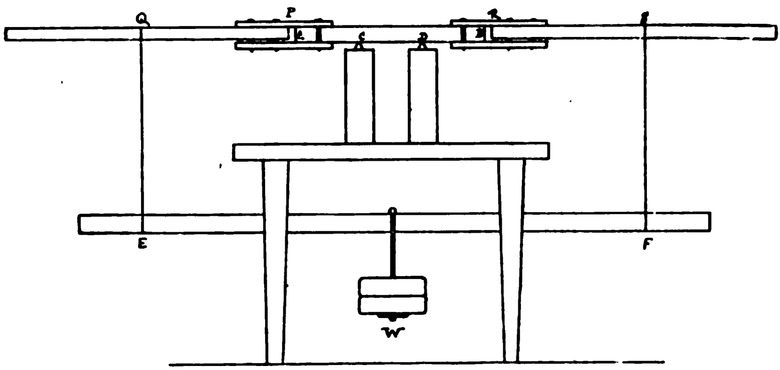


Fig. 1.

12 cms. apart. Long wooden arms *PQ, RS* were attached to the projecting parts *AC, BD* of the glass beam by a sort of fish-plate arrangement shown in Fig. 1.

Suspended from these arms by vertical strings hung a long wooden beam *EF*. When the latter was loaded in the middle with a weight *W*, the effect was to apply forces $W/2$ at *Q* and *S* and therefore a uniform bending moment $Wa/2$ throughout the part of the beam between the knife-edges, *a* being either distance *QC* or *SD*.

Besides this moment $Wa/2$ which could be applied at will there was always a permanent moment on the beam due to the weight of the arms *PQ, RS* and the projecting parts of the beam *AB*. This permanent moment, however, could be calculated, so that the total moment was always known.

The optical part of the arrangement is shown in plan in Fig. 2. Two glass beams such as the one described above were placed side by side. Light from a vertical slit *T* was first rendered parallel by a collimating lens *L*, then passed through a 60° prism *P* and finally focussed by a lens *L′*, after passing through a polarizing Nicol *M* and through the two beams on a scale *s* placed on the side of the front beam nearest the observer.

The scale *s*, which consisted of a network of squares of 5 mm. side, and carried besides one horizontal and one vertical millimetre

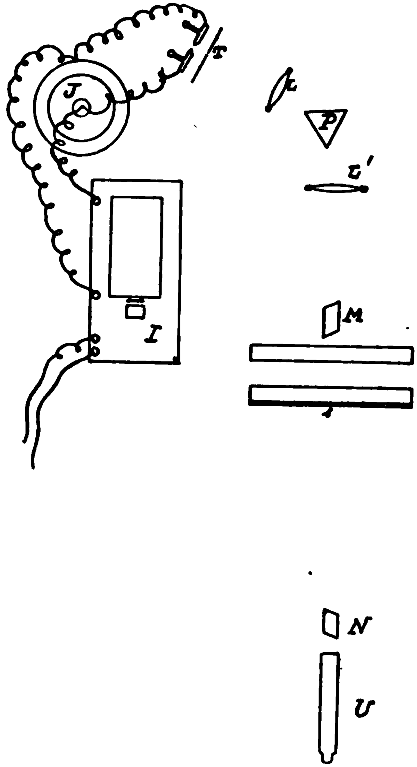


FIG. 2.

scale, was ruled for me on glass with a dividing engine by Mr Edwin Wilson, of Mill Lane, Cambridge. It was viewed through a telescope *U*, in front of which was placed an analysing Nicol *N*.

The slit *T* could be illuminated, either by a powerful arc lamp, or by a spark between metallic poles in air. An alloy of Lead, Tin and Cadmium was employed for this purpose. The use of this alloy was suggested to me by Mr H. C. Ramage, of St John's College, to whom I am also indebted for finding out for me the wave-lengths of a large number of characteristic lines in the spark spectrum of this alloy. This spectrum has a large number of bright lines fairly uniformly distributed from the red to the extreme violet and is accordingly very useful for comparison purposes. In the diagram (Fig. 2) the spark holder is in position

behind the slit, together with the induction coil I, and the Leyden jar J used to reinforce the discharge.

The focal length of the collimating lens L was about 19·5 cms.

The focal length of the collecting lens L' was about 56·5 cms.

The distance of the analyser and telescope from the scale was about 150 cms. The cone of light entering the Nicol and telescope was limited by a circular diaphragm of aperture 1 cm.

The beams used were cut from thick plate glass and were made for me by the London and Manchester Plate Glass Company. They exhibited very little imperfect annealing in the polariscope. The chemical composition of the glass was unknown, but its density was about 2·50.

3. The theory of the appearances seen was as follows. Suppose we have a very narrow pencil of homogeneous light, traversing horizontally, at a distance y above the central line a beam of thickness τ bent under a couple M, the relative retardation will be

$$r = C\tau T,$$

T being the horizontal tension (or pressure) in the beam at distance y from the axis. When the beam is under a pure couple this is the only stress, and it remains constant from cross-section to cross-section.

Now $$T = My/\Delta k^2,$$

Δ being the area of cross-section and k its radius of gyration about a horizontal line through its centroid in its plane.

Hence $$r = \frac{CyM\tau}{\Delta k^2}.$$

The difference of phase is therefore

$$\frac{2\pi r}{\lambda} = \frac{2\pi M\tau}{\Delta k^2}\frac{Cy}{\lambda}.$$

Now if the polarizer and analyser are crossed, with their principal planes at about 45° to the vertical, there will be darkness in the field of view if the difference of phase = a multiple of 2π, that is, when

$$yM\tau C/\lambda\Delta k^2 = n,$$

n being any integer. Accordingly if the narrow pencil be made to converge to a point on a transparent scale and this point be viewed through a telescope or otherwise, it will appear unilluminated.

Now if we can obtain a series of such thin pencils, corresponding to each point of the scale, we shall see, if the light be

homogeneous throughout, the scale crossed by a number of dark horizontal lines. One of these is central and corresponds to $n=0$: the others are parallel to it on either side at distances

$$\frac{\lambda}{C}\frac{\Delta k^2}{M_T},\quad \frac{2\lambda}{C}\frac{\Delta k^2}{M_T},\quad \text{\&c.}$$

If however the light be homogeneous only for each cross-section, but vary gradually in wave-length from cross-section to cross-section, then a continuous spectrum will appear in the field of view, crossed in the direction of its length by black bands, of which the central one is straight and horizontal, whereas the others are curves which close in upon the centre as we move from the red to the violet end.

The ordinates y of these black fringes measured from the straight central one as origin are proportional at each point to the λ/C corresponding to the line of the spectrum considered.

They accordingly map out continuously the variations of λ/C as we go along the spectrum; and if λ can be determined at each point, they give the variation of C.

4. In the actual case we can only approximate to an arrangement of this kind. First of all, in order to double the effect, *two* beams of glass were used, instead of one. These are not of exactly the same dimensions, nor are they exactly at the same vertical height; in both of them, however, the axes of polarization are throughout horizontal and vertical.

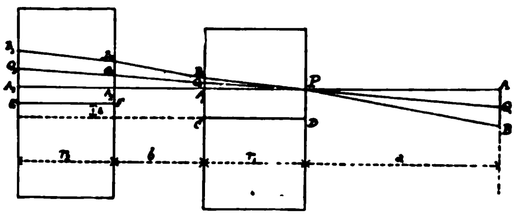


FIG. 3.

In the next place the pencils, which converge on the scale to form a spectrum, are not exactly normal to the sides of the beam, but the extreme pencils are more oblique than the others.

Finally the pencils are not indefinitely thin, but are cones containing a finite solid angle, namely that subtended at the scale s by the circular diaphragm in front of the analyser.

Let us examine therefore how the appearances should be affected by these various departures from the ideal arrangement discussed in the preceding paragraph.

Suppose that AB, Fig. 3, is the circular diaphragm (viewed sideways) in front of the analysing Nicol. Let CD, EF be the

beams, shown in cross-section, whose thicknesses are τ_1, τ_2. Let a be the distance from the scale to the diaphragm, b the horizontal distance between the two beams and ϵ the vertical height of the central axis of the second beam above that of the first.

Then if P be any point of the scale, the pencil of rays which ultimately form an image of P in the focal plane of the telescope is the broken cone bounded by the rays $APA_1A_2A_3$, $BPB_1B_2B_3$ in Fig. 2.

Now in investigating the relative retardations, all that we are concerned with is the path in the strained glass.

Now the paths in the glass may be found by the following construction.

Imagine the beam EF removed a distance $(\mu-1)b$ to the left, as $E'F'$ (Fig. 4), and similarly the diaphragm AB removed $(\mu-1)a$ to the right as $A'B'$. Then the paths of the actual rays in the glass will now lie inside a *straight* cone $A'PB'$, the paths in the two glass beams of any ray which emerges as PQ in the real case being given by the intercepts PQ_1, $Q_2'Q_3'$ of the corresponding ray PQ' in Fig. 4. Consider the relative retardation of the two oppositely polarized parts of the ray PQ'.

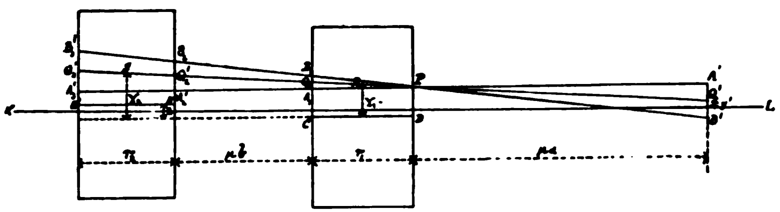


FIG. 4.

This can easily be shown to be

$$\int_{Q_1}^{P} CT_1 \cos^2\theta \sec\psi \, d\tau_1 + \int_{Q_2'}^{Q_3'} CT_2 \cos^2\theta \sec\psi \, d\tau_2,$$

$\frac{\pi}{2}-\theta$ being the angle which the ray PQ' makes with the direction of the axes of the beams and ψ the angle PQ' makes with the horizontal perpendicular to the sides of the beams.

This gives for the retardation

$$C\cos^2\theta \sec\psi \left[\int_{Q_1}^{P} \frac{M_1}{\Delta_1 k_1^2} y_1 d\tau_1 + \int_{Q_2'}^{Q_3'} \frac{M_2}{\Delta_2 k_2^2} y_2 d\tau_2\right],$$

y_1, y_2 being measured vertically from the neutral axes CD, $E'F'$ in the two beams.

This is obviously

$$C \cos^2\theta \sec\psi \left[\frac{M_1}{\Delta_1 k_1^2} Y_1 \tau_1 + \frac{M_2}{\Delta k_2^2} Y_2 \tau_2\right],$$

Y_1, Y_2 being the y_1 and y_2 of the mid-points of the intercepts PQ_1, $Q_2'Q_2'$ respectively.

Denote $\dfrac{M_1\tau_1}{\Delta_1 k_1^2}$ by c_1, $\dfrac{M_2\tau_2}{\Delta_2 k_2^2}$ by c_2

and write

$$Y_1 = Y_1' + \frac{\epsilon c_2}{c_1 + c_2},$$

$$Y_2 = Y_2' - \frac{\epsilon c_1}{c_1 + c_2},$$

so that Y_1', Y_2' are now measured from the same base KL (Fig. 4)[1].

Then the relative retardation

$$= C\cos^2\theta \sec\psi\, [c_1 Y_1' + c_2 Y_2'].$$

Now if y' = height of the point Q' above the same base, KL and y_0 = height of P above this base,

$$Y_1' - y' = (y_0 - y')\left(1 + \frac{\tau_1}{2\mu a}\right)$$

$$Y_2' - y' = (y_0 - y')\left(1 + \frac{2\tau_1 + 2\mu b + \tau_2}{2\mu a}\right).$$

Accordingly

$$c_1 Y_1' + c_2 Y_2' = y_0 \left(c_1 + c_2 + \frac{c_1\tau_1 + c_2(2\tau_1 + 2\mu b + \tau_2)}{2\mu a}\right)$$

$$- y' \left(\frac{c_1\tau_1 + c_2(2\tau_1 + 2\mu b + \tau_2)}{2\mu a}\right),$$

$$= py_0 - qy' \text{ say,}$$

where

$$p = c_1 + c_2 + \frac{c_1\tau_1 + c_2(2\tau_1 + 2\mu b + \tau_2)}{2\mu a},$$

$$q = \frac{c_1\tau_1 + c_2(2\tau_1 + 2\mu b + \tau_2)}{2\mu a}.$$

Thus

$$r = C\cos^2\theta \sec\psi\, [py_0 - qy'].$$

Now, in the experiments, both ψ and θ were small. The focal length of the condensing lens being about 56 cms., the total

[1] The length marked Y_2 in Fig. 4 is measured from CD as base and is equivalent to $Y_2 + \epsilon$ in the text.

length of the spectrum examined was about $4\frac{1}{2}$ cms., so that $\tan\theta$ never exceeded

$$\frac{1}{2}\frac{4\frac{1}{2}}{56} = \cdot 04 \text{ about.}$$

Accordingly $\cos^2\theta$ at most differed from unity by a quantity of the order ·0016 which was quite negligible, seeing the accuracy of the final measurements could certainly not be greater than $\frac{1}{2}$ per cent.

ψ was a quantity of the same order as θ and the effect of the term in ψ will have been to reduce the preceding error by nearly one-half, so that the total error introduced by putting the factor $\cos^2\theta \sec\psi = 1$ will be only about 1 in 1000. This we may certainly neglect.

If we do so

$$r = C(py_0 - qy').$$

Now the intensity due to a *very* thin pencil, whose cross-section at Q is $dx'dy'$, will be

$$I dx' dy' \sin^2 2\gamma \sin^2 \pi r/\lambda,$$

I being a constant and γ being the angle between the axes of the polarizer and analyser and the axes in the glass.

Hence the intensity due to the whole of the light from P which passes through the diaphragm

$$= I \sin^2 2\gamma \int dx' dy' \sin^2 \pi r/\lambda,$$

the integral being taken over the area of the diaphragm.

This gives: Intensity

$$= I\sin^2 2\gamma \int \frac{dx'dy'}{2}\Big(1 - \cos\frac{2\pi C}{\lambda}(py_0 - q\eta')\cos\frac{2\pi Cq}{\lambda}(y' - \eta')$$
$$- \sin\frac{2\pi C}{\lambda}(py_0 - q\eta')\sin\frac{2\pi Cq}{\lambda}(y' - \eta')\Big),$$

η' being the height of the centre of the diaphragm above KL.

The diaphragm being circular, the sine-integral vanishes and the others give

$$I\sin^2 2\gamma\left(\frac{\pi\rho^2}{2} - \cos\frac{2\pi C}{\lambda}(py_0 - q\eta')\frac{\pi\rho^2}{\alpha}J_1(\alpha)\right),$$

where
$$\alpha = \frac{2\pi Cq\rho}{\lambda},$$

ρ is the radius of the diaphragm and J_1 is the Bessel's function of order unity.

Now if $\alpha < \pi \times 1{\cdot}22$, $J_1(\alpha) > 0$.

In our case α is always less than this value, being in fact of the order $\pi/180$ and therefore small. But $J_1(\alpha) < \alpha/2$ and tends to $\alpha/2$ if α be small. So that the intensity never actually vanishes, but it falls to a very small minimum whenever

$$\frac{2\pi C}{\lambda}(py_0 - qy') = 2n\pi$$

or

$$py_0 - qy' = \frac{\lambda n}{C}.$$

This therefore corresponds to a black band in the field of view.

If now $y_0^{(n)}$, $y_0^{(n+1)}$ be the y_0's of two consecutive black bands

$$Cp\,(y_0^{(n+1)} - y_0^{(n)}) = \lambda.$$

This equation is of precisely the same form as in the ideal case previously discussed, except that the coefficient p is slightly different.

Hence, in so far as relative values of C for different λ's are concerned, $y_0^{(n+1)} - y_0^{(n)}$ being the quantity observed, no error whatever will be introduced by the obliquity, or by the finite solid angle of the pencils.

As a matter of fact p is known, as all the quantities which enter into its calculation can be easily measured: so the absolute values can be deduced when required.

5. It was now necessary to find the exact wave-length of the light corresponding to any vertical line on the scale.

To do this the arc lamp, which was used to give a continuous spectrum, was removed and the spark-holder with the poles made of the alloy mentioned above was placed behind the slit.

The spark spectrum of the alloy then appeared on the scale.

The positions of 25 of the most prominent lines of this spectrum, ranging from $\lambda 4000$ to $\lambda 6800$, were then carefully read off on the horizontal millimetre scale. The lines were identified and their accurate wave-lengths obtained.

The wave-lengths were then plotted on squared paper to the scale divisions. A smooth curve was drawn as nearly as possible through the 25 or so points obtained and from this the wave-length corresponding to any given scale division could be read off practically correct to the first three figures.

Before each experiment the "zero" was determined by reading off the position on the scale of a well-defined, easily recognizable, line of the comparison spectrum. The wave-lengths for each observation could then be deduced by means of the diagram.

The spectrum was carefully re-mapped and a new diagram plotted, at certain intervals, especially whenever the lenses, prism or beams had been moved or shaken in any way.

Special attention had to be given to the polarizing Nicol. The latter required to be displaced in order that the whole spectrum might be examined. Care had to be taken always to keep it parallel to itself. This was done by keeping the edge of the square stand of the Nicol flat against a fixed straight edge. In this way the motion of the Nicol could introduce no shift of the lines of the spectrum.

The vertical distances between the black bands were measured by means of a second glass scale which carried a vertical millimetre scale. This was placed in contact with the other scale, but could be shifted freely to the right and left so as to bring the vertical scale into any required horizontal position.

6. For the two beams actually used

$$\tau_1 = 2{\cdot}905 \text{ cms.}, \quad \tau_2 = 2{\cdot}885 \text{ cms.},$$
$$a = 152 \text{ cms.}, \quad b = 6 \text{ cms.}$$

The bending moments, after all the corrections had been applied, were found to be

$$M_1 = 676 \text{ kgms. weight cms.},$$
$$M_2 = 672 \text{ kgms. weight cms.}$$

The heights of the beams were given by

$$h_1 = 3{\cdot}67 \text{ cms.},$$
$$h_2 = 3{\cdot}65 \text{ cms.}$$

Whence c_1, c_2 were calculated to be

$$c_1 = 164{\cdot}1 \text{ kgms. weight per sq. cm.},$$
$$c_2 = 165{\cdot}9 \text{ kgms. weight per sq. cm.}$$

and

$$p = 340{\cdot}8 \text{ kgms. weight per sq. cm.}$$

Accordingly, λ being known and $y_0^{(n+1)} - y_0^{(n)}$ being measured in cms., the absolute value of C can be found in sq. cms. per kgm. weight.

As what is required, however, is the relative value of C for different colours, and as p remained the same throughout the whole series of experiments, the values of $(y_0^{(n+1)} - y_0^{(n)})/\lambda$ have not been reduced to absolute values by multiplying by p in each case, but the quantity studied has been kept in the form $1/Cp$.

The fringes in the field numbered 2 in the red, increasing to 3 in the violet, on either side of the central one. The distances $y^{(1)} - y^{(0)}$, $y^{(2)} - y^{(0)}$, $y^{(3)} - y^{(0)}$ were measured for a number of sections corresponding to divisions of the horizontal scale. A

Feb. 24			Feb. 25		
$y^{(n+1)}-y^{(n)}$ in cms.	λ in tenth-metres	$(Cp)^{-1}$	$y^{(n+1)}-y^{(n)}$ in cms.	λ in tenth-metres	$(Cp)^{-1}$
·670	6250	1070	·693	6440	1077
·645	6040	1070	·657	6210	1057
·615	5860	1050	·623	6000	1039
·595	5690	1045	·615	5830	1055
·570	5530	1030	·595	5650	1053
·555	5390	1030	·575	5500	1045
·540	5260	1025	·560	5360	1045
·525	5140	1020	·550	5240	1050
·510	5030	1015	·535	5120	1045
·495	4930	1005	·515	5010	1028
·485	4830	1005	·505	4910	1029
·475	4740	1000	·495	4810	1029
·470	4650	1010	·490	4720	1038
·462	4570	1010	·480	4630	1037
·452	4490	1005	·473	4550	1040
·448	4430	1010	·460	4480	1027
·436	4370	1000	·454	4420	1027
·430	4310	1000	·450	4360	1032
·424	4260	995	·442	4300	1028
·414	4200	985	·440	4250	1035
·410	4150	990	·426	4190	1017
·406	4100	990	·418	4140	1010
·400	4060	985			

weighted mean was obtained, thus $y^{(2)} - y^{(0)}$ was given double weight and $y^{(3)} - y^{(0)}$ triple weight, since the one represents *two* and the other *three* fringe intervals. This gave a good determination of a mean fringe interval $y_0^{(n+1)} - y_0^{(n)}$ in each case. The result was then divided by the calculated wave-length.

In calculating the latter a small correction has sometimes had to be put in on account of the fact that the spectrum lines were not, in all cases, perfectly straight, owing to spherical aberration in the lenses. In most cases this correction was too small to be applied. When sensible it was applied, so that the wave-lengths given in the table below are corrected.

The results obtained from the first two days' observations are shown in the table (page 488).

These show a steady increase of C from the red to the violet. In other words Wertheim's law does not hold, but the relative retardation in centimetres in air decreases as the wave-length increases.

The total variation is between 6 and 7 per cent. and could not, it seems, be accounted for by errors of observation. The fringes as seen by means of a powerful arc light were sharp and well defined, easily measurable to tenths of a millimetre division. The mean value of $y_0^{(n+1)} - y_0^{(n)}$ should be accurate to the second place, though even an error of one unit there would not account for the difference.

The experiments were repeated a large number of times, with a view to eliminating the accidental errors. The results are exhibited in the table (page 490).

The sets taken on Feb. 24 and 25, those on Feb. 28 (II), of March 5 and that of March 6 are from measures taken on the fringes in the upper part of the beam (under tension). The observations of March 1, 3, 4, 8, 10 and (I) of March 5 are made on the fringes visible in the lower part of the beam (under compression).

The latter do not indicate such a large variation of C as do the other observations. Those on March 1 and March 4 almost confirm Wertheim's law. But those of March 3 show a slight variation, tending to make C increase as λ diminishes; those of March 5 and 8 show a strong variation and those of March 10 a moderate variation in this sense.

All the results corresponding to measurements taken upon fringes in the upper half of the beam decidedly confirm this variation.

Feb. 28. I.		Feb. 28. II.		Feb. 28. III.		March 1		March 3		March 4	
λ	$(Cp)^{-1}$	λ	$(Cp)^{-1}$	λ	$(Cp)^{-1}$	λ	$(Cp)^{-1}$	λ	$(Cp)^{-1}$	λ	$(Cp)^{-1}$
6520	1085	6470	1074	6550	1073	6735	1044	6070	1071	6470	1046
5940	1048	5930	1088	5970	1051	6090	1073	5620	1079	5850	1058
5510	1039	5510	1062	5540	1060	5645	1069	5280	1054	5440	1054
5190	1026	5200	1067	5210	1060	5305	1056	4990	1062	5110	1050
4910	1034	4920	1047	4920	1042	5015	1064	4750	1060	4840	1047
4680	1020	4690	1055	4690	1029	4760	1071	4540	1072	4600	1058
4480	982	4490	1024	4480	1027	4550	1059	4370	1053	4410	1058
4330	970	4340	979	4330	987	4380	1052	4240	1038	4260	1064
4190	973	4200	988	4200	982	4250	1043	4110	1030	4130	1049
4070	977	4100	951	4100	963	4120	1048				

March 5. I.		March 5. II.		March 6		March 8		March 10	
λ	$(Cp)^{-1}$	λ	$(Cp)^{-1}$	λ	$(Cp)^{-1}$	λ	$(Cp)^{-1}$	λ	$(Cp)^{-1}$
6310	1057	6500	1062	6490	1058	6500	1072	6510	1034
5780	1067	5890	1064	5890	1058	5890	1070	5920	1042
5380	1047	5440	1036	5430	1050	5440	1085	5440	1035
5050	1050	5080	1050	5070	1045	5080	1063	5090	1028
4800	1035	4800	1014	4790	1037	4800	1069	4810	1040
4580	1030	4570	1021	4570	1021	4570	1047	4580	1037
4490	1006	4380	1012	4380	997	4380	1016	4390	1018
4230	1048	4220	979	4220	995	4220	1011	4220	995
4100	1015	4070	925	4070	983	4070	988	4080	1005

If in the table last given we take the mean of all the values of $1/Cp$ corresponding to wave-lengths lying inside intervals of 500 tenth-metres, we find

λ from	λ to	Mean value of $(Cp)^{-1}$	Corrg. value of C in sq. cms. per kg. wt.
6750	6250	1060	$2{\cdot}77 \times 10^{-6}$
6250	5750	1063	$2{\cdot}76 \times 10^{-6}$
5750	5250	1056	$2{\cdot}78 \times 10^{-6}$
5250	4750	1048	$2{\cdot}80 \times 10^{-6}$
4750	4250	1027	$2{\cdot}86 \times 10^{-6}$
4250	——	997	$2{\cdot}94 \times 10^{-6}$

It seems therefore probable that, although some hidden source of error (possibly a difference in the behaviour of the glass under tension and under pressure or, again, imperfect homogeneity of the glass) may have partly masked this effect in the observations on the lower fringes, and exaggerated it in the observations of the upper ones, yet the differences obtained are significant.

Wertheim's law is therefore not accurately true and the artificial double refraction due to strain does exhibit dispersion, the difference in the refractive indices being smaller in the red than in the violet by about 6 per cent. (taking mean values).

7. With regard to the law of variation of the coefficient C, the present observations are too rough to allow it to be determined: the table of mean values given above, however, would seem to indicate that C varies far more rapidly towards the violet than towards the red. But, having regard to the probable error of the observations, no definite conclusion can be drawn.

Since the above work was done, Herr F. Pockels has published an account (see *Wied. Ann.* 1902, Ser. IV. Vol. VII. p. 745, "Ueber die Aenderung des optischen Verhaltens verschiedener Gläser durch elastische Deformation") of some extremely interesting experiments on various glasses under compression. He has used light of three kinds, namely the Bunsen flame coloured with Na, Li and Tl-salt.

For heavy glasses Pockels finds a deviation from Wertheim's law of the same kind as the one indicated here, *i.e.* the coefficient C is greater (numerically) for the green and yellow rays than for the red. For lighter glasses this dispersion of double refraction appears to be insensible.

But here it should be noted that, besides the limited range of the observations, which are restricted to three wave-lengths and do not go beyond the green[1], Pockels' method appears open to one objection, when the dispersion of double refraction is small.

He uses a Babinet's quartz compensator: the observations are consequently affected by the dispersion of double refraction in quartz: it then hardly seems satisfactory to introduce, as he has done, a calculated correction for this, especially when, as in some of the cases he treats, the dispersion of double refraction only appears in the final result through this calculated correction.

Accordingly it seemed of interest to publish the present experiments, which are free from this source of error, if only as a confirmation by an independent method.

I am at present undertaking a series of observations, of which it is hoped eventually to publish an account, with a view to determining accurately, if possible, the law of variation of artificial double refraction.

In conclusion I wish to express my thanks to Professor Ewing, in whose laboratory this work has been carried out, and also to Professor J. J. Thomson for kindly letting me use some of the optical apparatus belonging to the Cavendish Laboratory.

I also wish to gratefully acknowledge the kind help which was given me by Mr H. C. Ramage, of St John's College, in connection with the arrangement and identification of the comparison spectrum.

[1] This, I think, accounts for his negative result for glasses of densities between 2 and 3, the greater part of the dispersion being then in the blue end.

EXPLANATION.

The numbers give the depth in fathoms. The letters *m*, *s*, *r* indicate the character of the bottom, *i.e.* mud, sand, rock. *A*, *B*, *D*, *E* are points referred to in the text.

The Coral Reefs of Zanzibar. By CYRIL CROSSLAND, B.A., Clare College.

[*Read* 19 May 1902.]

INTRODUCTION.

My opportunity for observing the facts upon which this paper is based arose from the enthusiasm for science of Sir Charles Eliot, K.C.M.G., H.M. Consul-General at Zanzibar, and Commissioner for the East African Protectorate, who took me out to assist in his work on Opisthobranch Mollusca. To our disappointment we found the neighbourhood of Zanzibar a poor collecting ground. For nearly a year I made my head-quarters on the east side of the island at Chuaka. At low spring tides Chuaka Bay is a wide expanse of bare sand, and though I found these banks extremely rich in both Polychaeta and Opisthobranchiata, I often wished to be nearer deep water for dredging, and to see the more typical conditions of life on the open coast. At Sir Charles Eliot's suggestion I first visited the Mnemba reef, and that journey made me feel the fascination of boat travel so that afterwards I undertook several other excursions. In this way I gained a knowledge of the reefs from Ras Nungwe, the northernmost point of the island, to Ras Mkunduchi[1], which is the south-east corner.

I wish to express my thanks to the Zanzibar Government for the loan of a boat during my stay.

PHYSICAL CONDITIONS, SEASONS.

The tides rise 15 feet at springs, 10 feet at neaps. On the east coast there is always a northerly current, varying from two to four miles an hour in the south-west monsoon, and from one to three in the north-east. This runs through the channel between Zanzibar and Pemba 'with great velocity'[2]. It is the result of

[1] My authority for place-names is T. J. Last, Esq., Slavery Commissioner of the Zanzibar Government, whose knowledge of the language and of the island exceeds that of any other resident.

[2] The African pilot.

the equatorial current of the Indian Ocean meeting the African continent[1]. The tidal currents cause variations in Zanzibar channel, as they run in from the north and south, meeting about half-way along the west coast. It is continuous to the north in the south-west monsoon, but in the north-east monsoon at springs the tidal currents overpower it in all the small channels and harbours.

The south-west monsoon commences in March, and its first two to three months are occupied by the greater rains. The lesser rains fall in October and November, the north-east monsoon beginning at their close. About a week of variable winds occurs at the change of monsoons. The south-west monsoon therefore lasts nine months, the north-east three. The annual rainfall averages sixty inches.

The wind is somewhat variable near the land, becoming easterly every afternoon, and as it reaches its greatest strength usually about this time most wind comes from the south-east.

A continuous heavy swell, such as is found on the west coast of Africa and on the Pacific Islands, does not occur. The surf is due mainly to local winds, and is nearly always present. Indeed I only remember two or three calm afternoons during my stay of a year, though the mornings were often calm. Only one hurricane is recorded, *i.e.* in April 1872, when a destructive cyclone came from the north-east, but the south end of the island escaped. Ordinary storms are so rare that a well-found open sailing-boat will hardly ever be weather-bound in these seas.

The 80° F. isothermal lines for the surface-water of the ocean enclose 40° of latitude in the Indian Ocean. Near the African coast this belt narrows to about 30° of latitude, and Zanzibar lies about its centre[2]. As the winds and currents are from the east warm surface-water bathes these shores continuously.

STRUCTURE OF THE ISLAND.

Zanzibar stands between latitudes 5° 42′ S. and 6° 28′ S., its length being 46 miles, its breadth 20 miles and its distance from the African mainland about the same.

The hundred fathom line lies one mile from the reef edge in the northern half of the east coast, and two and a half miles in the southern half. The chart records no soundings at a distance from land except in the channel between Zanzibar and Pemba island, where a depth of 450 fathoms is marked. The chart gives the bottom as rock or sand, usually the latter.

[1] Hugh R. Mill, "The Realm of Nature."
[2] Isothermal chart. Hugh R. Mill, "Realm of Nature."

Zanzibar is composed of (*a*) hard coral limestone, (*b*) white or yellow chalky deposits, (*c*) sand, (*d*) red earth.

A very soft sandstone is found occasionally, and harder beach sandstone occurs at a few places on the coast, and in particular near the town[1]. On the east this formation is insignificant.

The red earth is undoubtedly formed in the same way as in all other cases of its occurrence, viz. by the disintegration of coral rock, and the chalky deposits probably have a similar origin.

The bulk of these rocks is confined to the western and central areas. Coral limestone forms the mass of the island, including the whole of the eastern side. It also forms the nucleus of the sandbanks and reefs of the channel, *e.g.* Mwamba Bawe and Pange, which were doubtless formed by the removal of islands of this rock. The present small islands standing on large rock flats, such as Prison Island, Kebandiko and Chapani represent stages in island formation.

The distribution of cliffs of this rock on the coast is shewn by a black line in the appended map, the thinner shore line representing a sandy beach. In the latter regions remains of coral-rock cliffs occur frequently as a line of pinnacles between the beach and the reef flat, or as a line of undermined cliff more or less buried in the sand of the beach. At Muyuni the little village with its grove of coconut palms is situated on such an accumulation, and the top of the cliff is traceable for half a mile behind the village, emerging from the sand at either end. No such definite remains occur on the Paje and Jembiani beaches, though directly one goes inland the ground rises slightly and its surface is of coral rock.

I conclude therefore that when first the island was upheaved the east coast presented a uniform line of coral rock, the present variations being due to the accumulation of sand by the sea.

The cliffs are never high, 20 feet being the maximum in exposed situations, their height remaining very uniform. They are deeply undermined always, but falls of rock are rare. The hardness and density of the rock cause the whole overhanging mass to break off in one piece, and the fallen mass is then itself undermined. I have observed a casuarina tree, with a trunk four inches in diameter, which had grown on a fallen block the commencement of whose undermining was just visible.

This undermining is due to wave erosion, since the bare rock, continuous with that of the cliffs, is left as a level platform at the base.

The coral rock which rings to the hammer, is hard, crystalline, yellowish or white when broken, but on the surface dark,

[1] Rounded pebbles of quartz occur in these beds.

sometimes almost black. Empty spaces or pockets of red earth are often included in it. Fossil corals, shells and echinoderm spines are abundant[1], differing little from the remains of animals living in the adjacent seas. Corals are found embedded in the cliffs still in the position in which they grew. The surface is always pitted and covered with sharp points and ridges like the broken surface of a slag. Small caves are frequent along the coast, and many rocks have most fantastic shapes. The cliffs of Ras Juja in Chuaka Bay are cut to form an area covered by pinnacles close together, whose tops reach the usual height of the cliffs, and whose bases rise from a rock platform at low tide level. Mangroves, which often grow on the rock in Chuaka Bay, stand thickly among them.

The sand and chalk formations lie usually above the coral, forming most of the surface of the central plateau, and so far as I have seen, all of that of the highest hills, which attain a height of 440 feet. Hills composed mainly or entirely of coral rock only occur in the north, south and east, the highest of which is Kidoti Hill in the north-west, whose summit of coral rock is 250 feet above sea level. The extensive eastern plains are composed almost entirely of coral rock, and are interrupted by only a few isolated hills of the same from 70 to 200 feet in height.

The sheltered west coast is a succession of islands, shoals and long bays. The eastern, or ocean coast outline, would have a regular contour line but for Mnemba Reef and Chuaka Bay. The latter is five miles wide, and at low springs is mainly an expanse of sand. The creeks at its head run inland for a considerable distance along a depression which crosses the island to Menai Bay on the south-west coast.

Mnemba Reef is separated from Zanzibar by a channel, one and a quarter miles wide, having an average depth of 40 fathoms, being thus more deeply separated than is Zanzibar from the African mainland. The island of Mnemba, situated on the south-west of the reef, is about a quarter of a mile long, composed entirely of fine loose sand in which casuarinas and pandani grow.

The Reefs of the East Coast.

The outer edge of the whole reef lies at a height of 2 feet above the mean low tide level, being thus 3 feet above the lowest springs. The distance of its highest part to the lowest

[1] In this abundance of fossils this rock differs from the otherwise very similar raised coral rock described by Gardiner in the Lau group, Fiji, *Proc. Camb. Phil. Soc.* vol. IX. Part VIII. On Funafuti, Rotuma, and Fiji, p. 457.

tide level is about 10 yards, forming a regular slope, the corresponding slope to the boat channel being rather steeper. The outline of the reef edge is straight, in marked contrast to the irregularities and fissures characteristic of all growing reefs as described by Darwin[1], Dana[2], and Gardiner[3], and of the small ones in Zanzibar channel and off certain islands near the African mainland.

Beyond the breakers is a steep descent (Admiral Wharton[4] speaks of it as "wall-like") into from 8 to 30 fathoms, the latter depth being generally found within three-quarters of a mile of the edge. Though these depths are comparatively small, the existence of blue water close to the reef edge forms a striking contrast to the west coast.

Loose stones, up to 4 feet in diameter, formed of hard limestone absolutely similar to that which forms the cliffs may be present. The dotted line on the reef edge shews where they occur. If present they are numerous, especially on the highest point, though never covering the surface, and they occur on both slopes. They may be entirely absent over miles of the edge as indicated on the chart. They are never found in the boat channel or on the reef flat.

The surface of the edge is soft, even crushing underfoot sometimes. When broken it gives the impression of a shallow layer of rounded stones held together by a soft, gritty cement. At a depth of a few inches is hard rock similar to that of the cliffs.

A similar edge is found on the east and south sides of the Mnemba reef, but not on the sheltered western side, nor on the shore of the adjacent mainland.

Coral is practically absent, entirely so from the outer slope even among the breakers, and beyond to a depth of at least 5 fathoms are bare rock (on which grows a marine phanerogamous plant) and sand. I only saw clumps of a single species of coral at one point (*B*), two or three freshly dead fragments on the south end of the Mnemba reef, these being the only traces of coral living outside the reef edge.

Gardiner[5] having shewn the great importance of nullipores (*Lithothamnion*) on reefs, I observed carefully the extent of their occurrence here. Though encrusting forms are frequent on the reef edge they are far from covering its surface, and never form a thickness of more than a quarter of an inch, and even then

[1] Darwin, the Keeling Atoll, *Coral Reefs*, p. 2.
[2] Dana, *Corals and Coral Islands*, p. 136.
[3] Gardiner, *loc. cit.*; and The Formation of the Maldives, *Geog. Journal*, March, 1902.
[4] The African pilot.
[5] Funafuti, Rotuma, and Fiji, *loc. cit.*

the hard nodule may break underfoot, soft gritty sand being found underneath. They never cover more than half the area of the upstanding stones, where they grow so thinly that all the markings and roughness of the stone are visible through them as through a thin coat of paint. The rest of the stone is quite bare, having not even a covering of brown or green alga. The colour of the nullipores here is dull and unhealthy looking in contrast with the bright, clean appearance they have in certain places immediately under the cliffs and on the rocks of recent growth found in the boat channel. In any case the stones are partially exposed to erosion and must in course of time disappear. The addition made to the reef by nullipores is thus negligible; the reef edge is not growing but is formed of the same dead rock as forms the greater part of the island.

The colour of the outer slope is generally brown, from the presence of filamentous *Phaeophyceae*, or sometimes green predominates owing to the presence of scattered dwarf clumps of Enteromorpha. These algae protect the surface of the rock from further erosion by holding a layer of sand, matted together by their basal parts, against the rocks.

The Boat Channel.

There are but few depressions in the reef edge, so that at low tide water is held on the reef at a higher level than that of the water outside, causing strong tidal currents to run over the reef-flat to the openings. A channel has thus been hollowed out, which may be over 6 feet deep near an opening, but at a distance of a few miles dwindles and finally disappears. One such channel commences near Ras Mkunduchi, attains a depth of 6 feet at ordinary low tides opposite Jembiani, and empties at the point marked by a small arrow. Another commences at *A* and empties to the north of Ras Nungwe, and similarly along the whole reef.

Those openings between the channel and the sea which I saw at the points *D* and *E* were due to a collapse of the surface layers of rock, their surrounding slopes being cracked exactly as is the ice on a pond from which the water has been run off.

The rock flat has been eroded over the greater part of its breadth, though to a depth of 6 feet or more only near its outer side. But owing to deposition of fine and coarse sand, and to coral growths, the actual channel is usually much narrower.

Fine sand occurs in great quantity off Paje and Jembiani, forming the beach and the surface of the inner two-thirds of the reef. Through this winds the river-like channel of muddy water.

The sand is somewhat discoloured by organic matter on the reef, but on the beach it is almost white, and when dry the surface assumes the forms characteristic of that of dry powdery snow under the influence of the wind.

Further north, for several miles to the south of Ras Michamye, the sand almost disappears, except on the shore side. Here (Dingwe) the channel is from 4 to 8 ft. deep. On either side of it are here and there large flat-topped cylinders of coral (*Porites*) from 6 to 12 feet each way, growing at the sides but dead at the surface of the water. This genus flourishes where other corals are absent, *e.g.* also in the channel at Ras Nungwe. Beyond the channel is a gentle slope from 2 feet of water up to the undermined cliffs which form the shore.

A peculiarity of the reef to the north of Ras Michamve[1] is the way it is scooped out into big cylindrical basins, averaging 12 feet across by 6 deep, which are often so close together that their edges form only a narrow flat winding pathway across the reef. They are full of clear water, which flows from one to another or may be stationary. I expected corals to flourish on their sides, but they, and most other organisms, are quite absent. The shore side of the reef is a long flat of barren rock coated with slimy mud.

Chuaka Bay is at low springs mainly an expanse of sand-banks on which are large beds of *Zostera* (three species) and of a small stiff *Halimeda*. The central bank is covered with large, flat sponges, lying unattached, which one turns over in collecting as one would stones. A little coral is found below low tide mark at the mouth of the bay. All the surface which is not sand or mud is bare rock, comparatively barren of organisms. From the *Halimeda* and *Zostera* beds however I have collected about 100 species of Opisthobranchs, and the sand, but not the mud, is very rich in Polychaetes.

Halimeda is a factor of no importance in rock formation here. The dead "leaflets" cover the surface of the beds, but remain whole only to the depth of an inch or so, disintegrating below to a fine grey mud which in exposed situations would be swept away at once.

In many places north of Chuaka Bay the boat channel is more or less filled with coarse sand matted together by the roots of a dwarf *Zostera*. The sand is cut out into basin-like pools reaching down to the rock. This is so near Muyuni over the inner third of the reef flat, and to the south it forms the greater part of the reef surface for several miles.

[1] The reef edge is here again stony, the blocks being somewhat larger than I have seen elsewhere. It descends gradually below low tide level.

About Muyuni the boat channel tends to be filled up by blocks of stone formed by coral and nullipore growths. These occur mostly on the outer third of the reef, where they form as it were a giant pavement level with the surface of the water, the interstices of which may be 1 or more feet wide by 2 to 4 deep, containing clear water and floored with clean coarse sand which often but thinly covers the underlying rock. Scattered blocks may occur anywhere. Some are dead, but most are living round the edges, which overhang considerably. Nullipores flourish exceedingly, both on the surface and under the edges, encrusting and foliose forms covering every particle of non-living surface, down to the dead "leaflets" of *Halimeda*. Various genera of coral, *e.g. Pocillopora, Madrepora, Pavonia* and *Galaxea* are found on each block, though their interior appears homogeneous soft white rock bored by mollusca and worms. Caulerpas and Fucaceae also grow on it, giving this part of the reef a very rich appearance, in marked contrast to its usual barrenness.

Obviously if the growth of these blocks continues far enough, so that the spaces between them become obliterated, a bare flat surface of dead rock is again formed. A month or two later while travelling near Pongwe Bay I found that this obliteration had taken place. The inner third of the reef consists of old limestone and slimy mud banks. When the boat channel attains a depth of about three feet, scattered growing blocks occur similar to those near Muyuni, except that less coral and more nullipore in proportion grow on them. The smaller blocks at least, such as I was able to overturn, are not attached to the substratum. Further out these become more numerous and fewer are living, until a continuous surface is formed, having all the appearance of the ordinary reef flat of old rock, but for its softness and the presence of many deep holes and channels with overhanging edges. Close to the reef edge is an area of shingle—rounded by the growth of nullipores—partly loose, partly cemented. All this recently formed rock surface is level with that of the water in the boat channel, *i.e.* the same as that of the reef edge.

The Mnemba Reef has a continuous raised stony edge along its east and south sides. Its surface gives instances of everything shewn by the reefs of the main island, though no definite channels occur. The west side is of sand, similar to that at Jembiani but cleaner, descending gradually into several feet of water, after which is a steep slope down to about 50 fathoms. On the southern parts are growing blocks like those near Muyuni and Pongwe, and on the north rock-pools like those off Ras Michamve.

This reef shelters a few miles of the Zanzibar shore from the action of the surf, and consequently this shore is neither so broad

as usual, nor is it in the form of a regular reef. Its breadth is in fact only one-fifth of a mile and the reef edge ceases just south of Muyuni, commencing again at the point marked *B*. Between these points is an irregular shore of hummocky old rock and sand. No coral grows at low tide, though close to deep, clear water.

Zanzibar Channel.

The numerous islands and rocks of the channel are all formed of coral rock, the former being deeply undermined, standing on flats of the same rock whose extent shews the original size of the islands. In the course of time the islands will disappear, as in the case of the majority of the reefs and sandbanks of the channel, which have been formed in this way, thus the sandbank Pange still shews a nucleus of limestone. Coral flourishes round all these flats, *e.g.* on the sandbanks at the S.W. entrance to Kokotoni Harbour, Mungopwani, Prison Island, Pange, Murogo and Kisiki. Round the latter four its growth has formed a low precipice surrounding the sandbanks, whose top is a foot or two below low tide level, but little fresh surface has been formed. As nullipores are not abundant in any of these places it is doubtful whether real rock is being formed.

Conclusions.

The island of Zanzibar was formed as part of a great barrier reef on the coast of East Africa, which is here exceptionally distant from land in correspondence with the outward bend of the 100 fathom line. Coral exists at present at a level of 250 ft. above the sea, but the original elevation must have been much more, the commonness of the red earth and underground caves, being an indication of considerable rain erosion.

In addition to upheaval is the lowering of the level of the ocean, which Gardiner estimates at about 14 feet and to which he attributes the formation of many atoll islands[1]. It is interesting that if the ocean returned to this level large areas of the eastern parts of Zanzibar and Pemba islands and of the mainland would be submerged at high tide.

The fringing reefs of the east coast have not been formed by recent growth, but are due to the erosion of the raised rock by the waves.

[1] Gardiner, on Funafuti, Rotuma, and Fiji, *Proc. Camb. Phil. Soc.*, vol. IX. Pt. III.; Formation of the Maldives, *Geog. Journ.*, March 1902.

The height of the raised edge is that of the original shore platform, the boat channel being a secondary formation. Darwin[1] gives instances of sandstone formations which simulate coral reefs, *e.g.* a bar off Pernambuco, explaining the formation of their boat channels by the removal of the inner parts of reefs and sandbanks by the currents which return the water thrown upon them by the surf. The fewness of the depressions in so long a reef would make both these and the tidal current here exceptionally powerful. Near low tide while the edge of the reef is out of water the tidal current in the boat channel is running most strongly. Several times when travelling at spring tides where the boat channel is shallow or obstructed I have had to wait six hours for water to float the boat. This means that water (often muddy) is pouring over the reef flat for at least six hours longer than over the raised edge, and these hours are stated by Dana to be the period during which wave action is least powerful.

This explains the absence of stones similar to those on the reef edge from the channel and flat. These stones must have originally been evenly distributed over the platform, as they are remnants of the hardest portions of the rock which has been removed. It is impossible to believe that such waves as occur on this coast could break into this rock, which can be quarried for building purposes only with difficulty. Their absence from certain areas of reef edge is only explicable on the former theory of their origin. It is hardly possible that numerous stones should be broken from one part of the reef while none at all are from others which are exposed to the same surf. But it is conceivable that some parts of the original reef should contain specially hardened portions of rock, which might be absent from other areas.

It is remarkable that the reef edge should not be growing. All the physical conditions seem favourable to coral growths. Their absence from the Somali coast (north of the equator) is explained by the presence there of an off-shore current and a consequent rising of cold water from the depths to take its place. But here we have an on-shore current from the surface of the ocean under the equator. Mud is fatal to corals, unless kept in motion by a current. The bottom is visible here only to the depth of 5 fathoms, so that the water is dirty compared to that around the Pacific Islands[2]. But the good effect of the current one would expect to neutralise the evil of the mud, as it does on the sand-banks at the south entrance to Kokotoni harbour. The water here is positively thick, yet corals flourish, since strong tidal currents are continually flowing through these narrows.

[1] *Corals and Coral Islands*, pp. 72 and 73.
[2] Gardiner, on Funafuti, etc.

Another instance of the good effect of moving water upon corals is the fact that the only place where corals occur near Zanzibar town is in and around certain clear salt-water springs on the shore of Mbweni Bay, the rest of the shore being bare mud. Darwin[1] quotes similar inexplicable absences of coral reefs from large areas, viz. west coast of S. America, the Galapagos islands, *Gulf of Panama*, west coast of Africa and islands of the Gulf of Guinea, St Helena, *Ascension*, Cape Verdes, *St Paul's*, *Fernando Noronha*. He remarks that too great variation of temperature, recent volcanic action, may account for some of these cases, but he "cannot see that," in the cases printed in italics, "the absence of coral is explicable through any known cause."

The islands of Bermuda[2] afford a most interesting comparison to Zanzibar. They are composed of aeolian limestone whose properties are similar in many ways to those of raised coral, thus the forms of rocks exposed to erosion and their pitted surfaces, as shewn in Verrill's and Heilprin's[3] photographs, recall the rocks of East Africa very strongly. There is no question of the origin of these reefs by erosion, living coral forming only "an imperfect coating" (Verrill) to the aeolian limestone. Erosion has formed not only a semblance of fringing reefs, but has gone so far as to make what Verrill names a "pseud-atoll."

I wish to express my gratitude to Mr Stanley Gardiner, who has given much help by discussing and reading this paper.

[1] *Coral Reefs*, p. 81.
[2] Verrill, *Amer. Journ. Sci.*, Ser. 4, vol. IX. 1900, p. 313.
[3] Heilprin, *The Bermudas*, The Contemporary Publishing Co., Philadelphia.

On Induced Radio-activity. By Professor J. J. THOMSON.

[*Read* 3 March 1902.]

The investigation was undertaken with the intention of seeing whether the 'induced radio-activity' shown by a metal rod after long continued negative electrification in the open air would occur if the rod were placed in a closed vessel instead of outside in the open air. The closed vessel was a zinc gasometer 102 cm. high and 75 cm. in diameter; the vessel was insulated and used as one of the electrodes, the other electrode was a metal tube placed at the axis of the cylindrical gasometer. A potential difference of 800 volts between the cylinder and this rod was produced and the current between these electrodes was measured. This current was 'saturated' and was therefore a measure of the total ionization in the gas in the vessel; if the rod became radio-active the ionization and therefore the current would increase.

The current was measured in the morning, and the rod in the vessel kept connected with the negative terminal of a Wimshurst machine for 6 or 7 hours, when it was disconnected from the machine and the current again measured; if the gas in the vessel were not exposed to Röntgen rays whilst the rod was negatively electrified I was not able to detect any increase in the current through the gas as the result of the long negative electrification: if however the gas were exposed to Röntgen rays during the negative electrification of the rod, then a well-marked increase in the current took place—the increase being some 16 or 17 per cent.: this increase was due to some alteration in the rod and not to a change in the gas in the vessel, for if a rod similar to the one which had been electrified but which had not itself been electrified were substituted the current sank to its former value. No increase took place in the current if the rod were positively electrified.

A number of experiments were made on the currents through the vessel when the vessel was not exposed to rays and when the rod was not electrified. Rods of different sizes and different metals were tried—these all gave approximately the same current; if the rod were carefully wrapped round with dry filter paper the current showed a decided increase, while if the filter paper were damp the current was many times its value for the bare rod: the current in this case is greatest when the negative ions move up to the paper-covered rod—a large effect is also produced when the paper is wetted with brine or alcohol, but a solution of H_2O_2 produces by far the largest effect yet found.

On the increase in the electrical conductivity of air produced by its passage through water. By Professor J. J. THOMSON.

[*Read* 5 May 1902.]

In continuation of the experiments[1] brought before the Society last term the author investigated the effect produced on the conductivity of air by bubbling it through water. The air from a large gas-holder of about 350 litres capacity was bubbled vigorously through water by making the air in the vessel circulate through a water pump: this treatment increased the conductivity of the air, and when the bubbling had been going on for some time the conductivity of the air was 10 or 12 times the initial conductivity.

When once the air has been put in this highly conducting state it stays in it for a very considerable time; a large part of the conductivity produced by the bubbling remains in the air 48 hours after the bubbling has ceased; nor does it disappear when an intense electric force is kept applied to the gas. The effect produced by the passage of the air through water is similar to that which would be produced if the bubbling produced a radio-active 'emanation' similar in properties to those emitted by thorium and radium. The conducting gas can be passed from one vessel to another; it retains its conductivity after passing through a porous plug: passage through a long tube heated to redness destroys the conductivity; it takes however a very high temperature to do this, temperatures less than 300° or 400° C. seem to produce comparatively little effect; if the gas is passed very slowly through a long tube filled with beads moistened with sulphuric acid the conductivity is destroyed; unless however the stream of gas is very slow, the air retains a good part of its conductivity in spite of the sulphuric acid. Another point of resemblance between the 'emanation' from radio-active substances and a gas in this state, is that if a strongly negatively electrified conductor be kept in the gas for some time the conductor becomes radio-active: this activity was only reduced by about 20 % when the conductor was washed in water and then heated in the flame of a Bunsen burner; the radio-activity reduced in this way disappears in the course of a few hours.

[1] *On Induced Radio-activity*, page 504.

PROCEEDINGS AT THE MEETINGS HELD DURING THE SESSION 1901—1902.

ANNUAL GENERAL MEETING,

October 28th, 1901.

MR J. LARMOR, VICE-PRESIDENT, IN THE CHAIR.

The following were elected officers for the ensuing year:

President:

Professor A. Macalister.

Vice-Presidents:

Mr J. Larmor.
Mr W. Bateson.
Mr D. Sharp.

Treasurer:

Mr H. F. Newall.

Secretaries:

Mr A. E. Shipley.
Mr S. Skinner.
Mr H. M. Macdonald.

Other Members of the Council:

Sir G. G. Stokes.
Mr A. C. Seward.
Mr G. T. Walker.
Professor Liveing.
Mr F. Darwin.
Dr E. W. Hobson.
Mr A. Hutchinson.
Mr C. T. R. Wilson.
Mr J. Graham Kerr.
Professor J. J. Thomson.
Mr H. J. H. Fenton.
Mr A. Berry.

The names of the Benefactors were recited.

The following was elected a Fellow of the Society:

H. A. Wilson, B.A., Trinity College.

The following were elected Associates of the Society:

W. C. Baker, B.A. [Kingston], Non-Collegiate.
V. J. Blyth, B.A. [Glasgow], Emmanuel College.
J. A. Cunningham, B.A. [Dublin], St John's College.
J. J. Durack, B.A. [Sydney], Trinity College.
T. Lyman [Harvard University].
J. A. Patterson, B.A. [Toronto], Emmanuel College.

R. S. Whipple was re-elected an associate for a second period of three years.

The following Communications were made:

1. Notes on Minerals from the Lengenbach Binnenthal. By R. H. Solly, M.A., Downing College.

2. Some remarks on the notion of Number. By Dr E. W. Hobson, F.R.S., Christ's College.

3. The Hall Effect in Gases at Low Pressures. By H. A. Wilson, B.A., Trinity College.

4. On some problems in Electric Convection. By G. T. Walker, M.A., Trinity College.

5. On some phenomena connected with the combination of Hydrogen and Chlorine under the influence of light. By P. V. Bevan, B.A., Trinity College.

November 11*th*, 1901.

In the Optical Lecture Room.

Mr W. Bateson, Vice-President, in the Chair.

The following was elected an Associate of the Society:

Mr Edwin Wilson.

The following Communications were made:

1. The Unit of Classification for Systematic Biology. By H. M. Bernard, M.A. (Communicated by Mr A. E. Shipley.)

2. An Exhibition of Fishes and Amphibia to illustrate new methods of mounting specimens for Museums. By J. S. Budgett, M.A., Trinity College.

3. Notes on the Embryology of *Sagitta*. By L. Doncaster, B.A., King's College.

November 25*th*, 1901.

In the Cavendish Laboratory.

Mr J. Larmor, Vice-President, in the Chair.

The following were elected Fellows of the Society:

L. Doncaster, B.A., King's College.
L. N. G. Filon, B.A., King's College.
G. H. Hardy, B.A., Trinity College.
W. F. Lanchester, M.A., King's College.
F. H. A. Marshall, B.A., Christ's College.
O. W. Richardson, B.A., Trinity College.

The following Communications were made:

1. The negative radiation from hot Platinum. By O. W. Richardson, B.A., Trinity College.

2. On the ions produced by incandescent Platinum. By Professor J. J. Thomson, Trinity College.

3. On the action of incandescent metals in producing Electric Conductivity in Gases. By J. A. McClelland, B.A., Trinity College.

4. On the Seminvariants of Systems of Binary Quantics, the order of each quantic being infinite. By Major P. A. MacMahon.

5. On the Zeros of Polynomials. By J. H. Grace, M.A., St Peter's College.

6. The Type specimens of *Lyginodendron Oldhamium* (Binney). By E. A. N. Arber, B.A., Trinity College. (Communicated by Mr A. C. Seward.)

January 20th, 1902.

In the Optical Lecture Room.

PROFESSOR MACALISTER, PRESIDENT, IN THE CHAIR.

The following were elected Fellows of the Society:

K. C. Browning, M.A., St John's College.
C. Forster Cooper, B.A., Trinity College.
A. Young, M.A., Clare College.

The following Communications were made:

1. On the Question of Predisposition and Immunity in Plants. By Professor H. MARSHALL WARD, F.R.S., Sidney Sussex College.

2. On the Genito-urinary Organs of Dipnoan Fishes. By J. GRAHAM KERR, M.A., Christ's College.

3. Further observations on the Biological Blood Test. By G. H. F. NUTTALL, M.A., Christ's College.

February 3rd, 1902.

In the Chemical Laboratory.

PROFESSOR MACALISTER, PRESIDENT, IN THE CHAIR.

The following were elected Fellows of the Society:

J. F. Cameron, M.A., Gonville and Caius College.
R. S. Morrell, M.A., Gonville and Caius College.
H. W. Marett Tims, B.A., King's College.

The following Communications were made:

1. Oxidation in presence of Iron. By H. J. H. FENTON, M.A., Christ's College.

2. Decomposition of Hydrogen Peroxide by light. By R. F. D'ARCY, M.A., Gonville and Caius College.

3. Note on the decomposition of Oxalacetic Hydrazone. By H. O. JONES, B.A., Clare College, and O. W. RICHARDSON, B.A., Trinity College.

4. (1) The formation of di-nitro-phenoxazines. (2) The interaction of thiocyanates, picryl chloride and alcohols. By J. C. Crocker, B.A., St John's College.

5. Oxidation of Glucosone to Trioxybutyric acid. By R. S. Morrell, M.A., Gonville and Caius College.

6. Note on the Reduction of a Ternary Quantic to a symmetrical determinant. By Professor A. C. Dixon, Trinity College.

February 17*th*, 1902.

In the Cavendish Laboratory.

Professor Macalister, President, in the Chair.

The following were elected Fellows of the Society:

J. H. Jeans, B.A., Trinity College.
R. C. Punnett, B.A., Gonville and Caius College.
W. A. D. Rudge, B.A., St John's College.

The following Communications were made:

1. The Histology of the Endosperm during germination in *Tamus communis* and *Galium tricorne*. By W. Gardiner, M.A., Clare College, and A. W. Hill, M.A., King's College.

2. Demonstration on the Dimorphism of the Foraminifera (with Lantern Slides). By J. J. Lister, M.A., St John's College.

3. On the Differentiation and Integration of Divergent Series. By G. H. Hardy, B.A., Trinity College.

March 3*rd*, 1902.

In the Cavendish Laboratory.

Professor Macalister, President, in the Chair.

The following Communications were made:

1. On a method of increasing the sensitiveness of Michelson's Interferometer. By H. C. Pocklington, M.A., St John's College.

2. The influence of currents in metals on reflected and transmitted light. By P. V. BEVAN, B.A., Trinity College.

3. (*a*) On the conductivity of the vapours of the alkali metals. (*b*) On Induced Radio-activity. By Professor J. J. THOMSON, Trinity College.

4. On the Hall Effect in Gases at Low Pressures. (Second paper.) By H. A. WILSON, B.A., Trinity College.

5. On the extraction of the Gases from one cubic centimetre of Blood. By J. BARCROFT, M.A., King's College.

6. On the Coefficient of Mutual Induction between a circle and a circuit with two parallel sides of infinite length. By G. F. C. SEARLE, M.A., Peterhouse.

7. Notes on Semper's Larvae. By K. RAMUNNI MENON, B.A., Christ's College. (Communicated by Mr A. E. Shipley.)

May 5th, 1902.

In the Optical Lecture Room.

MR J. LARMOR, VICE-PRESIDENT, IN THE CHAIR.

The following Communications were made:

1. Regeneration in *Samia ailanthus*. By H. H. BRINDLEY, M.A., St John's College.

2. On the "Unit of Classification for Systematic Biology." A reply to Mr Bernard. By J. STANLEY GARDINER, M.A., Gonville and Caius College.

3. Remarks on Marconi's system of Telegraphy. By H. M. MACDONALD, M.A., Clare College.

4. Note on a general numerical connection between the atomic weights. By J. H. VINCENT, B.A., St John's College.

5. On Trinodal Quartics. By A. B. BASSET, M.A., Trinity College.

6. On a definite Integral. By T. J. I'A. BROMWICH, M.A., St John's College.

7. Reflexion and transmission of light by a charged metal surface. By P. V. BEVAN, B.A., Trinity College.

8. On Radio-active Rain. By C. T. R. Wilson, M.A., Sidney Sussex College.

9. On the increased conductivity of air produced by its passage through water. By Professor J. J. Thomson, Trinity College.

May 19*th*, 1902.

In the Optical Lecture Room.

Professor Macalister, President, in the Chair.

The following Communications were made:

1. Some observations on Protandry and Senescence in *Flabellum*. By J. Stanley Gardiner, M.A., Gonville and Caius College.

2. A note on the dispersive power of running water on Skeletons: with particular reference to the skeletal remains of *Pithecanthropus erectus*. By W. L. H. Duckworth, M.A., Jesus College.

3. The Coral Reefs of Zanzibar. By C. Crossland, B.A., Clare College.

4. On an attempt to detect the Ionisation of Solutions by the action of Light and Röntgen Rays. By J. A. Cunningham, St John's College.

5. On the influence of molecular attraction on collisions. By O. W. Richardson, B.A., Trinity College.

6. On the influence of Ultra-Violet Radiation on the discharge in a Vacuum Tube having a polished zinc electrode. By W. C. Baker.

7. On the variation of double refraction in strained glass with wave length. By L. N. G. Filon, B.A., King's College.

PERIODICAL PUBLICATIONS RECEIVED BY THE PHILOSOPHICAL SOCIETY.

BRITISH.

Belfast.	Natural History and Philosophical Society.	*Report and Proceedings.*
„	Queen's College.	*Calendar.*
Birmingham.	Philosophical Society.	*Proceedings.*
Bristol.	University College.	*Calendar and Report.*
Cambridge.	Observatory.	*Astronomical Observations.*
„	University Press.	*Reporter*, *Journal of Hygiene*, *Biometrika.*
Dublin.	Royal Dublin Society.	*Transactions*, *Proceedings*, *and Economic Proceedings.*
„	Royal Irish Academy.	*Transactions and Proceedings.*
„	Trinity College Observatory.	*Astronomical Observations and Researches.*
Edinburgh.	Edinburgh Medical Journal.	*Journal* (Dr Griffiths).
„	Mathematical Society.	*Proceedings.*
„	Royal Society.	*Transactions and Proceedings.*
„	Royal Physical Society.	*Proceedings.*
„	Royal Observatory.	*Astronomical Observations.*
„	Royal College of Physicians. (Research Laboratory.)	*Reports.*
„	Scottish Microscopical Society.	*Proceedings.*
Essex.	Field Club.	*Essex Naturalist.*
Glasgow.	Philosophical Society.	*Proceedings.*
„	Geological Society.	*Transactions.*
„	University.	*Calendar.*
Greenwich.	Royal Observatory.	*Astronomical Observations, &c.*
Leeds.	Yorkshire College.	*Report.*
„	Yorkshire Geological and Polytechnic Society.	*Proceedings.*

Liverpool.	Thompson Yates Laboratories.	*Report* (Professor Forsyth).
London.	Athenæum.	*Athenæum* (Professor (Newton).
"	British Association.	*Report.*
"	British Medical Journal.	*Journal* (Dr Griffiths).
"	Chemical News.	*Chemical News* (Professor Liveing).
"	Chemical Society.	*Journal and Proceedings.*
"	*Editor* Electrician.	*Electrician.*
"	Entomological Society.	*Transactions* (Mr W. Bateson).
"	Geological Society.	*Journal.*
"	Geologists' Association.	*Proceedings.*
"	Institution of Civil Engineers.	*Minutes of Proceedings.*
"	Institution of Electrical Engineers.	*Journal.*
"	Institution of Mechanical Engineers.	*Proceedings* (Mr Horace Darwin).
"	Lancet.	*Lancet* (Dr Humphry).
"	Linnean Society.	*Transactions, Proceedings, and Journal.*
"	Macmillan & Co.	*Nature.*
"	Mathematical Society.	*Proceedings.*
"	Meteorological Office.	*Weekly Weather Report.*
"	Patent Office.	*Journal.*
"	Physical Society.	*Proceedings.*
"	Royal Astronomical Society.	*Memoirs and Monthly Notices.*
"	Royal Institution.	*Proceedings.*
"	Royal Microscopical Society.	*Journal.*
"	Royal Society.	*Transactions, Proceedings, and Year-Book.*
"	*Editor* Science Abstracts.	*Science Abstracts.*
"	Society of Arts.	*Journal* (Professor Liveing).
"	University College.	*Calendar.*
"	Zoological Society.	*Proceedings and Transactions.*

London.	Zoologist.	*Zoologist* (Mr A. H. Evans).
Manchester.	Biological Laboratories of the Owens College.	*Studies.*
„	Literary and Philosophical Society.	*Memoirs and Proceedings.*
Newcastle-upon-Tyne.	Natural History Society.	*Transactions.*
„	University of Durham Philosophical Society.	*Proceedings.*
Oxford.	Radcliffe Library.	*Catalogue.*
„	Radcliffe Observatory.	*Astronomical Observations.*
Sheffield.	University College.	*Calendar.*

FOREIGN.

Aci Reale.	Accademia di Scienze, Lettere ed Arti.	*Atti e Rendiconti.*
Adelaide.	Royal Society of South Australia.	*Transactions and Memoirs.*
Amsterdam.	Akademie van Wetenschappen.	*Verslagen* (*Letterkunde* and *Natuurkunde*).
„	„ „	*Verhandelingen* (*Letterkunde* and *Natuurkunde*).
„	„ „	*Jaarboek, Proceedings of the Section of Sciences.*
„	Société Mathématique.	*Revue Semestrielle des Publications Mathématiques. Nieuw Archief voor Wiskunde. Wiskundige Opgaven.*
Austin.	Texas Academy of Science.	*Transactions.*
Baltimore.	Johns Hopkins University.	*American Chemical Journal.*
„	„ „	*American Journal of Mathematics.*
„	„ „	*American Journal of Philology.*
„	„ „	*University Circulars.*
„	„ „ (Biological Laboratory.)	*Memoirs.*
„	„ „ (Hospital.)	*Bulletin.*
„	Maryland Geological Survey.	*Annual Report.*

Baltimore.	Maryland Weather Service.	*Annual Report.*
"	Peabody Institute.	*Report.*
Basel.	Naturforschende Gesellschaft.	*Verhandlungen.*
Batavia.	Bataviaasch Genootschap van Kunsten en Wetenschappen.	*Notulen*, *Tijdschrift*, and *Verhandelingen.*
"	Royal Magnetical and Meteorological Observatory.	*Observations, &c.*
Bergen.	Museum.	*Aarbog.*
Berkeley.	University of California.	*University Chronicle, &c.*
Berlin.	Akademie der Wissenschaften.	*Abhandlungen* and *Sitzungsberichte.*
"	Deutsche Chemische Gesellschaft.	*Berichte* (Mr A. Hutchinson).
"	Gesellschaft Naturforschender Freunde.	*Sitzungs-Berichte.*
"	Physikalisch-Technischen Reichsanstalt.	*Wissenschaftliche Abhandlungen.*
Bern.	Schweizerische Naturforschende Gesellschaft.	*Neue Denkschriften.*
Bombay.	Government Observatory.	*Magnetical and Meteorological Observations.*
Bordeaux.	Société des Sciences.	*Mémoires* and *Procès-Verbaux.*
Boston (U. S. A.).	Society of Natural History.	*Memoirs, Proceedings and Occasional Papers.*
" "	American Academy of Arts and Sciences.	*Memoirs and Proceedings.*
Breslau.	Schlesische Gesellschaft.	*Jahresbericht.*
Brisbane.	Queensland Museum.	*Occasional Notes.*
"	Royal Society of Queensland.	*Proceedings.*
Brooklyn.	Museum of the Brooklyn Institute of Arts and Sciences.	*Science Bulletin.*
Brussels.	Académie Royale des Sciences.	*Mémoires Couronnés* (8vo.).
"	" " "	*Mémoires* and *Mémoires Couronnés* (4to.).
"	" " "	*Bulletin* and *Annuaire.*
"	Observatoire Royal.	*Annuaire* and *Annales.*
"	Société Belge de Microscopie.	*Bulletin* and *Annales.*
"	Université de Bruxelles. (*L'Institut Botanique.*)	*Recueil.*
Budapest.	Ungarische Akademie der Wissenschaften.	*Berichte.*
Buitenzorg.	's Lands Plantentuin.	*Mededeelingen, Bulletin, Verslag, &c.*

Cairo.	Egyptian Government School of Medicine.	*Records.*
Calcutta.	Geological Survey of India.	*Palæontologia Indica.*
„	„ „ „	*Memoirs and Report.*
Cambridge (Mass.).	Harvard Museum of Comparative Zoölogy.	*Memoirs, Bulletin, Report.*
„ „	Harvard Observatory.	*Annals, Report.*
„ „	Harvard Observatory; Henry Draper Memorial.	*Report.*
„ „	Harvard University.	*Annals of Mathematics.*
Cape of Good Hope.	Observatory.	*Annals.*
Cape Town.	South African Museum.	*Annals.*
„ „	South African Philosophical Society.	*Transactions.*
Cherbourg.	Société Nationale des Sciences Naturelles.	*Mémoires.*
Christiania.	Norwegische Meteorologische Institut.	*Jahrbuch.*
„	Videnskabs-Selskab.	*Forhandlinger.*
Cincinnati.	Lloyd Library.	*Bulletin.*
„	Society of Natural History.	*Journal.*
Colorado.	University of Colorado.	*Studies.*
Connecticut.	Academy of Sciences.	*Transactions.*
Copenhagen.	Danske Videnskabernes Selskab.	*Oversigt* and *Skrifter.*
Dantzig.	Naturforschende Gesellschaft.	*Schriften.*
Delft.	École Polytechnique.	*Annales.*
Des Moines.	Iowa Geological Survey.	*Annual Report.*
Easton.	American Association.	*Proceedings.*
„	American Chemical Society.	*Journal.*
Erlangen.	Physikalisch-Medicinische Societät.	*Sitzungsberichte.*
Florence.	Biblioteca Nazionale Centrale.	*Bollettino.*
Geneva.	Conservatoire et Jardin Botanique.	*Annuaire.*
„	Société de Physique et d'Histoire Naturelle.	*Mémoires.*
Genoa.	Museo Civico di Storia Naturale.	*Annali.*

Giessen.	Oberhessische Gesellschaft für Natur- und Heilkunde.	*Bericht.*
Gothenburg.	Kungl. Vetenskaps- och Vitterhets-Samhälles.	*Handlingar.*
Göttingen.	Gesellschaft der Wissenschaften.	*Abhandlungen* and *Nachrichten.*
Granville.	Scientific Laboratories of Denison University.	*Bulletin.*
Haarlem.	Musée Teyler.	*Archives* and *Catalogue.*
Halifax.	Nova Scotian Institute of Science.	*Proceedings and Transactions.*
Halle a. Saale.	K. Leopoldino-Carolinische Akademie.	*Nova Acta, Leopoldina* and *Katalog.*
Heidelberg.	Naturhistorisch-Medicinischer Verein.	*Verhandlungen.*
Helder.	Nederlandsche Dierkundige Vereeniging.	*Tijdschrift.*
Helsingfors.	Finska Vetenskaps-Societetens.	*Öfversigt, Bidrag* and *Acta.*
Ithaca, N. Y.	*Editor* Journal of Physical Chemistry.	*Journal of Physical Chemistry.*
" "	*Editor* Physical Review.	*Physical Review.*
Jassy.	Université.	*Annales Scientifiques.*
Jena.	Medizinisch-naturwissenschaftliche Gesellschaft.	*Jenaische Zeitschrift für Naturwissenschaft.*
Karlsruhe.	Technische Hochschule.	*Inaugural-Dissertation, &c.*
Kasan.	Université Impériale.	*Annales.*
Kharkow.	La Section médicale de la Société des Sciences expérimentales.	*Travaux.*
"	Université Impériale.	*Annales.*
Kiew.	Société des Naturalistes.	*Mémoires.*
Königsberg.	Physikalische Œkonomische Gesellschaft.	*Schriften.*
Lawrence.	Kansas University.	*Bulletin.*
Leipzig.	Königlich Sächsische Gesellschaft der Wissenschaften.	*Abhandlungen* and *Berichte.*
"	*Editor* Beiblätter zu den Annalen der Physik.	*Beiblätter.*
"	Deutsche Physikalische Gesellschaft.	*Verhandlungen.*
Liége.	Société Géologique de Belgique.	*Annales.*
Lille.	Université.	*Travaux et Mémoires.*
Lisbon.	Section des Travaux Géologiques du Portugal.	*Communicações* and *Memoirs.*

Lund.	University.	*Acta.*
Lyons.	Université.	*Annales.*
Madison.	Wisconsin Academy of Sciences.	*Transactions.*
"	Wisconsin Geological and Natural History Survey.	*Bulletin.*
Madras.	Government Museum.	*Bulletin.*
Magdeburg.	Der Naturwissenschaft. Verein.	*Jahresberichte u. Abhandlungen.*
Marseilles.	Faculté des Sciences.	*Annales.*
Mass. (U.S.A.).	Tufts College.	*Studies.*
Melbourne.	Royal Society of Victoria.	*Proceedings* and *Transactions.*
Metz.	Académie des Sciences.	*Mémoires.*
Mexico.	Asociacion de Ingenieros y Arquitectos.	*Anales.*
"	Instituto Geológico.	*Boletin.*
"	Sociedad Cientifica "Antonio Alzate."	*Memorias y Revista.*
Milan	Reale Istituto Lombardo di Scienze e Lettere.	*Rendiconti.*
"	" " " "	*Memorie (Classe di Lettere).*
"	" " " "	*Memorie (Matematiche e Naturali).*
Montreal.	McGill University.	*Papers.*
Moscow.	Société Impériale des Naturalistes.	*Bulletin* and *Nouveaux Mémoires*
Munich.	Bayerische Akademie der Wissenschaften.	*Abhandlungen, Sitzungsberichte.*
"	Sternwarte.	*Neue Annalen.*
Nantes.	Société des Sciences Naturelles de l'Ouest de la France.	*Bulletin.*
Naples.	Società di Naturalisti.	*Bollettino.*
"	Società Reale di Napoli.	*Rendiconto* and *Atti dell' Accad. delle Sc. Fis. e Matematiche, Atti della Reale Accademia di Scienze Morali e Politiche.*
New Haven.	Astronomical Observatory of Yale University.	*Transactions.*
" "	*Editor* American Journal of Science.	*American Journal of Science.*
" "	Connecticut Academy.	*Transactions.*

New York.	American Mathematical Society.	*Transactions and Proceedings.*
„ „	American Museum of Natural History.	*Memoirs, Bulletin, and Report.*
„ „	Academy of Sciences.	*Annals and Memoirs.*
Odessa.	Société des Naturalistes de la Nouvelle Russie.	*Mémoires.*
Ottawa.	Geological Survey of Canada.	*Annual Report.*
„	Literary and Scientific Society.	*Transactions.*
„	Royal Society of Canada.	*Proceedings and Transactions.*
Palermo.	Circolo Matematico.	*Rendiconti.*
Paris.	Muséum d'Histoire Naturelle.	*Nouvelles Archives.*
„	École Normale.	*Annales Scientifiques.*
„	École Polytechnique.	*Journal.*
„	Service Hydrographique de la Marine.	*Annales.*
„	Société Mathématique de France.	*Bulletin.*
Philadelphia.	Academy of Natural Sciences.	*Proceedings.*
„	American Philosophical Society.	*Transactions and Proceedings.*
„	University of Pennsylvania.	*Publications.*
„	Wagner Free Institute.	*Transactions.*
Pisa.	*Editor* Il Nuovo Cimento.	*Il Nuovo Cimento.*
Prague.	Česká Akademie Císaře Františka Josefa I.	*Rozpravy* and *Bulletin.*
„	Königliche Böhmische Gesellschaft der Wissenschaften.	*Jahresbericht*, *Abhandlungen*, and *Sitzungsberichte.*
Pulkowa.	Observatoire Central Nicolas.	*Publications.*
Rennes.	Société Scientifique et Médicale de l'Ouest.	*Bulletin.*
Rio de Janeiro.	Museu Nacional.	*Archivos* and *Revista.*
Rochester.	Academy of Science.	*Proceedings.*
Rome.	Reale Accademia dei Lincei.	*Atti (Classe di Scienze fisiche, matematiche e naturali).*
„	Società Italiana delle Scienze.	*Memorie di Matematica e di Fisica.*

Sacramento.	Lick Observatory of the University of California.	*Publications.*
Salem.	Essex Institute.	*Bulletin.*
San Fernando.	Instituto y Observatorio de Marina.	*Anales* and *Almanaque Náutico.*
San Francisco.	California Academy of Sciences.	*Proceedings.*
„	„ „ „	*Bulletin and Occasional Papers.*
Sassari.	Università.	*Studi.*
Sèvres (S. and O.)	Bureau International des Poids et Mesures.	*Travaux et Mémoires, Procès-Verbaux.*
St Louis.	Missouri Botanical Garden.	*Report.*
St Petersburg.	Académie Impériale des Sciences.	*Mémoires* and *Bulletin.*
„	Kaiserliche Akademie der Wissenschaften.	*Repertorium für Meteorologie.*
Stockholm.	*Editor* Acta Mathematica.	*Acta Mathematica.*
„	Kongliga Svenska Vetenskaps-Akademiens.	*Handlingar, Bihang, Öfversigt.*
Strassburg.	Kaiserliche Universität-Sternwarte.	*Annalen.*
Stuttgart.	Mathematisch-Naturwissenschaftliche Verein in Würtemberg.	*Mitteilungen.*
Sydney.	Australasian Association for the Advancement of Science.	*Report.*
„	Linnean Society of New South Wales.	*Proceedings.*
„	Royal Society of New South Wales.	*Journal and Proceedings.*
Tokio.	Imperial Earthquake Investigation Committee.	*Publications.*
„	University. (College of Science.)	*Memoirs and Journal.*
„	„ (Medicinische Facultät.)	*Mittheilungen.*
Toronto.	Canadian Institute.	*Proceedings and Transactions.*
„	University.	*Studies.*
Toulouse.	Faculté des Sciences de l'Université.	*Annales.*
Turin.	Reale Accademia delle Scienze.	*Memorie, Atti,* and *Osservaz. Meteorologiche.*
Upsala.	Geological Institution of the University of Upsala.	*Bulletin.*
„	Royal Society of Sciences.	*Nova Acta.*
Urbana.	Illinois State Laboratory of Natural History.	*Bulletin.*

Valparaiso.	Der deutsche wissenschaftliche Verein zu Santiago.	*Verhandlungen.*
Victoria.	Royal Society.	*Proceedings and Transactions.*
Vienna.	K. K. Geologische Reichsanstalt.	*Jahrbuch* and *Verhandlungen.*
„	K. K. Naturhistorische Hofmuseum.	*Annalen.*
Warsaw.	*Editor* Prace Matematyczno-Fizyczne.	*Prace.*
Washington.	Philosophical Society.	*Bulletin.*
„	Smithsonian Institution.	*Miscellaneous Collections, Contributions to Knowledge, Report, &c.*
„	„ „ (Bureau of Ethnology.)	*Report.*
„	„ „ (U.S. National Museum.)	*Report, Bulletin, Proceedings and Special Bulletin.*
„	„ „ (Astrophysical Observatory.)	*Annals.*
„	United States Department of Agriculture.	*Bulletin, Year-Book, and North American Fauna.*
„	United States Naval Observatory.	*Publications and Report.*
„	United States Geological Survey.	*Bulletin, Monographs, Reports, &c.*
„	National Academy of Sciences.	*Memoirs.*
Wellington.	Polynesian Society.	*Journal.*
Wiesbaden.	Nassauischer Verein für Naturkunde.	*Jahrbücher.*
Wisconsin.	*Editor* Astrophysical Journal.	*Astrophysical Journal.*
Würzburg.	Physikalische Medicinische Gesellschaft.	*Sitzungsberichte.*
Zagrab.	Hrvatsko Naravoslovno Društvo.	*Glasnik.*
Zürich.	Naturforschende Gesellschaft.	*Vierteljahrsschrift* and *Neujahrsblatt.*
„	Société Helvétique des Sciences Naturelles.	*Nouveaux Mémoires.*

PERIODICALS PURCHASED BY THE PHILOSOPHICAL SOCIETY.

Archiv für Mikroskopische Anatomie. (Schultze.) Hrsg. von O. Hertwig, &c.
Archiv für Naturgeschichte.
Archiv für Physiologie. Hrsg. von E. F. W. Pflüger.
Fortschritte der Physik.
Verhandlungen der k.k. Zoologisch-Botanischen Gesellschaft in Wien.
Zeitschrift für Wissenschaftliche Zoologie.
Zoologischer Garten.

PERIODICALS PURCHASED BY THE UNIVERSITY FOR THE PHILOSOPHICAL LIBRARY.

Annalen der Physik. (Wiedemann.) Hrsg. von Paul Drude.
Annales de Chimie et de Physique.
Annales des Sciences Naturelles; Botanique et Zoologie.
Annals and Magazine of Natural History.
Archiv für Anatomie und Physiologie. (Müller.) Hrsg. von W. His und Th. W. Engelmann.
Comptes Rendus.
Journal of Anatomy and Physiology.
Mémoires de la Société Géologique de France.
Messenger of Mathematics.
Observatory, The.
Palæontographical Society's Publications.
Philosophical Magazine.
Physikalische Zeitschrift.
Quarterly Journal of Mathematics.
Quarterly Journal of Microscopical Science.
Ray Society's Publications.

INDEX TO THE PROCEEDINGS,

with references to the Transactions.

CAMBRIDGE: PRINTED BY J. AND C. F. CLAY, AT THE UNIVERSITY PRESS.

PROCEEDINGS

OF THE

CAMBRIDGE PHILOSOPHICAL SOCIETY.

VOL. XI. PART I.

[MICHAELMAS TERM 1900.]

Cambridge:
AT THE UNIVERSITY PRESS,
AND SOLD BY
DEIGHTON, BELL & CO. AND MACMILLAN & BOWES, CAMBRIDGE.
C. J. CLAY AND SONS, AVE MARIA LANE, LONDON.
1901

Price Two Shillings and Sixpence.

30 *January* 1901.

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2. *Transactions.*

Volume XIX., Part I., was published on 19th June 1900. Fellows and Honorary Members of the Society, not having already applied, who desire that the various Parts of Volume XIX. should be sent to them as they appear are required to make application at the University Press, Cambridge. A separate application is, by order of the Council, necessary for each Volume, but not for each Part of a Volume.

3. Letters and Communications for the Society should be addressed to one of the Secretaries,

Mr H. F. Baker, St John's College,

Mr A. E. Shipley, Christ's College,

Mr S. Skinner, Christ's College.

Presents for the Library of the Society should be addressed to

The Philosophical Library,

New Museums,

Cambridge.

4. Members of the Society are requested to inform the Secretaries of any change of address. The Secretaries will be obliged also for any corrections of the list of Members which is sent out with this part.

CONTENTS.

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PROCEEDINGS

OF THE

CAMBRIDGE PHILOSOPHICAL

SOCIETY.

VOL. XI. PART VII.

[CONSISTING OF THE PROCEEDINGS OF THE SOCIETY FOR THE SESSION 1901—1902; PERIODICAL PUBLICATIONS RECEIVED BY THE SOCIETY; AND THE TITLE, CONTENTS AND INDEX TO VOLUME XI.]

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C. J. CLAY AND SONS, AVE MARIA LANE, LONDON.
1902

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1 *Dec.* 1902.

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